AN APPLICATION OF THE MATCHING LAW TO EVALUATE THE ALLOCATION OF TWO- AND THREE-POINT SHOTS BY COLLEGE BASKETBALL PLAYERS

TIMOTHY R. VOLLMER AND JASON BOURRET

UNIVERSITY OF FLORIDA

We applied the matching equation to evaluate the allocation of two- and three-point shots by male and female college basketball players from a large Division 1 university. The matching law predicts that the proportion of shots taken from three-point range should match the proportional reinforcement rate produced by such shots. Thus, we compared the proportion of three-point shots taken relative to all shots to the proportion of three-point shots scored relative to all shots scored. However, the matching equation was adjusted to account for the greater reinforcer magnitude of the three-point basket (i.e., 1.5 times greater than the two-point basket reinforcer magnitude). For players with substantial playing time, results showed that the overall distribution of two- and three-point shots was predicted by the matching equation. Game-by-game shot distribution was variable, but the cumulative proportion of shots taken from three-point range as the season progressed was predicted almost perfectly on a player-by-player basis for both male and female basketball players.

DESCRIPTORS: athletic performance, basketball, matching law, sports, quantitative analysis

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The matching law is a quantitative formulation stating that the relative rates of responding across two concurrently available alternatives tend to equal the relative reinforcement rates they produce (Catania, 1992). For example, if an animal receives food reinforcement for pressing Lever A on a variable-interval (VI) 30-s schedule and pressing Lever B on a VI 60-s schedule, the animal is likely to press Lever A twice as often as it presses Lever B (e.g., Herrnstein, 1961). This phenomenon is called *matching* (Herrnstein, 1961, 1970), and an equation representing the formulation can be expressed as follows:

$$\frac{R_1}{R_1 + R_2} = \frac{r_1}{r_1 + r_2}, \qquad (1)$$

where R_1 represents the rate of responding on one response alternative and R_2 represents the rate of responding on a second alternative; r_1 and r_2 represent the respective rates of reinforcement for those alternatives. A large body of empirical research with humans and nonhumans has shown the robust descriptive validity of the matching equation. Further, the basic formulation is flexible enough to incorporate variables such as reinforcer delay, reinforcer magnitude, and response bias (Baum, 1974b; McDowell, 1989). In this study, we applied the equation to the allocation of two- and three-point basket shooting in college basketball players.

Several studies have involved experimental manipulations of reinforcement rates to evaluate matching with human behavior in applied settings (Fisher & Mazur, 1997). For example, Conger and Killeen (1974) showed that allocation of conversation in college students was distributed in a manner proportional to the rate of reinforcement (positive reply statements) delivered by a confederate.

Similarly, Mace, Neef, and colleagues (e.g., Mace, Neef, Shade, & Mauro, 1994, 1996; Neef, Mace, & Shade, 1993; Neef, Mace, Shea, & Shade, 1992) conducted a series of studies evaluating matching relations with special education students working on academic tasks. The general preparation in this series of studies allowed students to choose between two academic activities equated along most dimensions while a variable of interest (such as rate of reinforcement) was manipulated. Collectively, the results of these studies showed that the relative rate, duration, amount, and delay of reinforcement between the two alternatives controlled response allocation on academic tasks. That is, responding was allocated in a manner predicted by the matching equation.

In almost all studies of matching to date, the reinforcement schedules were manipulated by the experimenters. Of course, experimental manipulation is the hallmark of behavior analysis and is a requisite for testing and evaluating matching principles. However, the question remains whether the matching relation holds in complex, naturally occurring human environments. This question arises for a number of reasons. First, the matching equation best describes response allocation when the two response alternatives are reinforced on concurrent VI schedules. However, true VI arrangements rarely exist in natural environments (Nevin, 1998). Second, most studies of concurrent choice involve quantitatively and qualitatively identical reinforcers for both alternatives. However, in natural environments, reinforcer amounts and qualities presumably vary across alternatives in a wide range of circumstances (Green & Freed, 1993). Third, behavior is often multiply controlled, so identifying a single reinforcer to evaluate from a perspective of matching often represents an incomplete analysis (Martens & Houk, 1989).

Despite the complications associated with evaluating naturally occurring behavior from a matching perspective, a handful of studies have successfully demonstrated matching relations under seminatural conditions. Perhaps the first evaluation of this sort was described by Baum (1974a), who monitored the activity of a flock of wild pigeons that was provided access to grain in a house. A version of standard laboratory equipment was placed in the attic of the home. Pigeons could obtain grain by pecking on either of two keys, and the keys were associated with different average intervals between grain access (i.e., concurrent VI schedules). As predicted by the matching law, the aggregate proportional response rates of the pigeon flock on the two response alternatives matched the proportional reinforcement rate associated with those alternatives. Analogous methods were reported by Case, Nichols, and Fantino (1995) to evaluate pigeons' preference for VI reinforcement under varied water budget arrangements. Finally, Hantula and Crowell (1994) showed that the matching law predicted response allocation on a computerized analogue investment task. In all of these prior reports, however, the degree to which naturally occurring response–reinforcer relations are represented is compromised by the fact that VI reinforcement schedules were programmed into the experiment.

