J. Anat. (1995) **187**, pp. 593–602, with 5 figures Printed in Great Britain

Fourier analysis of human soft tissue facial shape: sex differences in normal adults

VIRGILIO F. FERRARIO, CHIARELLA SFORZA, JOHANNES H. SCHMITZ, ALESSANDRO MIANI JR AND GIANLUCA TARONI

Laboratorio di Anatomia Funzionale dell'Apparato Stomatognatico, Istituto di Anatomia Umana Normale, Facoltà di Medicina e Chirurgia, Università degli Studi di Milano, Italy

(*Accepted 17 April 1995*)

ABSTRACT

Sexual dimorphism in human facial form involves both size and shape variations of the soft tissue structures. These variations are conventionally appreciated using linear and angular measurements, as well as ratios, taken from photographs or radiographs. Unfortunately this metric approach provides adequate quantitative information about size only, eluding the problems of shape definition. Mathematical methods such as the Fourier series allow a correct quantitative analysis of shape and of its changes. A method for the reconstruction of outlines starting from selected landmarks and for their Fourier analysis has been developed, and applied to analyse sex differences in shape of the soft tissue facial contour in a group of healthy young adults. When standardised for size, no sex differences were found between both cosine and sine coefficients of the Fourier series expansion. This shape similarity was largely overwhelmed by the very evident size differences and it could be measured only using the proper mathematical methods.

Key words: Human facial form; sexual dimorphism; Fourier analysis.

INTRODUCTION

The extraction of numerical information from complex biological structures is of extreme interest in both research and in clinical practice. The process always involves some kind of data reduction: not only does it miss the nonmetric characteristics, but it also has to simplify all the numerous metric details (Lestrel, 1989). These shortcomings are largely overwhelmed by the advantages of quantitative descriptions: numerical approaches allow the measurement of the differences existing between one individual and the norm, or between individuals, races, sex, or during development and growth. Measures of the craniofacial complex in living human beings are usually made directly in vivo, or from radiographs or photographs. Conventional linear and angular measurements are performed starting from sets of standardised hard and soft-tissue landmarks individualised on the cranium and face or on the films or prints, and their modifications in the different age, sex or race groups are analysed (Broadbent et al., 1975; Ricketts, 1981; Lestrel, 1989; Farkas, 1994). Unfortunately, the investigated morphological changes usually affect both the size and shape of craniofacial structures, and this conventional metric approach provides adequate quantitative information about size only, eluding the problems of shape definition and of shapes modifications (Lestrel et al. 1977; Diaz et al. 1989; Lestrel, 1989; Halazonetis et al. 1991; Lowe et al. 1994). Moreover, the relative positions of the craniofacial landmarks are not independent one from the other, and they mutually interact during growth and development (Enlow, 1990). Separate analyses for each point (such as bivariate analysis for the mean location of each landmark) do not seem to be appropriate because shape should be analysed in its entirety.

More sophisticated mathematical methods allow a quantitative analysis of shape and of its changes, whilst providing a more ready account of size. In the analysis of craniofacial forms, both homologous point representations (Cheverud et al. 1983, 1992;

Correspondence to Professor Virgilio F. Ferrario, Istituto di Anatomia Umana Normale, via Mangiagalli 31, I-20133 Milano, Italy.

Bookstein, 1984, 1991; Moss et al. 1985; Richtsmeier, 1988; Lestrel, 1989; Richtsmeier et al. 1990; Lele & Richtsmeier, 1991, 1992; Fine & Lavelle, 1992; Ferrario et al. 1993*a*, *c*, 1994*a*, *b*, 1995*b*), and boundary representations (Lu, 1965; Lestrel et al. 1977; Lestrel, 1989; Johnson et al. 1990, 1992; Ferrario et al. 1991, 1992; Halazonetis et al. 1991; Lowe et al. 1994) have been used. Anthropometric, radiographic or photographic landmarks are directly used in homologous point methods, while in boundary methods they are used to reconstruct a profile mathematically (Lestrel, 1989).

Boundary representations deal with the outlines of objects: a curve-fitting procedure (usually a harmonic analysis such as the Fourier series) is used to compute a mathematical function which will describe the object's profile and which will be used to compare different objects, or the same object through time. Complex forms are thus decomposed into a series of cosine and sine functions of increasing frequency (Johnson et al. 1992). Both classic Fourier series and elliptic Fourier analysis have already been successfully used for quantitative studies of biological forms in several fields: neurology, dentistry, osteology, haematology (Lu, 1965; Lestrel et al. 1977; Johnson et al. 1985, 1990, 1992; Diaz et al. 1989; Casanova et al. 1990; Ferrario et al. 1991, 1992, 1994*c*; Halazonetis et al. 1991; Fine & Lavelle, 1992; Lowe et al. 1994). These methods analyse the global shape characteristics of the object, independently from its size, spatial orientation, or relation to reference planes.

Size could be a confounding factor in the analysis of shape changes, because its modifications are often of greater magnitude than those of the corresponding shape (Lestrel et al. 1977; Bookstein, 1991). The control for size variability can be achieved in several ways. Both local methods, i.e. limited to the analysed sample (for example subtraction of the mean, division by the largest value, or ranging; see the review by Sneath & Sokal, 1973), and general methods, i.e. that can be used within different samples (such as division by a constant) could be applied. In the analysis of 2-dimensional regular forms, the major axis, even if not completely appropriated because a 1-dimensional datum has to control for 2-dimensional variability, could still be a suitable size measure, but with irregular forms the area becomes a proper size measure (Lestrel et al. 1977). Indeed, the size standardisation should be constructed using the analysed measures themselves, and not using external standards of age, body length or mass (Bookstein, 1991).

A method for the reconstruction of outlines starting from selected landmarks, for their Fourier analysis and for their comparison between different groups has been developed. The method has been derived from Lu (1965) and Lestrel et al. (1977) and modified for a computerised application. The method is described and applied to analyse the sex differences in shape of the soft tissue facial contour in a group of healthy young adults.

