JOURNAL OF APPLIED BEHAVIOR ANALYSIS 2002, **35,** 13–27 NUMBER 1 (SPRING 2002)

AN APPLICATION OF THE MATCHING LAW TO SEVERE PROBLEM BEHAVIOR

JOHN C. BORRERO AND TIMOTHY R. VOLLMER

UNIVERSITY OF FLORIDA

We evaluated problem behavior and appropriate behavior using the matching equation with 4 individuals with developmental disabilities. Descriptive observations were conducted during interactions between the participants and their primary care providers in either a clinical laboratory environment (3 participants) or the participant's home (1 participant). Data were recorded on potential reinforcers, problem behavior, and appropriate behavior. After identifying the reinforcers that maintained each participant's problem behavior by way of functional analysis, the descriptive data were analyzed retrospectively, based on the matching equation. Results showed that the proportional rate of problem behavior relative to appropriate behavior approximately matched the proportional rate of reinforcement for problem behavior for all participants. The results extend prior research because a functional analysis was conducted and because multiple sources of reinforcement (other than attention) were evaluated. Methodological constraints were identified, which may limit the application of the matching law on both practical and conceptual levels.

DESCRIPTORS: matching, descriptive analysis, severe problem behavior, functional analysis

---

Choice may be defined as the emission of one of two or more alternative and usually incompatible responses (Catania, 1998). In operant laboratories, choice has been evaluated typically in concurrent-schedule arrangements (e.g., Herrnstein, 1961) under which two or more schedules operate simultaneously but independently, and consequences are delivered for different response alternatives. Herrnstein provided a quantitative description of behavior on concurrent schedules of reinforcement now known as the matching law. The matching law states that organisms will distribute their behavior among concurrently available response alternatives in the same proportion that reinforcers are distributed among those alternatives. The matching law may also be expressed mathematically as follows:

$$\frac{R_1}{R_1 + R_2} = \frac{r_1}{r_1 + r_2}, \qquad (1)$$

where R_1 represents the rate of Response 1, R_2 represents the rate of Response 2, r_1 represents the reinforcement rate for R_1, and r_2 represents the reinforcement rate for R_2.

A number of basic and applied studies on choice have shown that both nonhuman and human behavior generally conform to the relation described by the matching equation when reinforcement schedules are precisely controlled (Baum, 1979; Davison & McCarthy, 1988; Mace, Neef, Shade, & Mauro, 1994; McDowell, 1988; Neef & Lutz, 2001; Neef, Mace, & Shade, 1993). In addition, transformations of Equation 1 allow the incorporation of additional variables known to influence response allocation, such as mag-

Preparation of this manuscript and portions of the data collection were supported in part by Grant HD38698 from the National Institute of Child Health and Human Development. We thank Jennifer Asmus, Jesse Dallery, and Brian Iwata for their assistance with an earlier version of this manuscript. We also thank Jason Bourret, Anibal Gutierrez, and Carrie Wright for their assistance with various aspects of this study.

Reprints may be obtained from Timothy R. Vollmer, Department of Psychology, The University of Florida, Gainesville, Florida 32611.

nitude of reinforcement, quality of reinforcement, delay to reinforcement, and duration of reinforcement (Baum, 1974b). Although it is clear that each of these variables may control response allocation, in the current investigation we have chosen to analyze rate of reinforcement during interactions between primary care providers (usually parents) and individuals who engage in severe problem behavior.

Historically, the matching law has been evaluated in experimental analyses (i.e., direct manipulation of an independent variable). In a seminal experiment, Herrnstein (1961) measured the allocation of responses among concurrent variable-interval (VI) schedules for 3 pigeons. Results showed that the proportion of responses that were emitted on either the left or the right key equaled the proportion of reinforcers delivered as a result of pecks on the respective keys.

Results of experimentation with nonhumans have been extended to humans. For example, Conger and Killeen (1974) demonstrated that college students distributed statements within a conversation to a confederate in direct proportion to the rate of positive statements delivered by the confederate. In a similar study, Beardsley and McDowell (1992) evaluated time allocation based on naturalistic social reinforcement during interactions between college students and an experimenter using the single alternative formulation of the matching equation. Results showed that the matching equation accounted for a significant proportion of the variance in the distribution of time allocation. In a novel evaluation of matching in humans, Schroeder and Holland (1969) arranged reinforcers (i.e., signals) in a concurrent-schedule arrangement contingent on macrosaccadic eye movements. With the addition of a changeover delay between response alternatives, participants matched the relative rate of eye movements based on the relative rates of reinforcement available for each response alternative.

Collectively, the experiments by Conger and Killeen (1974), Beardsley and McDowell (1992), and Schroeder and Holland (1969) demonstrated the robustness of the matching law with humans under tightly controlled experimental conditions. However, there have also been attempts to describe naturally occurring behavior–environment interactions in terms of matching. In what may be the first evaluation of matching outside the basic operant laboratory, Baum (1974a) measured response allocation in a flock of wild pigeons using a standard laboratory apparatus. Pigeons could receive grain by pecking one of two response keys that were controlled by separate VI counters. Results showed that the combined proportional response rates of the flock on the two response alternatives matched the proportional reinforcement associated with those alternatives. In a recent study evaluating athletic performance, Vollmer and Bourret (2000) applied a version of the matching equation to evaluate the allocation of two- and three-point shots made by male and female basketball players from college teams. As predicted, the overall distribution of two- and three-point shots matched the overall distribution of reinforcement associated with the two alternatives when reinforcer magnitude was weighted into the matching equation.