A second approach to evaluating the matching law under naturally occurring situations involves describing response–reinforcer relations as they occur in the absence of programmed experimentation. For example, McDowell (1988) evaluated the relation between a young boy's self-injury and maternal attention that occurred contiguously with the self-injury. Results showed that the matching equation closely predicted the proportion of time the boy spent engaging in self-injury. There was some experimental evidence presented in the study to suggest that maternal attention was a rein-

forcer for self-injury, but other potential sources of reinforcement were not ruled out. Similarly, Martens and Houk (1989) evaluated the relations among disruptive behavior, on-task behavior, and teacher attention with an adult woman with mental retardation. Again, the matching law proved to be useful in predicting response allocation toward disruptive behavior as a function of teacher attention in a classroom setting. However, just as in the McDowell study, attention was presumed to be the sole reinforcer for problem behavior. Further, momentary fluctuations in reinforcer value could not be addressed given that attention took a variety of forms and the establishing operations for attention as reinforcement were not taken into account. Nonetheless, the descriptive validity of the matching equation for complex human interactions was supported by these studies.

It is likely that in many human situations the amount of a reinforcer associated with one schedule differs from the amount of the reinforcer associated with another schedule. When the amount of reinforcement differs, the matching equation can be adjusted (known as the "concatenated" matching equation) to take into account differing amounts and rates of reinforcement (Davison, 1988; Davison & Hogsden, 1984; Davison & McCarthy, 1988), as follows:

$$\frac{R_1}{R_1 + R_2} = \frac{r_1(A)}{r_1(A) + r_2}. \qquad (2)$$

In this equation, A represents the amount by which the reinforcer for one alternative differs from the amount of the reinforcer for the other alternative. For example, if two times the amount of the reinforcer is available for one alternative, A would be 2. This equation can be converted algebraically to the concatenated generalized matching equation (Baum, 1974b; Davison & Hogsden, 1984), but for the purposes of the current study it is mainly important to recognize that reinforcer amount can be weighted into the equation mathematically. Equation 2 will be applied to college basketball shooting.

In college basketball, a team receives three points for baskets shot and scored from outside an arc extending from the top of the key down to the baseline in the corners of the court. The team receives two points for shots scored inside that arc. Basketball shooting was selected for analysis for four reasons. First, a shot taken is a clearly defined operant, and statistics for each college game are readily available. Second, a basket scored is a clearly defined outcome with a reinforcer value that stays virtually constant throughout the game. Third, the two- and three-point shot rule in college basketball provides a reasonable approximation of a concurrent choice arrangement because on any shooting opportunity a player could take either type of shot. Fourth, the specific point amount of a shot scored is known. A term representing differential reinforcer amount (i.e., for three-point shots, 1.5 times the value of two-point shots) can be inserted into the matching equation. In the analyses herein, we will compare the utility of the matching equation with and without weighting for the additional value of three-point shots. As such, this is the first evaluation of matching in naturally occurring human behavior with reinforcer amounts incorporated in the equation. Thus, basketball shooting in a real-game context provides a useful format to test the robustness of the matching law in uncontrolled (i.e., not experimentally manipulated) human environments.

A secondary reason for this analysis is that it may provide a forum for behavior analysts to further evaluate choice making in athletic competition. If basic principles of behavior analysis apply to complex performance in the athletic arena, a better understanding of factors that influence athletic performance should emerge. In turn, behavior analysts

would be better equipped to explain and enhance athletic performance.

METHOD

Data were obtained from the sports information office at the university for the basketball season running from late 1998 through early 1999, either via an Internet Web site or via direct communication. Statistics were evaluated from 13 players on the men's team and 13 players from the women's team of a large National Collegiate Athletic Association (NCAA) Division 1 school. In the concatenated matching equation, R_1 represents the rate of shots taken from three-point range and R_2 represents the rate of shots taken from the two-point range; r_1 and r_2 represent the rate of baskets scored (i.e., reinforcement) from the three- and two-point range, respectively. Note that in the equation time is canceled out as a result of being equivalent across all terms (hence, terms representing rate of baskets made become number of baskets made). Also, r_1 in the equation (relative rate of reinforcement for three-point shots) is multiplied by the value 1.5 because the three-point shot has a value 1.5 times that of the two-point shot; thus,

$$\frac{R_1}{R_1 + R_2} = \frac{r_1(1.5)}{r_1(1.5) + r_2}. \tag{3}$$

To illustrate application of Equation 3, suppose that for an entire season a player scored on 30 shots from three-point range and scored on 55 shots from two-point range. If the reinforcer values are held constant, the matching equation would predict that 35.3% of the player's shots should thus be allocated to the three-point alternative (30 divided by 85 total shots equals .353). However, because the value of the reinforcer is 1.5 times greater for three-point shots scored, the adjusted (concatenated) matching equation would predict that 45% of the

player's shots (i.e., a proportion of .45) would be taken from three-point range, as follows:

$$\frac{R_1}{R_1 + R_2} = \frac{30(1.5)}{30(1.5) + 55} = \frac{45}{100}$$
$$= .45. \tag{4}$$

Equation 3 was used to evaluate shot allocation in the following ways: (a) The men's and women's (group) overall shot allocation for the entire season was compared to the overall predicted shot allocation for the entire season. (b) Each player's overall shot allocation for the entire season was compared to the overall predicted shot allocation for the entire season for that player. (c) Each player's overall shot allocation for the entire season up to game X was compared to the overall predicted shot allocation for the entire season up to game X. In other words, as each game occurred, a new predicted level of shot allocation was calculated and compared to the actual shot allocation up to that point in the season. (d) The individual-game shot allocation for each player was compared to the predicted shot allocation up to that particular game.