MATERIALS AND METHODS

Sample and data collection (3-dimensional facial landmarks)

Some of the data used in the present investigation had already been collected during previous studies of 3-dimensional facial morphometry (Ferrario et al. 1994*b*, *d*, 1995*a*, *b*). From a group of 180 white Caucasian dental students who volunteered for the investigation, a smaller subgroup of 32 men and 30 women aged 19–32 y (mean 21) was selected according to rigorous criteria of dentofacial normality (Scheideman et al. 1980): sound dentitions, with bilateral Angle Class I first permanent molar relationship, absence of anterior or lateral cross-bite, no previous craniofacial trauma or surgery, no previous or current orthodontic treatment, and no mandibular or craniocervical disorder.

For each subject, the 3-dimensional coordinates of a set of standardised facial landmarks were collected automatically using the Three-Dimensional Facial Morphometry method, which has been described in detail elsewhere (Ferrario et al. 1994*a*, *b*, *d*, 1995*a*, *b*). In brief, a single operator located and traced the landmarks on the face of each subject with a black eye-pencil (Fig. 1). On the centre of each point a 2 mm hemispheric reflective marker was fixed using a bi-adhesive plaster.

The subjects were seated in an upright position on a stool in front of 2 CCD cameras pointed to obtain a stereometric view of the subjects' faces with their jaws in centric occlusion (i.e. natural dental contact without clenching force). The cameras, interfaced with an automatic image analyser (ELITE, BTS, Milan, Italy), lit up the reflective markers with an infrared stroboscope; the centres of gravity of these points were automatically recognised and recorded by the analyser and the spatial X, Y, Z coordinates of the points were provided. The system was calibrated before each acquisition, thus providing real metric data. In each face, the nasion-subnasale line coincided with the vertical axis.

The coordinates were projected on a frontal plane passing through the eye lateral canthi; thus the X, Y

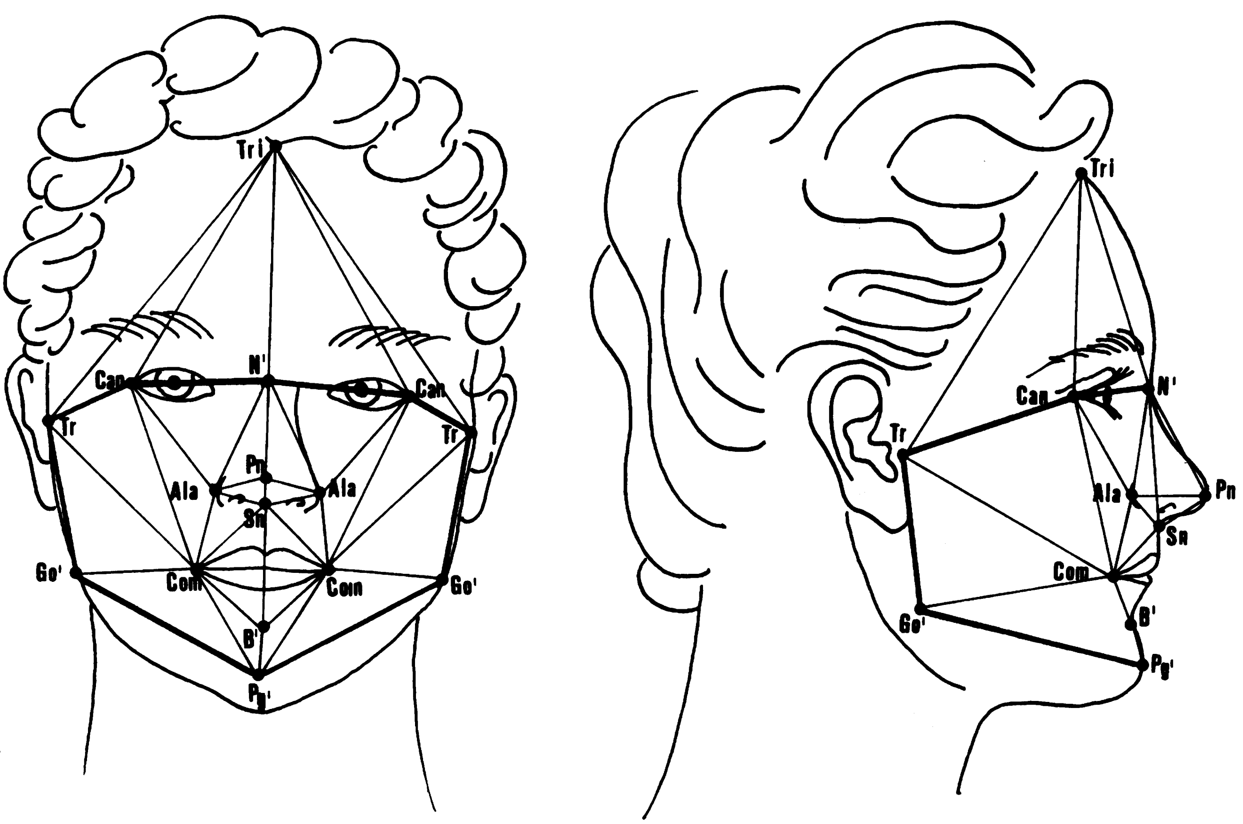

Fig. 1. Digitised 3-dimensional facial landmarks, frontal and lateral views. In both views, the landmarks used in the analysis are connected by heavier segments. The landmarks are defined in Table 1.

Table 1. *Definition of points (landmarks) used in the analysis*

Median points (on the midsagittal plane)	
Trichion	Tri
Soft tissue nasion	N′
Subnasale	Sn
Pronasale	Pn
Soft tissue B′ point	B′
Soft tissue pogonion	Pg′
Lateral points (right and left sides)	
Eye lateral canthus	Can
Tragus	Tr
Nasal ala	Ala
Labial commissure	Com
Soft tissue gonion	Go′

components of nasion, left and right eye lateral canthi, left and right tragi, left and right gonia and pogonion were further analysed (Table 1).

Fourier analysis

The whole of the subsequent analysis was performed automatically by a computer (Z-Station 466 Xn, Zenith Data System Co., St Joseph, MI) programmed by one of the authors (V.F.F.)