Although the potential therapeutic significance of the matching law has been noted (McDowell, 1981; Myerson & Hale, 1984; Noll, 1995; Pierce, Epling, & Greer, 1981), very few demonstrations of the matching law with clinically significant behavior problems have been reported in the literature. In an attempt to extend the generality of the matching law to socially significant human behavior, McDowell (1981) evaluated data collected on the self-injurious scratching of a normally developing 11-year-old boy. McDowell reevaluated data that were originally reported by Carr and McDowell (1980) on the naturally occurring rates of self-injurious behavior (SIB) and verbal reprimands. In the initial investigation, Carr and McDowell conducted four 20- to 30-min observations in the participant's home and identified verbal reprimands as a reinforcer for the participant's SIB. Results showed that the matching equation accounted for 99.7% of the variance in the observed rates of SIB.

Martens and Houk (1989) also conducted a study on severe behavior problems to assess matching relations in the natural environment. Martens and Houk evaluated matching based on time allocation between disruptive behavior and on-task behavior in an 18-year-old woman with developmental disabilities. The investigators conducted observations in the participant's classroom during academic work periods with either the participant's teacher or a classroom aide, using both teacher attention and aide attention as presumed reinforcers. Results showed that the matching equation provided a useful description of response allocation between the two alternatives as a function of adult attention.

More recently, Oliver, Hall, and Nixon (1999) evaluated time allocation between communicative behavior and problem behavior in an individual with Down syndrome. First, the authors conducted descriptive observations of the individual during various school-related activities. Next, the authors conducted an experimental antecedent assessment of two variables hypothesized to maintain problem behavior, alternated with a control condition. Finally, the authors evaluated time allocation between appropriate behavior and problem behavior across 1-hr observations using a variation of Equation 1. Results showed that problem behavior and communicative behavior conformed to the proportion of reinforcement made contingent on each response.

Current research on matching relations and severe problem behavior may be extended in at least three ways. First, previous evaluations of matching in the natural environment have assumed the reinforcing efficacy of attention (e.g., Martens & Houk, 1989). Given the recent increase in research using functional analysis methods to identify reinforcers, it is no longer necessary to assume that some events serve as reinforcers; reinforcement effects can be identified empirically. Second, matching evaluations of problem behavior have not considered the possibility of multiply controlled problem behavior in an experimental analysis of consequent events (e.g., McDowell, 1981; Oliver et al., 1999). Several studies have shown that problem behavior reinforced by attention can be maintained by other sources of reinforcement, such as access to tangible stimuli or removal of instructed activities (e.g., Iwata, Pace, Dorsey, et al. 1994; Piazza et al., 1997; Smith, Iwata, Vollmer, & Zarcone, 1993). Third, there has been an implicit assumption that reinforcement rate drives (i.e., controls) response rate. In this study, we provide evidence that in some cases, high reinforcement rates may drive high response rates, but high response rates may also drive high reinforcement rates. Thus, the purpose of the current investigation was to extend prior research on problem behavior and the matching law by identifying reinforcers via functional analyses and by identifying limitations of a matching account during descriptive analyses of human behavior.

METHOD

Participants

Participants were 4 individuals with varying degrees of developmental disabilities who had been referred for the assessment and treatment of severe problem behavior.[1] To participate in the analysis, individuals had to display both inappropriate and appropriate behavior under some circumstances. Appropriate behaviors of interest were selected based on the function of problem behavior identified via functional analysis. For example, if problem behavior was shown to be reinforced by the contingent presentation of tangible stimuli, the functionally equivalent appropriate response would be a vocal or a gestural request for the tangible item.

Linda was a 14-year-old girl who had been diagnosed with mild mental retardation. Her primary target behaviors were aggression and disruption, which consisted of hitting, kicking, property destruction, and spitting. Appropriate behaviors for Linda included vocal requests for attention (e.g., "Mom, I have a question") and vocal requests for tangible items (e.g., "Can I have my radio, please?").

Mandy was a 24-year-old woman who had been diagnosed with Sticklers syndrome and mild mental retardation. Her primary target behavior was self-injurious behavior (SIB), which consisted of head banging, nose punching, chin punching, and head hitting. Appropriate behaviors for Mandy included vocal requests for attention (e.g., "Mom, let's talk about weddings") and vocal requests for tangible items (e.g., "May I have that magazine?").

Max was a 7-year-old boy who had been diagnosed with autism and moderate mental retardation. Max had been referred for the assessment and treatment of aggression, which consisted of hitting, slapping, and pulling the hair of others. Appropriate behaviors for Max included requests for attention (e.g., gently tapping the arm of a person in the room), requests for tangible items (e.g., pointing to an item, then to himself), and compliance, which was defined as completion of a specified instruction.

Dan was a 9-year-old boy who had been diagnosed with moderate mental retardation and cerebral palsy. Dan had been referred for the assessment and treatment of aggression and disruptive behavior, which consisted of hitting and kicking others, banging against walls, throwing materials, and property destruction. Appropriate behaviors included vocal requests for tangible items (e.g., "toys please").

Data Analysis

To evaluate matching relations, we conducted two prerequisite analyses. First, we conducted descriptive observations and recorded data on the occurrence of each participant's behavior and subsequent environmental events (potential reinforcers). Next, we conducted functional analyses for each participant using methods similar to those described by Iwata, Dorsey, Slifer, Bauman, and Richman (1982/1994). The descriptive analysis was conducted before the functional analysis to capture behavior during natural interactions without previous exposure to test contingencies during a functional analysis. Finally, when reinforcers had been identified via functional analysis, we were able to evaluate the descriptive data using the matching equation.