To evaluate the men's and women's (group) overall shot allocation for the entire season, we calculated the weighted (concatenated) and unweighted matching equations following every game based on the cumulative shots scored from two- and three-point range up to that point in the season. Recall that the weighted equation is Equation 2 (i.e., it takes into account the greater reinforcer amount of a three-point shot scored) and the unweighted equation is Equation 1 (i.e., it does not take into account the greater reinforcer amount of a three-point shot scored).

To evaluate each individual player's shot allocation we also calculated the predicted and obtained proportion of shots taken from three-point range using the weighted and

unweighted matching equations. These data were plotted in several ways to examine the extent to which the obtained proportion of shots was predicted by the weighted matching equation. In addition, the log response ratios were plotted against the log reinforcer ratios (Baum, 1974b). The advantage of such analyses using log ratios is that they permit treatment of the data in terms of sensitivity (i.e., the slope of the log-log function) and bias (i.e., the intercept of the function). Perfect matching with no bias would yield the following linear equation:

$$y = 1.0x + 0. \qquad (5)$$

On a log-log plot, a line described by this equation represents perfect matching because changes in x (the ratio of log three-point shots scored to log two-point shots scored) correspond perfectly to changes in y (the ratio of log three-point shots taken to log two-point shots taken). That is, if the ratio of reinforcement for one alternative matches the ratio of responding for that alternative, the slope of the regression line is 1.0. In addition, if there were no bias toward one alternative or the other, the y intercept would be at 0. A bias in responding indicates that, for reasons independent of reinforcement, response allocation is shifted toward one alternative.

RESULTS

Table 1 shows for each player the number of two-point shots attempted and scored, the number of three-point shots attempted and scored, the proportion of attempted shots from three-point range, and the weighted prediction derived from Equation 2. For the men's team, Players M1 through M9 had the most playing time and attempted the most shots. For these players, the matching equation closely predicted actual shot allocation in terms of proportion of shots taken from three-point range. For Players M10 through M13, three-point shots were never reinforced (i.e., they never scored a three-point shot in their limited minutes played); thus, any shooting from three-point range created a proportion of shot allocation greater than the proportion of reinforcement for that alternative. Players M1 and M2 on the men's team shot frequently from three-point range, which was predicted based on the relative rate of reinforcement (multiplied by differential point value). Players M9 and M10 never made three-point shots and, accordingly, rarely took shots from that range. Players M11, M12, and M13 each played a total of less than 30 min during the course of the season, so the fact that their behavior does not reflect matching may merely reflect an insufficient sample of behavior (alternatively, it is possible that the poor response allocation is the reason they rarely played). For the women's team, Players W1 through W5 had the most playing time and attempted the most shots. For these players, the matching equation was highly predictive of actual shot allocation from three-point range. W10 and W13 both attempted 98 or more shots, but never attempted a shot from three-point range. All of the other players attempted relatively few shots, and some of them (W7, W9, and W11) never scored from three-point range.

Figure 1 shows scatter plots for the proportion of responses (three-point shots over total shots taken) against the proportion of obtained reinforcers (three-point shots scored over total shots scored, multiplied by 1.5), with each player contributing a single point based on end-of-season performance. However, only those players who attempted more than 100 shots were included on the scatter plot because the variance in the estimation of proportions is unacceptably large with smaller numbers. The matching law predicts that each point would fall on the diagonal line extending through the frame.

Although the tendency toward matching

Table 1
Overall Shooting Statistics

	M1	M2	M3	M4	M5	M6
2 pt. attempted	67	69	67	100	67	146
3 pt. attempted	178	170	72	118	55	87
2 pt. scored	39	32	29	56	42	84
3 pt. scored	78	64	24	45	19	31
Prop. 3 pt. attempted	.727	.711	.518	.541	.451	.373
Weighted prediction	.75	.75	.554	.547	.404	.356
	W1	W2	W3	W4	W5	W6
2 pt. attempted	90	279	356	380	263	57
3 pt. attempted	177	110	59	30	1	19
2 pt. scored	37	113	178	177	140	26
3 pt. scored	53	39	24	12	0	3
Prop. 3 pt. attempted	.663	.283	.142	.073	.038	.25
Weighted prediction	.682	.341	.168	.092	0	.148

can be seen readily in Table 1 and Figure 1, most current research on the matching equation involves evaluations of the data as log response ratios against log reinforcer ratios (see Baum, 1974b, for a discussion of the generalized matching law). The upper two panels of Figure 2 depict best fit lines (solid lines) derived from the weighted formulations of the matching equation compared to perfect matching (dashed lines). Again, only data from those players who attempted more than 100 shots are represented in the figure. In addition, data from those who never scored a three-point shot are omitted (because there can be no log of zero). For both the men and the women, the regression lines closely approximate the dashed diagonal line, indicating a high degree of matching, with only small degrees of bias against taking three-point shots (the y intercept is $-.023$ for men and $-.079$ for women). The lower two panels depict the regression lines for the same data but without weighting the reinforcer value for three-point shots. It is noteworthy that the y intercept shifts in a direction that indicates bias toward the three-point shots in both cases ($+.137$ for men and $+.097$ for women). The matching relation observed for the men using the

weighted equation is disrupted when the unweighted equation is used. Although the women's data appear to reflect near matching with or without weighting, a perhaps-false bias or preference toward taking three-point shots is obtained if the differential reinforcer amount is not taken into account.