Centroid calculation. The contiguous landmarks were connected by segments (Fig. 2), and the centroid (or centre of gravity, CG) of the area of the form thus delimited was calculated from the cartesian coordinates of the selected landmarks. The CG of the form was set as the origin of cartesian axes.

Normalisation. The form was size-normalised by setting its area equal to a constant of 5000 mm^2. Size normalisation was performed mathematically with expansions or contractions of the area that did not modify the shape (continuous line in Fig. 2). Normalisation of the orientation relative to the coordinate axes was not necessary because all coordinates were already referred to the same internal axes (nasion–subnasale line, and eye lateral canthi).

Curve fitting and harmonic analysis. From the CG (origin of axes) a vector was rotated counterclockwise for 360° with a 1° step to intersect the contour of the form, and 360 segments were produced. The length of each vector (modulus, or distance from the CG to 1 of the 360 points of the contour), together with the relevant vector angle relative to the horizontal, were used to generate a curve in a cartesian coordinate system: y, vector modulus, x, angle (from 1° to 360°). The method is similar to that employed by Lestrel et al. (1977) and Halazonetis et al. (1991), but it was

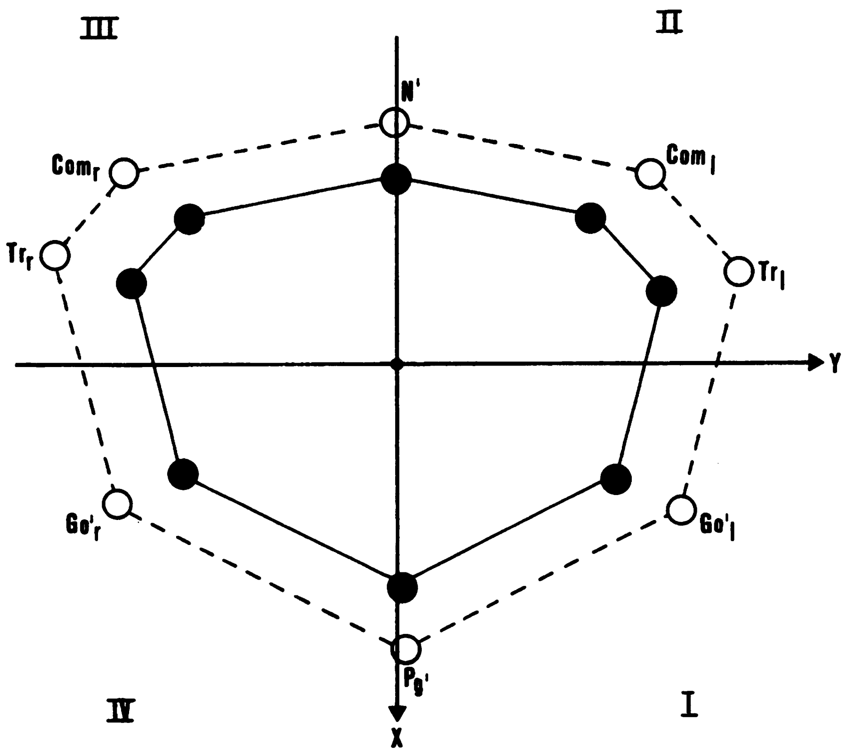

Fig. 2. Facial landmarks of one woman. The landmarks are the endpoints of the analysed form. Open circles and interrupted line: digitised landmarks and form with its original size. Closed circles and continuous line: size standardised form used for shape analysis only. The centre of gravity and the coordinate axes are also indicated.

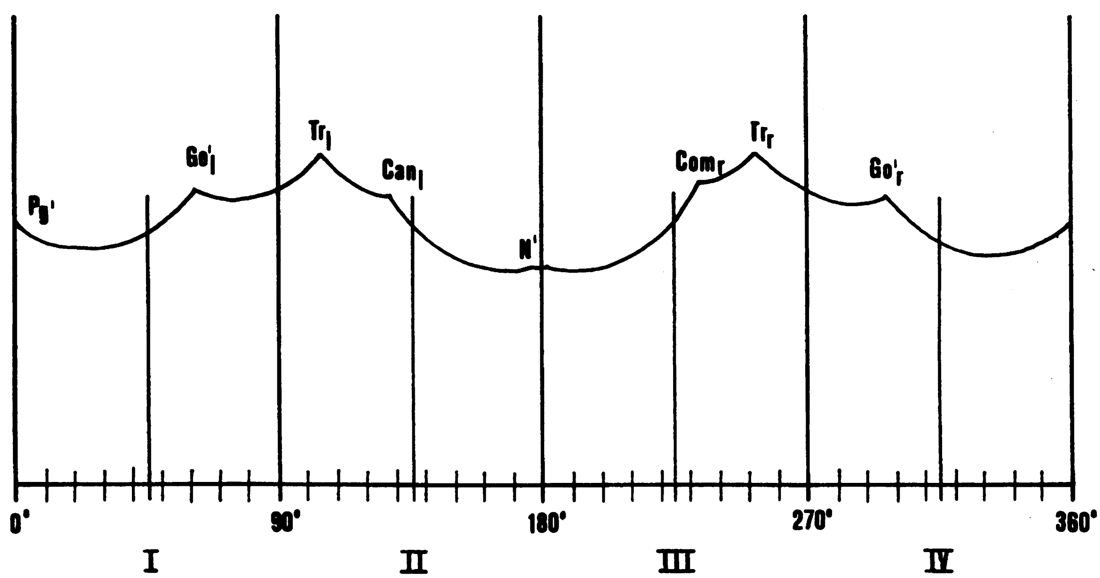

Fig. 3. Vector modulus (Y axis) as a function of the vector angle (X axis) in the same woman of Figure 2.

performed automatically, thus reducing the digitisation errors, and it covered 360°. Figure 3 gives the vector modulus as a function of the vector angle in the same woman shown in Figure 2.