Descriptive Analysis and Settings

Descriptive analyses were conducted for all participants using the methods described by Vollmer, Borrero, Wright, Van Camp, and Lalli (2001). Briefly, observers used a computerized data-collection program to record three potential reinforcers (attention, instruction termination, and access to tangible items), problem behavior (see individ-

[1] Descriptive data for Linda, Mandy, and Max were derived from the raw data from Vollmer, Borrero, Wright, Van Camp, and Lalli (2001). Vollmer et al. calculated conditional probabilities of reinforcement given the occurrence of problem behavior but did not report response rate or reinforcement rate. Computerized printouts of the data were examined to calculate reinforcer rates.

ual definitions provided above), and appropriate behavior (see individual definitions provided above). Potential reinforcers were recorded as duration measures for all participants (e.g., duration of attention delivered). Problem behavior (e.g., SIB) and appropriate behavior (e.g., requests for attention) were recorded as frequency measures. In the event that a potential establishing operation (e.g., low attention) was not observed, parents or staff were prompted to set up a particular environmental event. For example, if a primary care provider had not exposed a participant to an instructional demand situation by the end of the scheduled descriptive analysis, the care provider was asked to instruct the child to complete a self-care task or a similar task.

In addition, it was possible for up to three potential reinforcers to be scored for a single instance of behavior. For example, if a participant hit his mother after being instructed to place his chair beneath a table, and the mother responded by reprimanding the child, removing the instructional demand, and providing the child with a cup of milk, attention, instruction termination, and access to tangible items were scored. Whether those consequences were counted as reinforcers was determined after the reinforcers that maintained problem behavior were identified via functional analysis. *Attention* was defined as physical or verbal interaction between the participant and primary care providers, including hugging, manual restraining, comfort statements, reprimands, and so forth. *Instruction termination* was defined as removal of demands and instructional materials for longer than 3 s, or the absence of instructions if the participant stopped engaging in the previously specified task for at least 3 s. *Access to tangible items* was defined as the availability of previously restricted tangible items or previously requested items for manipulation or consumption.

Descriptive observations were conducted during interactions with their primary care providers (parents, staff from their group homes) for Linda, Mandy, and Max. The care providers were asked to come to the hospital for these observations. Hospital rooms were equipped with a couch, a table, chairs, fold-away beds, and additional materials such as televisions, videocassette recorders, and radios when requested or upon availability. Observers were bachelors and masters behavior analysis interns and clinical specialists. All observers received at least 20 hr of training in behavioral observation, attended a 2-hr seminar on descriptive analysis data-recording methods, completed at least 5 hr of training in a natural setting, and had high interobserver agreement scores (>90%) with previously trained observers. Observers were seated behind a one-way mirror and used laptop computers to collect data. Total descriptive observation times were 2.1 hr, 2.5 hr, and 1.7 hr, respectively, for Linda, Mandy, and Max. The number of sessions conducted per day ranged between two and four, and the duration of sessions ranged between 10 and 40 min. Observations were conducted when primary care providers were available. These observations were a standard component of the assessment process at the hospital.

Descriptive observations for Dan were conducted during interactions between his mother and siblings throughout various locations of the participant's home (e.g., living room, kitchen). Observers were undergraduate students enrolled in an applied behavior analysis laboratory class and graduate behavior analysis students. All observers attended five 1-hr training seminars on descriptive analysis data-recording methods, completed at least 2 hr of training in a natural setting, and had high interobserver agreement scores (>90%) with previously trained observers. Observers were seated as unobtrusively as possible throughout various locations of the home. Total descriptive observation time for Dan was 2.4 hr. Descriptive observations for all participants were arranged based on care provider availability, but usually occurred three times per week for 30 min per day (range, 10 to 60 min).

Functional Analysis

Functional analyses were conducted using procedures similar to those described by Iwata et al. (1982/1994). Clinical specialists, behavior analysis interns, and graduate students in behavior analysis served as therapists. Functional analysis sessions were conducted in hospital rooms similar to those described above (Linda, Mandy, and Max) or in a vacant classroom at a local elementary school (Dan). Three test conditions and one control condition were alternated in a multielement design. A fourth test condition (the alone condition) was included for Mandy.

During the attention condition, the participants had access to an activity but did not have access to attention unless problem behavior was observed. The therapist diverted his or her attention from the participant and proceeded to engage in paperwork. Occurrences of problem behavior resulted in a brief reprimand followed by statements of concern totaling 30 s of attention. The attention condition was designed to test whether problem behavior was reinforced by adult attention.

During the tangible condition, participants were permitted to engage with a preferred tangible item for approximately 1 to 2 min before the item was withheld. Stimuli incorporated in the tangible condition of the functional analysis were identified via a free-operant preference assessment using methods similar to those described by Roane, Vollmer, Ringdahl, and Marcus (1998). The therapist interacted with the participant on a fixed-time (FT) 30-s schedule, and occurrences of problem behavior resulted in 30 s of access to the preferred tangible item. The tangible condition was designed to test whether problem behavior was reinforced by preferred tangible items.

During the escape condition, participants were asked to complete an instructed activity. Instructions were delivered approximately every 30 s using a three-prompt sequence (Horner & Keilitz, 1975). Contingent on problem behavior, instructional materials were removed, and a 30-s escape period was provided. The escape condition was designed to test whether problem behavior was reinforced by escape from instructed activities.