Figure 3 compares the predicted and actual shot allocation using the weighted equation for Players M2, M6, W1, and W3. The results for these 4 players are generally representative of all players with significant playing time. The data path representing the predicted shot allocation (proportion of shots taken from three-point range) was made by using the concatenated matching equation following every game, based on the cumulative data reported for shots taken and shots scored from two- and three-point range up to that point in the season, again with three-point shots given a 1.5 times greater value. The data path for actual shot allocation was based on the cumulative data for shots actually taken from two- and three-point range up to that point in the season. Overall, M2 took fewer shots from three-point range than was predicted by the weighted matching equation. M6, W1, and W3 showed almost perfect matching. How-

Table 1
(*Extended*)

M7	M8	M9	M10	M11	M12	M13
135	90	192	56	5	2	1
41	31	2	1	1	3	3
64	46	117	30	3	0	1
19	9	0	0	0	0	0
.233	.256	.01	.018	.167	.6	.75
.308	.227	0	0	0	0	0
W7	W8	W9	W10	W11	W12	W13
5	11	11	106	77	45	98
0	7	0	0	3	16	0
1	1	6	54	36	17	43
0	1	0	0	0	4	0
0	.389	0	0	.0375	.262	0
0	.6	0	0	0	.261	0

ever, W3 took proportionally fewer shots from three-point range than predicted by the equation. These data indicate that W3 started shooting more three-point shots about halfway through the season and also started scoring on some of those shots. Had she started shooting more from three-point range but rarely or never scored on the shots, the predicted path would have been at or near zero. However, the predicted proportion of shots from three-point range very closely matches the actual proportion, indicating that her shot allocation was sensitive to the rate of reinforcement (multiplied by 1.5 for the differential reinforcer amount).

Figure 4 shows individual-game response allocations for the same 2 male players (M2 and M6) and the same 2 female players (W1 and W3). The data path representing the predicted shot allocation was made by using the concatenated matching equation following every game, based on the cumulative data reported for shots scored from two- and three-point range up to that point in the season, with three-point shots given a 1.5 times greater value. However, the data path for the actual proportion of shots taken represents the allocation within a single game rather than cumulatively up to a certain point in

the season. Given the relatively small number of shots taken in any particular game, it is not surprising that the actual shot allocation within a particular game is highly variable around the predicted shot-allocation data path for all 4 players. The most unexpected finding was for Player W3, who took proportionally more three-point shots than predicted in 12 of the last 15 games. This finding does not seem to correspond with the findings reported in Figure 3 for W3. However, a close examination of W3's data revealed that the lower-than-predicted shot taking over the course of the entire season (see Figure 3) was produced by her performance during the first half of the season, during which time she took zero shots from three-point range in 12 of 14 games.

The upper panel of Figure 5 shows the results of the comparison of predicted (with and without weighting for the three-point alternative) versus actual shot allocation for the entire season for the men's team. The data were pooled across all players, resulting in an aggregate number of shots taken and shots scored. The data paths for the actual and predicted shot allocation (with weighting) converge and virtually overlap after about the sixth game of the year. The data
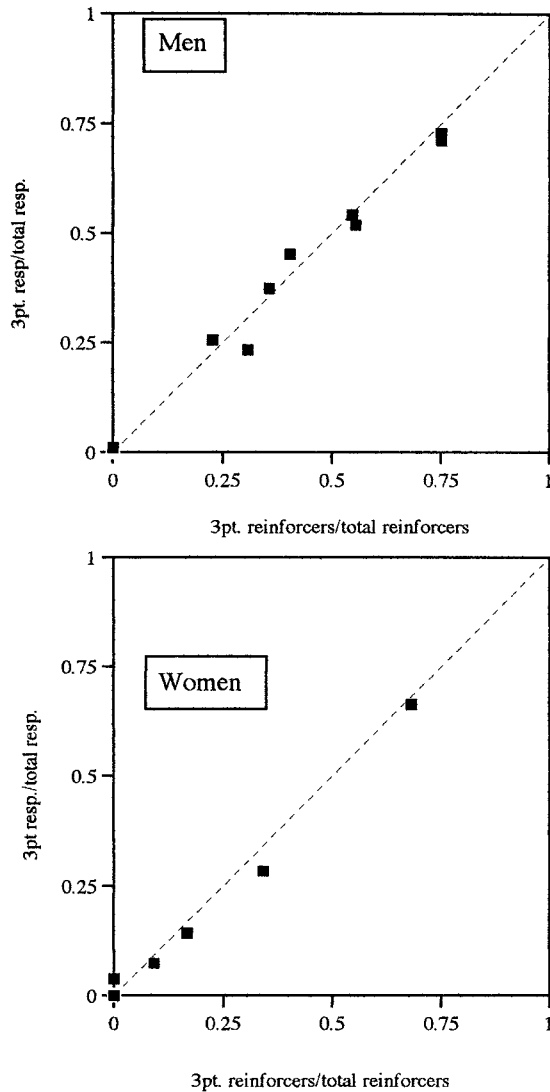
Figure 1. Scatter plots of predicted and actual three-point shot allocation for male (upper panel) and female (second panel) players on an individual basis for those players who attempted more than 100 shots. Data used to calculate predicted and actual shot allocation are taken from the entire season. The data are plotted against a diagonal line representing perfect matching.

paths for actual and predicted shot allocation without weighting do not match. The lower panel of Figure 5 shows the results of the same type of analysis for the women's team. The results in both panels show that the players tended to allocate responding in

concordance with the relative reinforcement rates for two- and three-point shots. In addition, although more clearly so for the men, these results generally demonstrate the utility of taking into account the reinforcer amount of three-point shots made. That is, imperfect matching is obtained to a greater extent when the unweighted equation is used.