Using the least squares method, a Fourier analysis of the curve was thus performed, with period $\tau = 360$ (Lu, 1965). The mathematical expression of the Fourier series is reported in the Appendix. Series were truncated at the 20th harmonic, because the higher degree coefficients were negligible. The percentage contribution of sine and cosine coefficients to the mathematical description of the outline was also computed.

The Fourier reconstruction of the same female

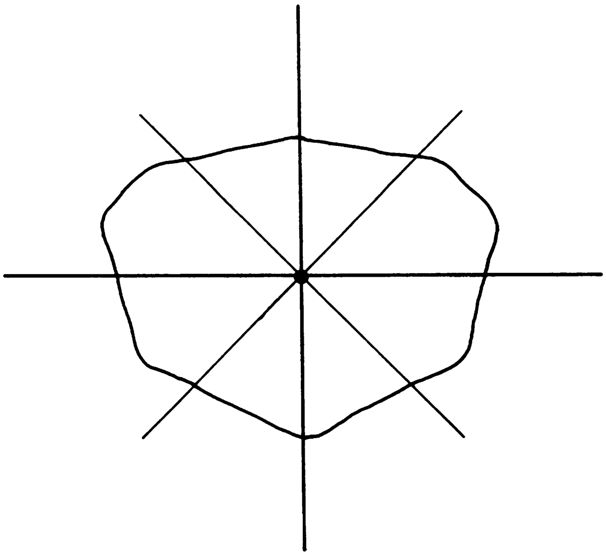

Fig. 4. Size-normalised Fourier reconstruction of the female plot of Figure 2. Series were truncated at the 20th harmonic. The figure also shows the CG (origin of axes), the X and Y axes, as well as the 45° axes. The relevant Fourier coefficients are listed in Table 2.

Table 2. *Fourier coefficients of the female plot shown in Figures 2–4. The series were truncated at the 20th harmonic*

H	*a*	*b*	*c*
0	43.0815		
1	0.4374	−0.0486	0.440
2	−6.9557	0.4400	6.969
3	2.5694	0.0850	2.570
4	0.8083	−0.0980	0.814
5	−0.4235	0.1670	0.455
6	1.7673	0.0528	1.768
7	0.4186	−0.1125	0.433
8	−0.4598	0.1328	0.478
9	−0.1824	0.1187	0.217
10	0.2979	−0.0839	0.309
11	0.3647	0.0227	0.365
12	0.1250	0.0808	0.148
13	−0.0424	−0.0311	0.052
14	0.1409	0.0195	0.142
15	−0.1432	0.0702	0.159
16	−0.0108	0.0005	0.010
17	0.3151	−0.0187	0.315
18	0.0627	0.0190	0.065
19	−0.1562	0.0366	0.160
20	0.0923	0.0155	0.093

H, harmonics; *a*, cosine coefficient; *b*, sine coefficient; *c*, amplitude.

plot is reported in Figure 4. The relevant Fourier coefficients of the first 20 harmonics are listed in Table 2.

Goodness of fit. The goodness of fit for each individual curve, i.e. the agreement between the observed/digitised values and the values estimated according to the Fourier series, was calculated as proposed by Lu (1965) from the variances of the estimated and observed data.

Statistical comparisons. Descriptive statistics (mean and S.D.) for each sine and cosine coefficient were calculated within sex from the coefficients of each facial outline. The significance of the coefficients was evaluated with a factorial analysis of variance (Lu, 1965). Mean values within sex were also computed for the percentage sine and cosine components, as well as for the original area (i.e. before size normalisation) of the facial outline.

The mean values between sexes were compared using Student's t tests for independent samples.

Data collection and method error

The reproducibility of landmark identification and marker positioning has already been tested with the same set of landmark data, and the within-operator error has been found to be about 2 mm for all the 3 point coordinates (X, Y and Z), while the reproducibility of the data collection procedure has been estimated to be about 0.1 mm for all the 3 point coordinates (Ferrario et al. 1994*a*, *b*, 1995*b*). Other negligible errors could derive from the mathematical procedure, in particular from the approximation algorithms.

RESULTS

In all cases the coefficient of agreement (Lu, 1965) was higher than 0.99 when the series were truncated at the 20th harmonic, and higher than 0.91 when the series were truncated at the 6th harmonic. Moreover, this excellent superimposition between the original plot and its mathematical reconstruction was also tested qualitatively with the graphical subroutine. All the further sex comparisons were thus made with the first 6 Fourier coefficients only. In the example shown in Figures 2–4, the original area was 9801.18 mm^2 (interrupted line in Fig. 2), which was set to 5000 mm^2 for the shape analysis (continuous line in Fig. 2), and the coefficient of agreement was 0.998 with a 20-harmonic truncation. The sines explained 0.52% of the contour, while the cosines explained 99.48%.

The mean area delimited by the soft-tissue facial outline was 11607.3 mm^2 (S.D. 1031.24) in men, and 10166.59 mm^2 (S.D. 838.52) in women. The outline area was significantly higher in men than in women, with a difference of about 14% (Student's $t = 5.915$, 60 degrees of freedom, $P < 0.001$).

The Fourier coefficients were then size-standardised, and shape alone was analysed. Statistics and within-group significance of the cosine (a) and sine (b) coefficients of the first 6 harmonics are given in Table

Table 3. *Fourier analysis of male and female soft tissue facial outlines*

H	*a*	S.D.	*b*	S.D.	*c*
Men					
0	42.99829	0.2676955			
1	0.32644*	0.1668424	0.01312	0.0616908	0.327
2	−5.20136*	1.5167517	−0.04683	0.6408892	5.202
3	2.40593*	1.0721506	0.16203	0.6079539	2.411
4	0.08834	0.7584686	0.09525*	0.2580757	0.130
5	−0.73558*	0.4430984	0.04464	0.2325251	0.737
6	1.70664*	0.3368717	0.00190	0.3395094	1.707
Women					
0	42.98980	0.3342121			
1	0.27127*	0.1071680	0.03286*	0.0683275	0.273
2	−5.23169*	1.5632102	−0.12027	0.7900853	5.233
3	2.06394*	0.6136120	0.25525*	0.4953307	2.080
4	0.36653*	0.6881885	0.00092	0.1964968	0.367
5	−0.68790*	0.4361983	−0.05316*	0.2027733	0.690
6	1.64244*	0.2214494	0.15725*	0.3160170	1.650

Mean and S.D. of the cosine and sine coefficients of the first 6 harmonics, S.D. and relevant harmonic amplitudes. The within-group significant coefficients ($P \leqslant 0.05$) are noted (*).
H, harmonics; *a*, cosine coefficient; *b*, sine coefficient; *c*, amplitude.