During the control condition, the participants had access to preferred tangible items (e.g., magazines, milk), attention was continuously available, and no demands were presented throughout the session. During the alone condition, Mandy was observed in an austere environment, and no programmed consequences were provided for SIB. All functional analysis sessions lasted 10 min.

Interobserver agreement. Interobserver agreement was assessed by having a second observer simultaneously but independently score each potential reinforcer (e.g., attention), problem behavior (e.g., aggression), and appropriate behavior (e.g., compliance) during 50% of all descriptive observations for a total of 4.7 hr of observation time. Each observation was divided into consecutive 10-s bins, and the smaller number of observed responses was divided by the larger number of observed responses (Iwata, Pace, Cowdery, & Miltenberger, 1994). The smaller number of seconds was divided by the larger number of seconds within the 10-s bins and the obtained values were averaged for the entire observation session for all duration measures. Agreement averaged 81.1% (range, 69.5% to 94.3%) for attention, 88.6% (range, 83.5% to 93.6%) for escape from instructed activities, and 85.4% (range, 68% to 95.4%) for access to tangible items. Agreement for Linda's aggression and disruptive behavior averaged 93.5%, whereas agreement for appropriate communicative behavior averaged 94%. Agreement for Mandy's SIB averaged 84% and averaged 96.4% for appropriate communication. Agreement for Max's aggression averaged 96% and averaged 83.4% for appropriate behavior. Agreement for Dan's aggression and disruptive behavior averaged 96% and averaged 89.7% for appropriate communication.

Interobserver agreement was assessed during 23.3% of all functional analysis sessions for Linda and averaged 97.3% (range, 81.1% to 100%) for aggression and disruptive behavior. Agreement was assessed during 26.7% of all functional analysis sessions for Mandy and averaged 92.4% (range, 85% to 100%) for SIB. Agreement was assessed during 20% of all functional analysis sessions for Max and averaged 99.2% for aggression. Agreement was assessed during 81.3% of all functional analysis sessions for Dan and averaged 87.6% for aggression and disruptive behavior.

Data Preparation

We calculated the rate of responding for both problem behavior and appropriate behavior as well as the reinforcement rates for both problem behavior and appropriate behavior from the descriptive observation data. An instance of problem behavior or appropriate behavior was scored as "reinforced" if the reinforcer identified via functional analysis was delivered within 10 s of the behavior. Although it is unknown how long after a response occurs an event might function as a reinforcer, 10 s was considered a reasonable starting point for analysis, and other values could be subjected to similar analyses. To calculate the rate of both problem behavior and appropriate behavior, we divided the total number of problem behaviors by the total observation time. For example, if 60 instances of problem behavior were observed over a period of 2 hr, the rate of problem behavior would be 30 per hour. Rate of reinforcement was calculated using a similar procedure. Next, we calculated the proportional rate of problem behavior versus appropriate behavior and the proportional rate of reinforcement for those alternatives in the context of Equation 1.

RESULTS

Functional Analysis

Figure 1 displays the functional analysis results for all participants. Results of Dan's functional analysis showed the highest levels of aggression and disruptive behavior in the tangible condition, suggesting that problem behavior was reinforced by access to tangible items (M = 1.10 responses per minute).

Max's aggression occurred at high rates in the attention, tangible, and escape conditions of the functional analysis, whereas zero instances of aggression were observed during the control condition. These results suggested that Max's aggression was reinforced by adult attention, access to tangible items, and escape (*M*s = 1.38, 1.67, and 1.20 responses per minute, respectively).

Although Linda engaged in problem behavior to some extent across all assessment conditions, the highest rates were observed in the attention and tangible conditions (*M*s = 2.97 and 3.84 responses per minute, respectively), suggesting that problem behavior was reinforced by adult attention and access to tangible items.

Similar results were obtained for Mandy, in that problem behavior (i.e., SIB) occurred to some extent across all conditions; however, the highest rates of SIB were observed in the attention and tangible conditions (*M*s = 3.95 and 3.48 responses per minute, respectively), suggesting that problem behavior