DISCUSSION

A variation of the matching equation closely predicted two- and three-point shot allocation by male and female college basketball players at a NCAA Division 1 university. The near constancy of relative reinforcer value for two- and three-point shots allowed the insertion of a reinforcer-amount term into the equation (i.e., for three-point shots, 1.5 times the value of two-point shots). Although the equation accurately predicted shot allocation when a large number of shots were taken into account across the entire season, individual-game shot allocation was more variable.

One clear deviation from other matching analyses is that basketball shooting is not maintained on a pure VI schedule, whereas most research on the matching equation has involved VI schedules. The reinforcement schedule for basketball shooting represents a blend of VI and variable-ratio schedules. The schedule is ratio-like in the sense that the more one shoots, the higher the rate of reinforcement. Thus, at one level, it should not be surprising that proportional reinforcement rate closely matches proportional response rate. In ratio schedules, the matching relation is "forced" in the sense that the higher the response rate, the higher the reinforcement rate. However, basketball shooting is unlike performance under concurrent ratio schedules insofar as concurrent ratio schedules yield nearly exclusive responding to the schedule that yields richer reinforcement (e.g., Herrnstein & Loveland, 1975).
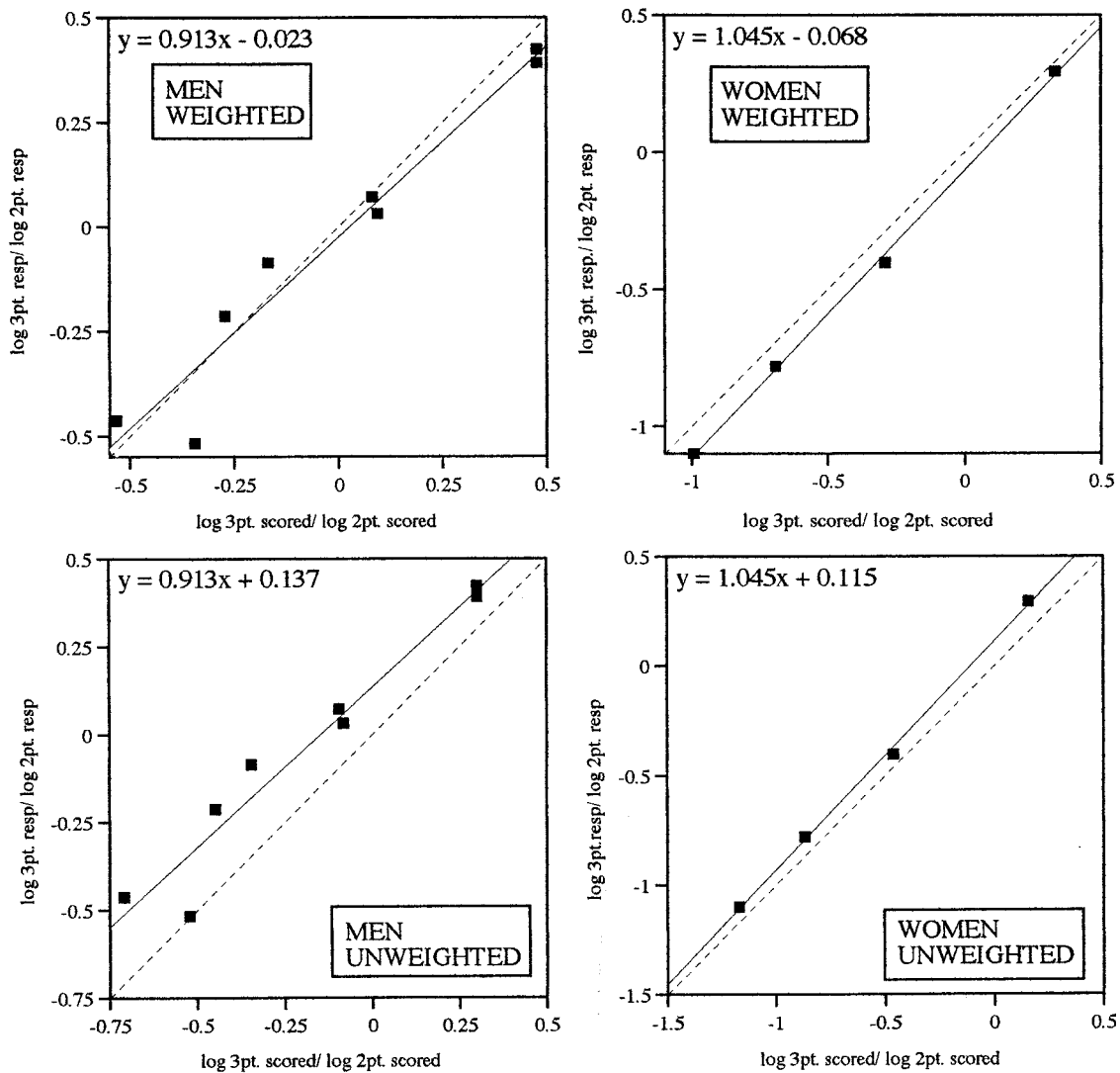
Figure 2. Log response ratios plotted against log reinforcer ratios for players who attempted more than 100 shots and scored on at least one three-point shot. The upper two panels depict data for the male and female players using the weighted matching equation. The lower two panels depict data for the male and female players using the unweighted matching equation. The linear equation in each panel depicts sensitivity (slope) and bias (y intercept).