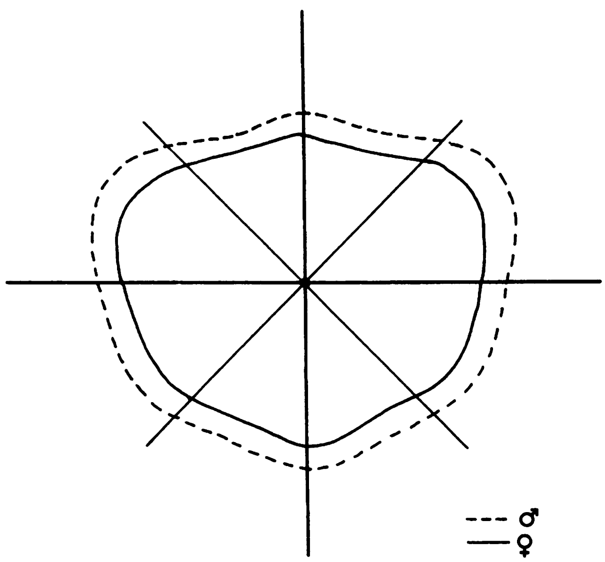

Fig. 5. Fourier reconstruction of the mean male (interrupted line) and female (continuous line) soft tissue facial outline. Series were truncated at the 6th harmonic. The size of the profiles is proportional to the original male:female area ratio in order to show the size difference.

3. In men, only half of the coefficients were statistically significant ($P \leqslant 0.05$): all the cosine coefficients (except that of the 4th harmonic), and the sine coefficient of the 4th harmonic. In women, a larger number of coefficients was significant: all cosine coefficients, and the sine coefficients of the 1st, 3rd, and 6th harmonics. No sex differences were found between Fourier coefficients (Student's *t* test for independent samples). Moreover, the relative contribution of the sine and cosine components to the description of the facial outline was similar in both sexes: in men, on average, cosines explained 96.85% of the shape of the contour, sines 3.15%, with an S.D. of 3.3669; in women, cosines explained 95.87%, sines 4.13%, S.D. 4.2106. The S.D. was the same for both components (sine and cosine), being their sum equal to 100 in all subjects.

The 2 relevant 6-harmonic plots are shown in Figure 5: in the plots, the 2 outlines have been drawn with the original male:female area ratio in order to appreciate the size difference. Indeed, the 2 outlines are practically identical, and a size-standardised plot would have shown only one outline.

DISCUSSION

In this selected sample of healthy young human adults, the size of the soft tissue facial contour was significantly higher in men than in women, but its shape showed no sexual dimorphism: Fourier analysis allowed a correction for the size discrepancy, and separated the size and shape contributions to global morphology. Indeed, morphological analyses often deal with individuals with great size differences, but size plays a distorting role in the estimation of morphology (Lestrel et al. 1977). The conventional metric approach does not allow the separation of the size and shape contributions to the overall morphology (Lestrel et al. 1977; Johnson et al. 1985; Lestrel, 1989; Lowe et al. 1994). Size can mask the more subtle shape differences, and ratios or ranging of data (Sneath & Sokal, 1973) are thus used to control for size differences. Unfortunately, ratios have been demonstrated to be inadequate, also making the interpretation of results difficult (Albrecht et al. 1993).

In addition, the indices such as width-to-length commonly used in morphological analyses for the quantification of form differences are arbitrary constructs because they are made from the combination of isolated metric measurements (Lestrel et al. 1977). Conversely, Fourier analysis represents the global morphology of the contour of a form.

Whenever closed forms are considered, the classic Fourier series could be replaced by the elliptic Fourier series (Kuhl & Giardina, 1982; Diaz et al. 1989; Lestrel, 1989; Ferrario et al. 1991, 1994*c*; Lowe et al. 1994). Unfortunately, in this case the outline of the form has to be codified using a rather elaborate and complex algorithm. The method proposed in this investigation seems to be simpler and easier to understand, and it can be applied both to open and closed forms. Moreover, whenever groups of forms have to be compared, the analysis of variance for the comparison of Fourier coefficients (which satisfy the property of orthogonality, and can thus be further analysed with conventional statistics) has already been exemplified in detail by Lu (1965) and Lestrel et al. (1977) for the classic Fourier series only.

In this investigation, the female group scored a larger number of coefficients with a within-group significance (Table 3) than the male group (6 coefficients of 12 in men, 9 of 12 in women). It thus seemed that the men had a larger variability in facial shape than the women. The variability in size was similar regardless of sex: the coefficients of variation (i.e. the percentage ratio of standard deviation to mean) of the areas delimited by the analysed soft tissue profile were 8.25% in men, and 8.88% in women.

A further advantage of the classic series is the geometric meaning of the coefficients of the harmonics: the numerical differences in the Fourier coefficients can be related to actual differences in the observed morphology (Lu, 1965; Lestrel et al. 1977). In each harmonic, the sine term measures asymmetry, while the cosine term measures symmetry with respect to the X-axis (Lu, 1965; Lestrel et al. 1977; Johnson et al. 1985, 1990, 1992). The cosine coefficient of the O-harmonic measures the size of the form and therefore should be constant or similar in all cases when all the outlines are standardised for size. The 1st harmonic is a circle shifted towards the region of major area; the 2nd harmonic is a lemniscate; the 3rd harmonic is a 3-leaved figure, and each further harmonic adds one more leaf to the figure, thus a high frequency harmonic resembles a circle (Lu, 1965; Lestrel et al. 1977).