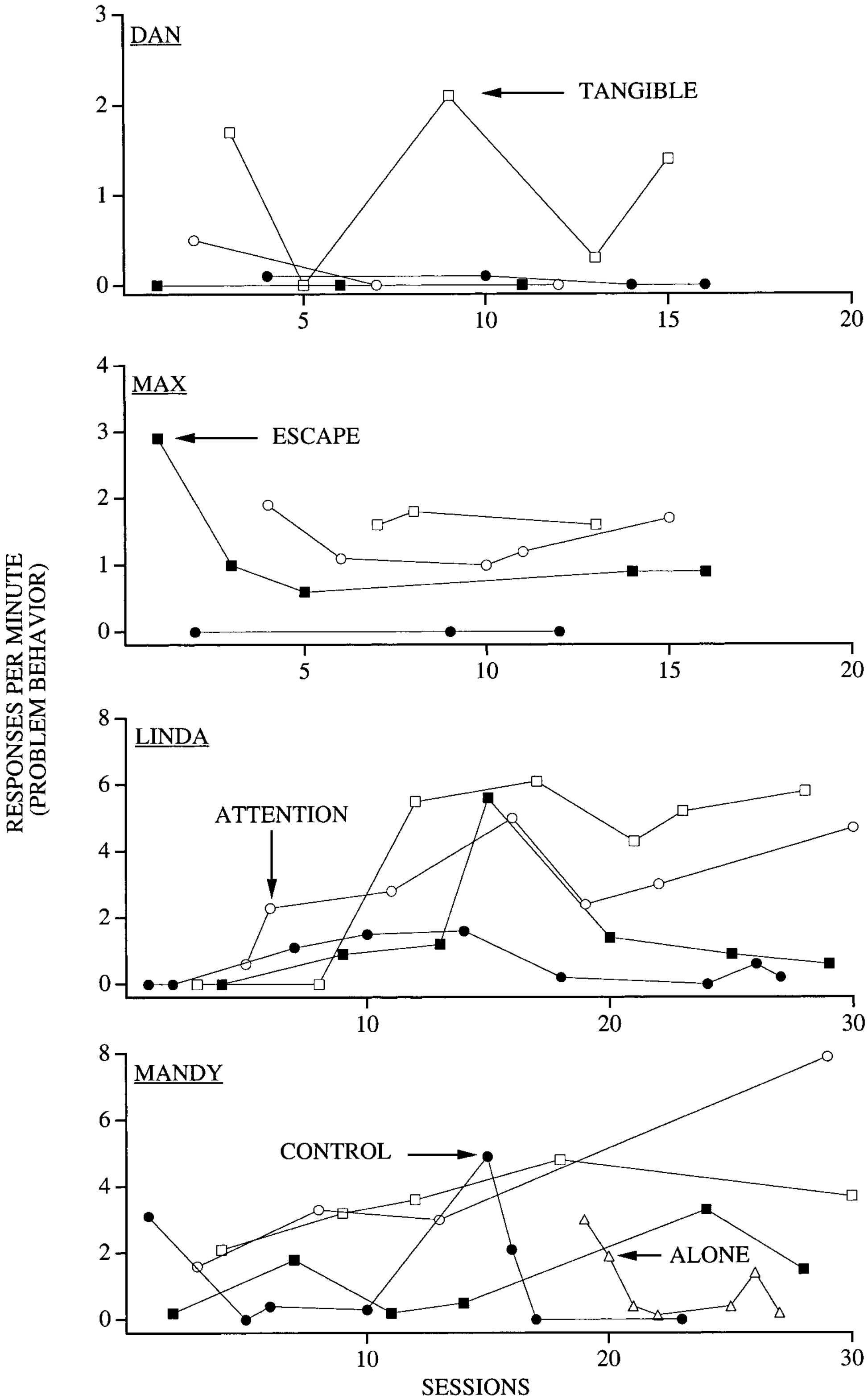


Figure 1. Functional analysis results for Dan (upper panel), Max (second panel), Linda (third panel), and Mandy (bottom panel).

was reinforced by adult attention and access to tangible items.

After identifying the reinforcers that maintained problem behavior for all participants, we were able to evaluate the descriptive analysis data according to the matching equation. We could now identify the relative response rates for problem behavior and appropriate behavior as well as the relative reinforcement rates for problem behavior and appropriate behavior.

Matching Analysis

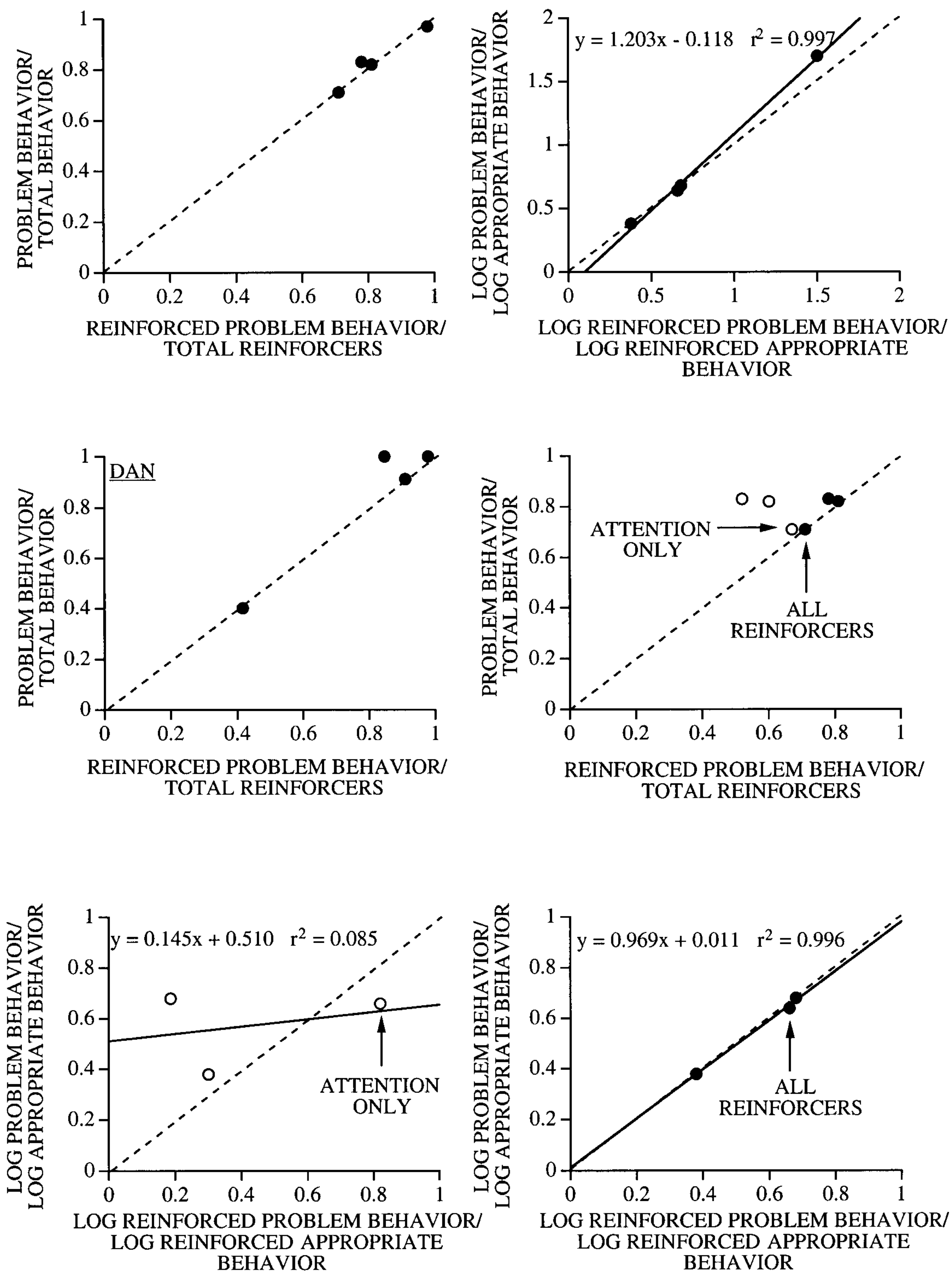
Figure 2 shows various graphic representations of the matching analysis for all participants. The upper left panel shows scatter plots for the proportion of problem behavior over total behaviors emitted (i.e., problem behavior plus appropriate behavior) against the proportion of reinforcers obtained (i.e., reinforcers for problem behavior over reinforcers for problem behavior plus appropriate behavior) for all participants. The broken diagonal line represents perfect matching as described by Equation 1. Each data point represents the data for 1 participant. The relative rate of responding for problem behavior closely matched the relative rate of reinforcement for problem behavior for all participants. To remain consistent with previous analyses of matching, we plotted the data as log response ratios as a function of log reinforcer ratios (upper right panel) (Baum, 1974b; McDowell, 1989; Vollmer & Bourret, 2000). Again, the broken diagonal represents perfect matching. The solid line is a best fit line, which closely matched the dashed diagonal line, indicating close adherence to the matching equation (r^2 = .997).