It may be that the probability of reinforcement increases as time elapses (as it does with interval schedules) because the opposing player may gradually defend the three-point zone less carefully when the shooting player has not taken a shot from three-point range for a period of time. In any case, the reinforcement schedule for basketball shooting in a real-game situation exemplifies the complex blend of ratio- and interval-like schedule characteristics in most human situations (see Nevin, 1998, for a recent discussion). Despite uncertainty about the underlying schedules, the utility of the matching equation was demonstrated in the current study in several ways. Perhaps most uniquely, the analysis contributed to the understanding of naturally occurring matching
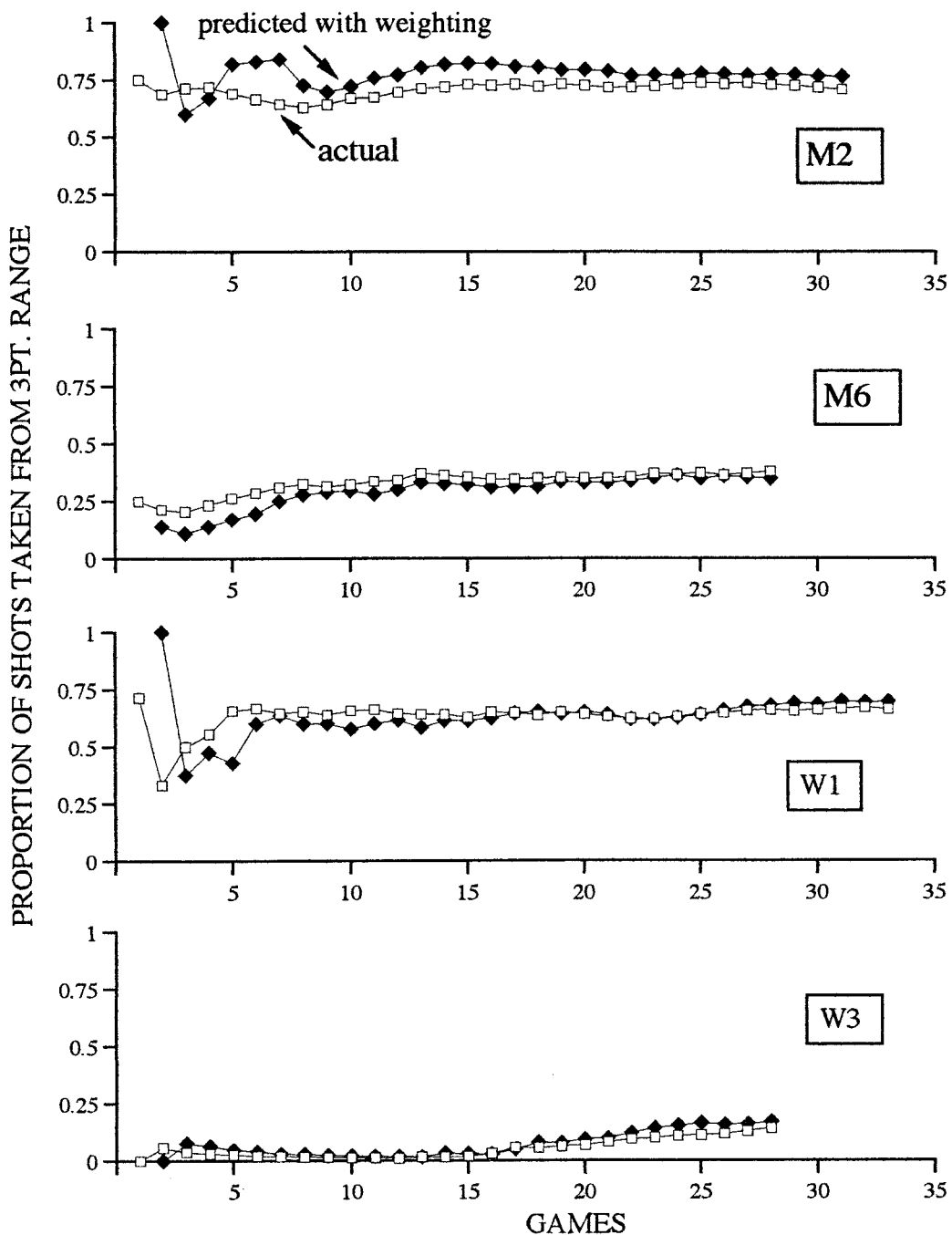