Indeed, the amplitude of the 1st harmonic was lower than 1 in all cases. The small amplitude of this harmonic is explained by its geometric meaning (a circle shifted towards the region of major area): the analysis used the centroid of the facial outline as the origin of axes, and the area was equally distributed around the centroid itself. Nevertheless, this 1st harmonic cannot be neglected because all the low-frequency coefficients are needed to reconstruct the original outline (Johnson et al. 1985, 1990, 1992).

Two limitations to the present method can be recognised. First, the method is suitable for every complex open or closed form provided that the intersection between the contour of the form and the vector is unique, i.e. the profile of the form should not be excessively concave relative to its centroid. Secondly, according to Lestrel (1989) the method is not really coordinate-free, because a centroid is needed and a starting point should be defined (Lestrel et al. 1977; Johnson et al. 1985). In this method, the orientation and starting point were necessarily identical for all the forms, and defined starting from the intrinsic facial characteristics of each subject (nasion–subnasale line and lateral eye canthus plane) and the geometric characteristics of each form independently from size (the centroid, as used by Lestrel et al. 1977 and Johnson et al. 1985). In this way, a kind of internal orientation was used, according to the homology between the collected landmarks across the subjects (Lu, 1965; Lestrel et al. 1977; Johnson et al. 1985).

The number of landmarks that can be used in this method is unlimited. The more their number grows, the more accurate the outline of the form becomes, but the subsequent calculations are independent from the landmark number. The only limitations are biological, such as significance of the landmarks and uniqueness of definition (Bookstein, 1984, 1991; Ferrario et al. 1993*a*). Indeed, the number of landmarks selected in this investigation seemed to be adequate. In all cases, the coefficient of agreement was higher than 0.99 with a 20-harmonics truncation, and higher than 0.91 with a 6-harmonics truncation. According to Lu (1965), this measure of agreement should be 0.6 at least, thus indicating a formal correlation of about 0.9 between the observed and estimated data. L should be 1 for a perfect fit (coincidence between the observed and estimated data).

Fourier analysis can be a true shape analysis, because normalised Fourier coefficients are position, size and orientation-invariant (Shen et al. 1994). Size was standardised by dividing all Fourier coefficients by an arbitrary value of 5000 mm^2, corresponding to the area of the enclosed profile, which allowed a convenient graphical representation. Position relative to the coordinate axes influences the coefficients of the first harmonic only: for each case, the origin of axes was set in the centroid of the contour. Orientation and starting point influence the phase Φ of the harmonics (see Appendix), but the amplitude c is unchanged (Lestrel et al. 1977; Johnson et al. 1985; Shen et al. 1994). Orientation and starting point for Fourier coefficients computation were standardised by orienting all the facial coordinates according to internal axes, as already discussed. The X axis was set vertical, and thus coincided with the midsagittal plane of each face (Lu, 1965).

The presence of homologous landmarks thus allowed the standardisation of Fourier coefficients in a way comparable in all subjects. Indeed, in the present study landmark data were used to reconstruct an outline, and the shape of this outline was then analysed in its entirety. This procedure avoided the dispersion of information across the landmarks, whose reciprocal positions are not ontogenetically independent. Another landmark-based shape analysis has recently been proposed by Bookstein (1991), but its relatively complex mathematical base did not seem to be suitable for the present investigation. Moreover, according to the same author, this method is not completely appropriate for curving forms, where the position of landmarks does not express all the shape information, and additional information (e.g. local curvature) is necessary (Bookstein, 1991). In this case, Fourier series seem the best choice.

The sex differences in the facial size and shape of healthy young adults have already been investigated by several 2 and 3-dimensional studies (Scheideman et al. 1980; Enlow, 1990; Genecov et al. 1990; Ferrario et al. 1992, 1993*b*, *c*, 1994*a*, 1995*b*; Farkas, 1994), but no studies have analysed the soft-tissue facial contour in the frontal plane.

The size difference between the soft-tissue contour of adult male and female faces obtained in this investigation was somewhat smaller than that obtained in previous studies performed using both separate 2-dimensional frontal and lateral projections of facial coordinates collected using conventional photographic or radiographic equipment (Scheideman et al. 1980; Enlow, 1990; Genecov et al. 1990; Ferrario et al. 1993*b*, *c*), and real 3-dimensional data (Ferrario et al. 1994*a*, 1995*b*). In the present study the area of the male face was 14% larger than the relevant female area, corresponding to a 3.7% difference in the linear distances, while in the other investigations differences were around 6–7%. It has to be mentioned that in the present study only the frontal projections (on the eye lateral canthi plane) of facial landmarks were used.

The present 3-dimensional study did not find a significant sex difference in the shape of the soft tissue facial contour, and it contrasted with the highly significant sexual dimorphism found in our precedent global 2-dimensional investigation (Ferrario et al. 1993*c*), and in our conventional metric analysis (Ferrario et al. 1993*b*). The data collected in the 2- and 3-dimensional studies are not directly comparable, even though similar landmarks and the same selection criteria were employed. The 2-dimensional studies were performed in the frontal plane only, and projected the landmark coordinates on a conventional photographic plane which was *external* to the subject, and which did not take the postural modifications of the subject into account. Conversely, the present study analysed the projection of real spatial points on an *intrinsic* frontal plane passing through the subject's eye lateral canthi. Moreover, the sexual dimorphism (shape plus size) of the facial contour found in the conventional metric investigation (Ferrario et al. 1993*b*) was not tested by a formal global statistical analysis; indeed, only partial analyses were made, and significant differences found between 2-dimensional chin and cheek angles.

When separate lateral, posteroanterior and submental-vertex radiographic or photographic 2-dimensional projections are used, two kinds of errors can result: errors of projection and errors of landmark identification. Different head orientations (especially in anteroposterior views) will result in different linear and angular dimensions (El Mangoury et al. 1987; Schmid et al. 1991; Pirttiniemi, 1992; Tng et al. 1993). Moreover, some landmarks may be hidden by other structures, or their definition may depend on head position. Many of these problems are avoided in the 3-dimensional technique adopted in this investigation which allowed the direct identification of the landmarks on the subject's face, and the calculation of undistorted 3-dimensional coordinates.