The middle left panel is a scatter plot of the proportion of problem behavior over total behaviors emitted against the proportion of reinforced problem behavior and appropriate behavior, obtained across four 30-min observations. This analysis was conducted for Dan because he engaged in a minimum of 10 behaviors across multiple 30-min observations. These data also show that the relative rate of responding for problem behavior closely matched the relative rate of reinforcement for problem behavior, even on a session-by-session basis.

The middle right panel of Figure 2 shows a more fine-grained matching analysis for the 3 individuals with multiply controlled problem behavior (Max, Linda, and Mandy). The open circles represent the results of the matching analysis if attention was considered to be the sole reinforcer. Recall that previous research had either identified one reinforcer (e.g., McDowell, 1981) or assumed the efficacy of attention as a reinforcer (e.g., Martens & Houk, 1989). With attention considered to be the only reinforcer that maintained problem behavior, the relative rate of responding for problem behavior consistently exceeded the relative rate of reinforcement obtained for problem behavior. When all sources of reinforcement are entered into the matching equation (filled circles), the data points fall close to the matching line. The lower left panel shows the results of the analysis presented in the middle right panel for attention only, presented as response ratios. When expressed as response ratios, the regression line does not match the dashed diagonal line, indicating poor adherence to the matching equation (r^2 = .085). The lower right panel shows the results of the analysis presented in the middle right panel for all reinforcers, presented as response ratios. When expressed as response ratios, the regression line closely matched the dashed diagonal line, indicating adherence to the matching equation (r^2 = .996).

DISCUSSION

The allocation of responding between problem behavior and appropriate behavior


PROBLEM BEHAVIOR/
TOTAL BEHAVIOR
REINFORCED PROBLEM BEHAVIOR/
TOTAL REINFORCERS
LOG PROBLEM BEHAVIOR/
LOG APPROPRIATE BEHAVIOR
y = 1.203x - 0.118 r² = 0.997
LOG REINFORCED PROBLEM BEHAVIOR/
LOG REINFORCED APPROPRIATE
BEHAVIOR
DAN
PROBLEM BEHAVIOR/
TOTAL BEHAVIOR
REINFORCED PROBLEM BEHAVIOR/
TOTAL REINFORCERS
ATTENTION
ONLY
ALL
REINFORCERS
PROBLEM BEHAVIOR/
TOTAL BEHAVIOR
REINFORCED PROBLEM BEHAVIOR/
TOTAL REINFORCERS
LOG PROBLEM BEHAVIOR/
LOG APPROPRIATE BEHAVIOR
y = 0.145x + 0.510 r² = 0.085
ATTENTION
ONLY
LOG REINFORCED PROBLEM BEHAVIOR/
LOG REINFORCED APPROPRIATE
BEHAVIOR
LOG PROBLEM BEHAVIOR/
LOG APPROPRIATE BEHAVIOR
y = 0.969x + 0.011 r² = 0.996
ALL
REINFORCERS
LOG REINFORCED PROBLEM BEHAVIOR/
LOG REINFORCED APPROPRIATE
BEHAVIOR

was evaluated from a matching perspective for 4 participants who engaged in severe problem behavior during interactions with primary care providers. Collectively, these results provide further support for the linear relation between relative rate of reinforcement and relative rate of responding described by the matching equation during natural human interactions. The matching equation accurately described response allocation between appropriate and inappropriate behavior for all participants.