Figure 3.   Predicted and actual shot allocation for some individual players from the men's and women's teams. The predicted and actual shot-allocation data paths are based on the equation using all data up to a particular game.
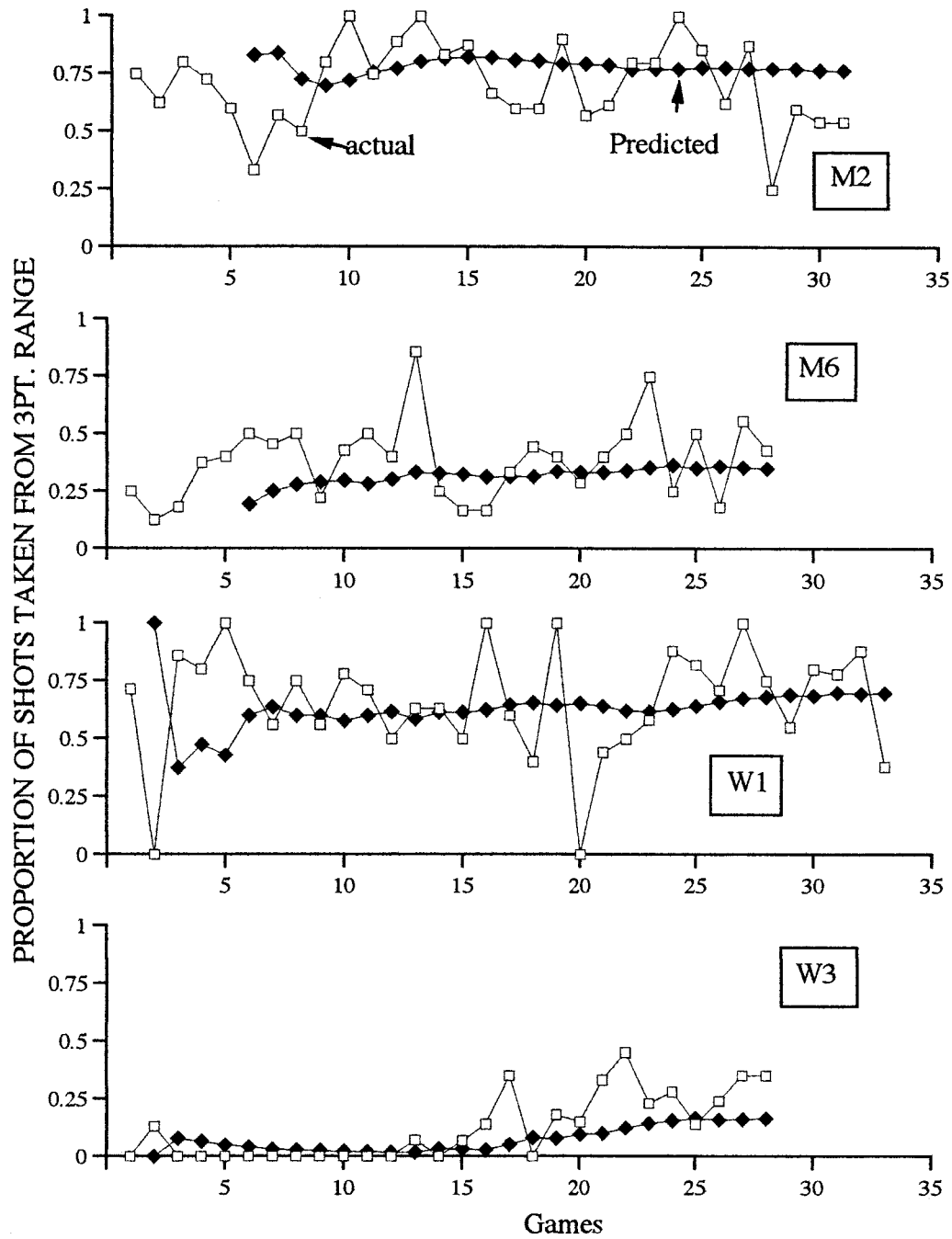
Figure 4. Predicted and actual shot allocation for some individual players from the men's and women's teams on a game-by-game basis. The predicted shot-allocation data paths are based on the equation using all data up to a particular game. The actual shot allocation is the observed shot allocation during a single game.
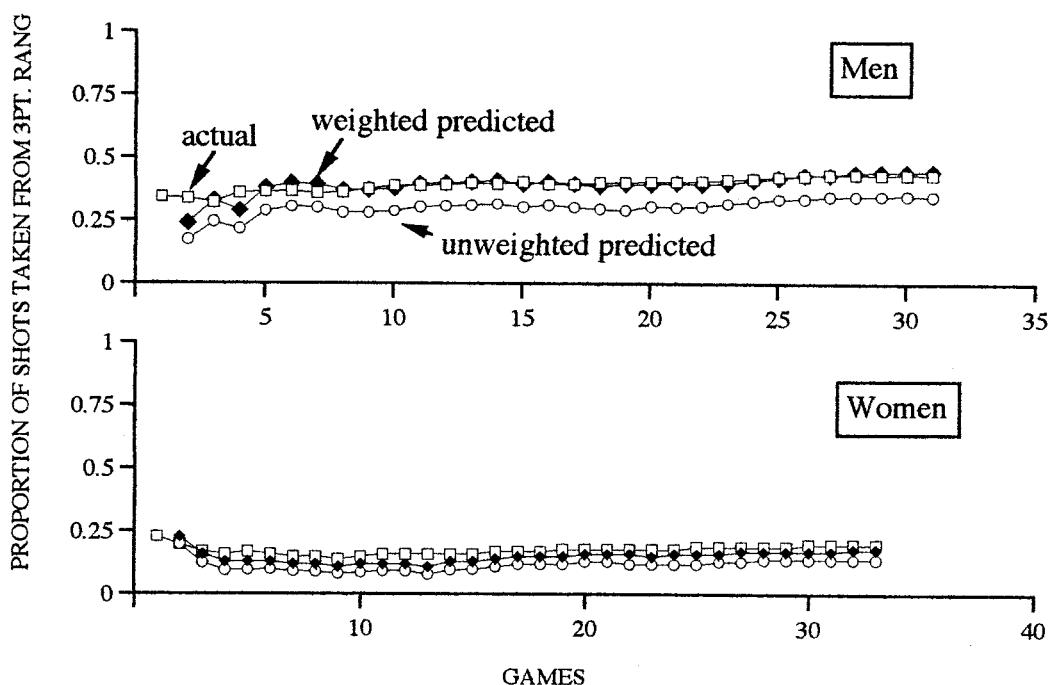
Figure 5. Predicted and actual three-point shot allocation for the entire season for men's (upper panel) and women's (lower panel) teams as groups. The predicted and actual shot-allocation data paths are based on the equation using all data up to a particular game both with and without weighting for the enhanced value of the three-point shot.