When 3-dimensional coordinates were used, no significant sexual dimorphism in shape was found in both formal shape analyses (Ferrario et al. 1994*a*, 1995*a*) and conventional metric analyses (Ferrario et al. 1995*b*). In this last investigation, all the 3-dimensional linear distances were significantly larger in men than in women, but no differences were found for the angular data, which are more sensitive to the shape than to the size differences. Unfortunately, there are no other formal analyses of human facial contour, but a simple size difference in facial dimensions, with the men sized larger than the women, without conspicuous shape differences, has already been hypothesised by some investigators (Peck et al. 1992; Ferrario et al. 1995*a*). A last consideration pertains to facial symmetry. In both sexes, most of the mean facial outline was explained by the cosine component of the Fourier series, which measures the symmetry with respect to the X axis, and, in our coordinate system, the symmetry with respect to the midsagittal plane as determined by the nasion–subnasale line. The results therefore indicate a high degree of symmetry in the faces, in apparent contrast with our previous investigations (Ferrario et al. 1994*b*, 1995*a*), where a significant asymmetry in the reciprocal arrangement of facial landmarks was found regardless of sex. Indeed, the sine component of this investigation measured a mean characteristic of the soft tissue facial outlines: the individual asymmetries previously found in the reciprocal arrangement of facial landmarks cancelled each other, and while mean symmetric faces resulted (small mean sine components), a relatively large variability was found in both sexes (S.D.s larger than the relevant means). The relative contribution of sine and cosine coefficients to an outline could thus be used to measure the degree of symmetry of the outline itself (Lestrel et al. 1977; Lu, 1977; Johnson et al. 1985).

The described method of shape analysis could be further applied in the field of craniofacial growth and development to analyse the age and sex variations in patients with a specific skeletal classification, or to compare different skeletal or aesthetic types (Ferrario et al. 1991, 1992; Lowe et al. 1994). In particular, the reciprocal contributions of the sine and cosine components to the total outline could be used to quantify the degree of symmetry of a particular shape.

ACKNOWLEDGEMENTS

The artwork by Mrs Paola Raselli is gratefully acknowledged.

REFERENCES

Albrecht GH, Gelvin BR, Hartman SE (1993) Ratios as a size adjustment in morphometrics. *American Journal of Physical Anthropology* **91**, 441–468.

Bookstein FL (1984) A statistical method for biological shape comparisons. *Journal of Theoretical Biology* **170**, 475–520.

Bookstein FL (1991) *Morphometric Tools for Landmark Data. Geometry and Biology*. New York: Cambridge University Press.

Broadbent BH Sr, Broadbent BH Jr, Golden WH (1975) *Bolton Standards of Dentofacial Developmental Growth*. St Louis: CV Mosby.

Casanova MF, Sanders RD, Goldbert TE, Bigelow LB, Christison G, Torrey EF et al. (1990) Morphometry of the corpus callosum in monozygotic twins discordant for schizophrenia: a magnetic resonance imaging study. *Journal of Neurology, Neurosurgery and Psychiatry* **53**, 416–421.

Cheverud J, Lewis JL, Bachrach W, Lew WD (1983) The measurement of form and variation in form: an application of three-dimensional quantitative morphology by finite-element methods. *American Journal of Physical Anthropology* **62**, 151–165.

Cheverud JM, Kohn LAP, Konigsberg LW, Leigh SR (1992) Effects of fronto-occipital artificial cranial vault modification on the cranial base and face. *American Journal of Physical Anthropology* **88**, 323–345.

Diaz G, Zuccarelli A, Pelligra I, Ghiani A (1989) Elliptic Fourier analysis of cell and nuclear shapes. *Computers and Biomedical Research* **22**, 405–414.

El Mangoury NH, Shaheen S, Mostafa YA (1987) Landmark identification in computerized posteroanterior cephalometrics. *American Journal of Orthodontics and Dentofacial Orthopedics* **91**, 57–61.

Enlow DH (1990) *Facial Growth*, 3rd edn. Philadelphia: W. B. Saunders.

Farkas LG (1994) *Anthropometry of the Head and Face*, 2nd edn. New York: Raven Press.

Ferrario VF, Sforza C, Miani A Jr, d'Addona A, Todisco M (1991) Cephalometrics and facial shape: new thresholds by an overall approach to classic standards. *International Journal of Adult Orthodontics and Orthognathic Surgery* **6**, 261–269.

Ferrario VF, Sforza C, Miani A Jr, Poggio CE, Schmitz J (1992) Harmonic analysis and clustering of facial profiles. *International Journal of Adult Orthodontics and Orthognathic Surgery* **7**, 171–179.

Ferrario VF, Sforza C, d'Addona A, Miani A Jr, Poggio CE (1993*a*) ANB skeletal types correlated to facial morphology: Euclidear distance matrix analysis. *International Journal of Adult Orthodontics and Orthognathic Surgery* **8**, 181–190.

Ferrario VF, Sforza C, Miani A Jr, Tartaglia G (1993*b*) Craniofacial morphometry by photographic evaluations. *American Journal of Orthodontics and Dentofacial Orthopedics* **103**, 327–337.

Ferrario VF, Sforza C, Pizzini G, Vogel G, Miani A (1993*c*) Sexual dimorphism in the human face assessed by euclidean distance matrix analysis. *Journal of Anatomy* **183**, 593–600.

Ferrario VF, Sforza C, Poggio CE, Serrao G, Miani A Jr (1994*a*) A three-dimensional study of sexual dimorphism in human face. *International Journal of Adult Orthodontics and Orthognathic Surgery* **9**, 303–310.

Ferrario VF, Sforza C, Poggio CE, Tartaglia G (1994*b*) Distance from symmetry: a three-dimensional evaluation of facial asymmetry. *Journal of Oral and Maxillofacial Surgery* **52**, 1126–1132.

Ferrario VF, Sforza C, Serrao G, Frattini T, del Favero C (1994*c*) Shape of the human corpus callosum: elliptic Fourier analysis on midsagittal magnetic resonance scans. *Investigative Radiology* **29**, 677–681.