In addition, the present results suggest that a failure to identify multiple reinforcers maintaining problem behavior may lead to inaccurate predictions using the matching law. The relative rates of attention following problem behavior erroneously suggested that participants engaged in proportionally more problem behavior than would have been predicted by the proportional reinforcer rates. However, when all sources of reinforcement were included in the analysis, the matching law provided an accurate description of response allocation for all participants. It is possible that deviations from matching with humans (see Pierce & Epling, 1983, for a review) may be explained by additional variables (i.e., reinforcers) that were not taken into account.

It should be noted that several difficulties arise when conducting matching analyses in uncontrolled environments. In laboratory research, matching has been most effective in describing behavior reinforced on concurrent VI schedules. However, the schedules of reinforcement in the course of human interactions are not pure VI schedules (Nevin, 1998).

Findings of the current investigation must be evaluated with caution for several reasons. First, the procedures described in the current investigation are limited in that the reinforcers that maintained appropriate behavior (e.g., compliance, requests for attention) were not empirically determined. However, the functional equivalence of problem behavior and appropriate communicative behavior has been demonstrated repeatedly in the behavior-analytic literature (Durand & Carr, 1991, 1992; Northup et al., 1991). We are currently evaluating procedures to identify reinforcers for appropriate communicative behavior when problem behavior is not amenable to standard functional analysis procedures.

Second, the possibility of reactivity effects during the descriptive analysis portion of the investigation must be considered. However, the problem of reactivity is a limitation of not only the current investigation but also of previous investigations in which descriptive observations were conducted (Martens & Houk, 1989; McDowell, 1981; Oliver et al., 1999; Thompson & Iwata, 2001; Vollmer et al., 2001).

A third potential limitation of the current investigation is the possibility that the rate

←

Figure 2. Scatter plots of predicted and observed response allocation between problem behavior and appropriate behavior for all participants (upper left panel). Log response ratios plotted against log reinforcer ratios for all participants. The linear equation depicts slope and bias (upper right panel). Scatter plots of predicted and observed response allocation between problem behavior and appropriate behavior for Dan with the reinforcer identified via functional analysis (middle left panel). Scatter plots of predicted and observed response allocation between problem behavior and appropriate behavior for participants whose problem behavior was multiply controlled (Mandy, Linda, and Max) with attention as the sole reinforcer (middle right panel). Log response ratios plotted against log reinforcer ratios for participants whose problem behavior was multiply controlled (Mandy, Linda, and Max) with attention as the sole reinforcer. The linear equation depicts slope and bias (lower left panel). Log response ratios plotted against log reinforcer ratios for participants whose problem behavior was multiply controlled (Mandy, Linda, and Max) with all reinforcers taken into account. The linear equation depicts slope and bias (lower right panel).

of responding might have driven the rate of obtained reinforcement. In concurrent-ratio arrangements, the relation between response rate and reinforcer rate is fixed. That is, reinforcer rate is determined by response rate. Therefore, it is not clear whether the rate of reinforcement drives the rate of responding or if the rate of responding drives the rate of reinforcement.

In one case, Dan, it is quite likely that a high response rate drove a high reinforcer rate to some degree (e.g., Baum, 1973). In other words, if the schedules in the natural environment were conceptualized as concurrent-ratio schedules, the rate of responding observed would directly influence the rate of reinforcement obtained. The ratio of reinforcers for problem behavior to reinforcers for appropriate behavior was 32.3:1, and the ratio of problem behavior to appropriate behavior was 32.6:1. Presumably, a similar phenomenon would be observed if the environmental event was someone saying "bless you" and the target response was sneezing. On the other hand, the 3 individuals with multiply controlled problem behavior demonstrated that the reinforcer must drive response rate to some degree. Most likely, some combination is true in most natural settings (Vollmer & Bourret, 2000).

In the current investigation, we did not directly manipulate the schedules of reinforcement derived from the descriptive analysis data. However, a critical test of the matching law will be to evaluate how behavior is allocated among concurrently available response alternatives when the approximated schedules of reinforcement from descriptive analyses are reversed, or otherwise altered, in an *experimental* analysis. We are currently conducting experiments to test this hypothesis. For example, if the approximated interval schedule of reinforcement in place for SIB was VI 20 s and the approximated schedule of reinforcement in place for appropriate behavior was VI 40 s, the relative proportion of problem behavior and appropriate behavior should switch when the reinforcement rates are switched. Also, if the interval schedules for both appropriate and problem behavior are equated, biases toward one response alternative could be identified. Similarly, preference among reinforcing consequences (e.g., attention and tangible items) may be assessed by arranging equal VI schedules in an experimental analysis. If the stimuli are equally preferred, one may conclude that similar rates of responding would be observed. However, if one stimulus were more preferred than another, differential response rates would identify subtle preferences.

The current study also suggests implications for evaluations of treatment integrity with care providers. Previous evaluations of treatment integrity have evaluated the effectiveness of functional communication training under optimal (i.e., 100% integrity) and less than optimal (e.g., 25% integrity) treatment values using arbitrarily selected schedules of reinforcement (Vollmer, Roane, Ringdahl, & Marcus, 1999; Worsdell, Iwata, Hanley, Thompson, & Kahng, 2000). Similar analyses may be conducted using the procedures described in the current investigation by evaluating response allocation under schedules derived from descriptive analyses while systematically identifying a concurrent schedule that favors appropriate behavior. Although care providers would not be encouraged to reinforce problem behavior at any time, treatment integrity could be addressed proactively.