relations because the reinforcer values were different (two points vs. three points) for the two concurrent operants, and this adjustment to the equation was necessary to predict response allocation more accurately.

A limitation of the current study is that both teams evaluated were good teams, both of which made postseason national invitational tournaments. It is possible that poor teams are less likely to show matching relations with shot allocation. For example, the first author casually observed a street game at a local park and noted a high percentage of shots taken from three-point range despite virtually no reinforcement (i.e, almost no points were scored) from that range. Presumably some poor teams would similarly misallocate shots to the three-point range in a manner akin to impulsive behavior (Logue, 1995). The behavior is impulsive-like in that it yields an overall lower density of reinforcement although the immediate, albeit inter-

mittent, "extra" reinforcer of an additional point presumably maintains responding.

It is hoped that the matching analysis for basketball shooting will contribute to the literature on application of behavioral principles to athletic performance. Numerous studies have been published that demonstrate how principles of reinforcement and stimulus control can be used to improve athletic performance (e.g., Anderson, Crowell, Doman, & Howard, 1988; McKenzie & Rushall, 1974; Osborne, Rudrud, & Zezoney, 1990), and a few studies have been published that show how basic behavioral mechanisms might influence naturally occurring athletic performance (e.g., Mace, Lalli, Shea, & Nevin, 1992). The implications of the current study for direct application may not be immediately obvious, but we suggest that such analyses could contribute to an understanding of athletic performance in a number of ways. Coaches might be trained to

make judgments about the efficiency of shot selection on a game-by-game basis, defensive strategies could be adjusted based on inter-response times for shooting from a particular location, and so on.

If the reinforcers for choice making in other athletic arenas could be identified, the matching law may prove to be useful in predicting response allocation across a range of athletic responses. For example, on any given pitch, a baseball pitcher chooses from among an array of operants consisting of curveballs, fastballs, sliders, and change-ups. Presumably, these choices are controlled by relative reinforcement rates (e.g., getting a strike or an out) under highly constricted circumstances (e.g., a specific batter, fatigue level, etc.), but the choices should be amenable to complex analyses based on the matching equation. Similar analyses could be made of quarterbacks throwing long or short passes in a football game, and so on.

In theory, the matching equation is equally useful for evaluating response allocation of other forms of complex human behavior including verbal interactions (Conger & Killeen, 1974), problematic behavior (Martens & Houk, 1989; McDowell, 1988), and academic performance (e.g., Mace et al., 1994; Neef et al., 1993). The challenge for the application of the matching law in naturally occurring behavioral interactions is in identifying the relevant reinforcers that maintain behavior, identifying the relative value of that reinforcer in comparison to other concurrently available reinforcers, and identifying momentary fluctuations in reinforcer value. Although the current analysis lends further support to the matching law in complex human interactions, the more difficult analyses seem to lie ahead.

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STUDY QUESTIONS

1. What are the general predictions of the matching law, and what are some difficulties in applying the law to naturally occurring behavior?

2. The authors suggested that VI arrangements rarely exist in nature. Describe at least one naturally occurring response–reinforcer relation that seems to conform to a VI schedule.

3. List four reasons why the authors chose basketball shooting as an appropriate behavior for a matching analysis.

4. What variables entered into the concatenated matching equation, and what adjustment was made to account for the difference in points earned for the two different types of shots?

5. Describe the four different ways in which the data were analyzed.

6. How well did the weighted and unweighted matching equations describe response allocation between three- and two-point shots?

7. The matching equation best describes responding under concurrent VI schedules. How is basketball shooting similar to and different from concurrent VI schedules?

8. How might an understanding of shot allocation in basketball be used to enhance team performance?

Questions prepared by Eileen Roscoe and Rachel Thompson, The University of Florida