Ferrario VF, Sforza C, Serrao G, Puletto S, Bignotto M, Tartaglia G (1994*d*) Soft tissue facial morphometry in occlusal class I vs occlusal class II. *International Journal of Adult Orthodontics and Orthognathic Surgery* **9**, 187–194.

Ferrario VF, Sforza C, Miani A Jr, Serrao G (1995*a*) A three-dimensional evaluation of human facial asymmetry. *Journal of Anatomy* **186**, 103–110.

Ferrario VF, Sforza C, Poggio CE, Serrao G (1995*b*) Facial three-dimensional morphometry. *American Journal of Orthodontics and Dentofacial Orthopedics*, in press.

Fine MB, Lavelle CLB (1992) Diagnosis of skeletal form on the lateral cephalogram with a finite element-based expert system. *American Journal of Orthodontics and Dentofacial Orthopedics* **101**, 318–329.

GENECOV JS, SINCLAIR PM, DECHOW PC (1990) Development of the nose and soft tissue profile. *The Angle Orthodontist* **60**, 191–198.

HALAZONETIS DJ, SHAPIRO E, GHEEWALLA RK, CLARK RE (1991) Quantitative description of the shape of the mandible. *American Journal of Orthodontics and Dentofacial Orthopedics* **99**, 49–56.

JOHNSON DR, O'HIGGINS P, MCANDREW TJ, ADAMS LM (1985) Measurement of biological shape: a general method applied to mouse vertebrae. *Journal of Embryology and Experimental Morphology* **90**, 363–377.

JOHNSON DR, O'HIGGINS P, MCANDREW TJ (1990) The effect of diet on bone shape in the mouse. *Journal of Anatomy* **172**, 213–220.

JOHNSON DR, O'HIGGINS P, MCANDREW TJ, KIDA MY (1992) The inheritance of vertebral shape in the mouse. I. A study using Fourier analysis to examine patterns of inheritance in the morphology of cervical and upper thoracic vertebrae. *Journal of Anatomy* **180**, 507–514.

KUHL FP, GIARDINA CR (1982) Elliptic Fourier features of a closed contour. *Computer Graphics and Image Processing* **18**, 236–258.

LELE S, RICHTSMEIER JT (1991) Euclidean distance matrix analysis: a coordinate-free approach for comparing biological shapes using landmark data. *American Journal of Physical Anthropology* **86**, 415–427.

LELE S, RICHTSMEIER JT (1992) On comparing biological shapes: detection of influential landmarks. *American Journal of Physical Anthropology* **87**, 49–95.

LESTREL PE (1989) Some approaches toward the mathematical modeling of the craniofacial complex. *Journal of Craniofacial Genetics and Developmental Biology* **9**, 77–91.

LESTREL PE, KIMBLE WH, PRIOR FW, FLEISCHMANN ML (1977) Size and shape of the hominoid distal femur: Fourier analysis. *American Journal of Physical Anthropology* **46**, 281–290.

LOWE BF JR, PHILLIPS C, LESTREL PE, FIELDS HW JR (1994) Skeletal jaw relationships: a quantitative assessment using elliptical Fourier functions. *The Angle Orthodontist* **64**, 299–310.

LU KH (1965) Harmonic analysis of the human face. *Biometrics* **21**, 491–505.

MOSS ML, SKALAK R, PATEL H, SEN K, MOSS-SALENTIJN L, SHINOZUKA M et al. (1985) Finite element method modeling of craniofacial growth. *American Journal of Orthodontics* **87**, 453–472.

PECK S, PECK L, KATAIA M (1992) Some vertical lineaments of lip position. *American Journal of Orthodontics and Dentofacial Orthopedics* **101**, 519–524.

PIRTTINIEMI P (1992) *Associations of Mandibulofacial Asymmetries, with Special Reference to Glenoid Fossa Remodelling*. Oulu, Finland: University of Oulu.

RICHTSMEIER JT (1988) Craniofacial growth in Apert syndrome as measured by finite-element scaling analysis. *Acta Anatomica* **133**, 50–56.

RICHTSMEIER JT, CHEVERUD JM, MORRIS R (1990) Comparison of three-dimensional form. *Journal of Anatomy* **170**, 221–223.

RICKETTS RM (1981) Perspectives in the clinical application of cephalometrics. *The Angle Orthodontist* **51**, 115–150.

SCHEIDEMAN GB, BELL WH, LEGAN HL, FINN RA, REISCH JS (1980) Cephalometric analysis of dentofacial normals. *American Journal of Orthodontics and Dentofacial Orthopedics* **78**, 404–420.

SCHMID W, MONGINI F, FELISIO A (1991) A computer-based assessment of structural and displacement asymmetries of the mandible. *American Journal of Orthodontics and Dentofacial Orthopedics* **100**, 19–34.

SHEN L, RANGAYYOU RM, DESAUTEL JEL (1994) Applications of shape analysis to mammographic calcifications. *IEEE Transactions on Medical Imaging* **13**, 263–274.

SNEATH PHA, SOKAL RR (1973) *Numerical Taxonomy*. San Francisco: W. H. Freeman.

TNG TTH, CHAN TCK, COOKE MS, HAGG U (1993) Effect of head posture on cephalometric sagittal angular measures. *American Journal of Orthodontics and Dentofacial Orthopedics* **104**, 337–341.

APPENDIX

The Fourier series expansion is defined as:

$$y = \frac{a_0}{2} + \sum_{m=1}^{20} [a_m \cos(\theta m x) + b_m \sin(\theta m x)] \qquad (1)$$

where m = harmonic, a_m = cosine coefficient of m harmonic, b_m = sine coefficient of m harmonic, $\theta = 2\pi/\tau$, with τ = normalised profile length.

Equation (1) can also be written as

$$y = \frac{a_0}{2} + \sum_{m=1}^{20} [c_m \cos(\theta m x + \Phi_m)] \qquad (2)$$

where $c_m = \sqrt{(a_m^2 + b_m^2)}$ amplitude of the m harmonic, $\Phi_m = \arctan(b_m/a_m)$ phase of the m harmonic.