We evaluated one variable that has been shown to influence response allocation on concurrent schedules of reinforcement (i.e., reinforcement rate). Future research could replicate the procedures described in the current investigation and evaluate additional variables shown to influence choice, such as quality of reinforcement, delay to reinforcement, or magnitude of reinforcement. Under some circumstances (e.g., life-threatening SIB), it may be extremely difficult, if not impossible, to alter the rate of reinforcement for SIB. However, it may be possible to alter other variables known to influence choice, such as duration or quality of a reinforcing consequence.

The data from the current study provide support for a matching account of human choice responding. However, the data also highlight the issue of correlation versus causation when applying the matching equation without experimental manipulation. Despite the limitations inherent to naturalistic and descriptive accounts of human behavior, such research is necessary to assess the generality of behavioral principles outside the controlled laboratory environment.

REFERENCES

Baum, W. M. (1973). The correlation-based law of effect. *Journal of the Experimental Analysis of Behavior, 20,* 137–153.

Baum, W. M. (1974a). Choice in free-ranging wild pigeons. *Science, 185,* 78–79.

Baum, W. M. (1974b). On two types of deviation from the matching law: Bias and undermatching. *Journal of the Experimental Analysis of Behavior, 22,* 231–242.

Baum, W. M. (1979). Matching, undermatching, and overmatching in studies on choice. *Journal of the Experimental Analysis of Behavior, 32,* 269–281.

Beardsley, S. D., & McDowell, J. J. (1992). Application of Herrnstein's hyperbola to time allocation of naturalistic human behavior maintained by naturalistic social reinforcement. *Journal of the Experimental Analysis of Behavior, 57,* 177–185.

Carr, E. G., & McDowell, J. J. (1980). Social control of self-injurious behavior or organic etiology. *Behavior Therapy, 11,* 402–409.

Catania, A. C. (1998). *Learning* (4th ed.). Upper Saddle River, NJ: Prentice Hall.

Conger, R., & Killeen, P. (1974). Use of concurrent operants in small group research. *Pacific Sociological Review, 17,* 399–416.

Davison, M., & McCarthy, D. (1988). *The matching law: A research review.* Hillsdale, NJ: Erlbaum.

Durand, V. M., & Carr, E. G. (1991). Functional communication training to reduce challenging behavior: Maintenance and application in new settings. *Journal of Applied Behavior Analysis, 24,* 251–264.

Durand, V. M., & Carr, E. G. (1992). An analysis of maintenance following functional communication training. *Journal of Applied Behavior Analysis, 25,* 777–794.

Herrnstein, R. J. (1961). Relative and absolute strength of response as a function of frequency of reinforcement. *Journal of the Experimental Analysis of Behavior, 4,* 563–573.

Horner, R. D., & Keilitz, I. (1975). Training mentally retarded adolescents to brush their teeth. *Journal of Applied Behavior Analysis, 8,* 301–309.

Iwata, B. A., Dorsey, M. F., Slifer, K. J., Bauman, K. E., & Richman, G. S. (1994). Toward a functional analysis of self-injury. *Journal of Applied Behavior Analysis, 27,* 197–209. (Reprinted from *Analysis and Intervention in Developmental Disabilities, 2,* 3–20, 1982)

Iwata, B. A., Pace, G. M., Cowdery, G. E., & Miltenberger, R. G. (1994). What makes extinction work: An analysis of procedural form and function. *Journal of Applied Behavior Analysis, 27,* 131–144.

Iwata, B. A., Pace, G. M., Dorsey, M. F., Zarcone, J. R., Vollmer, T. R., Smith, R. G., Rodgers, T. A., Lerman, D. C., Shore, B. A., Mazaleski, J. L., Goh, H., Cowdery, G. E., Kalsher, M. J., McCosh, K. C., & Willis, K. D. (1994). The functions of self-injurious behavior: An experimental-epidemiological analysis. *Journal of Applied Behavior Analysis, 27,* 215–240.

Mace, F. C., Neef, N. A., Shade, D., & Mauro, B. C. (1994). Limited matching on concurrent-schedule reinforcement of academic behavior. *Journal of Applied Behavior Analysis, 24,* 719–732.

Martens, B. K., & Houk, J. L. (1989). The application of Herrnstein's law of effect to disruptive and on-task behavior of a retarded adolescent girl. *Journal of the Experimental Analysis of Behavior, 51,* 17–27.

McDowell, J. J. (1981). On the validity of and utility of the Herrnstein's hyperbola in applied behavior analysis. In C. M. Bradshaw, E. Szabadi, & C. F. Lowe (Eds.), *Quantification of steady-state operant behaviour* (pp. 311–324). Amsterdam: Elsevier/North Holland.

McDowell, J. J. (1988). Matching theory in natural human environments. *The Behavior Analyst, 11,* 95–108.

McDowell, J. J. (1989). Two modern developments in matching theory. *The Behavior Analyst, 12,* 153–166.

Myerson, J., & Hale, S. (1984). Practical implications of the matching law. *Journal of Applied Behavior Analysis, 17,* 367–380.

Neef, N. A., & Lutz, M. N. (2001). A brief computer-based assessment of reinforcer dimensions affecting choice. *Journal of Applied Behavior Analysis, 24,* 57–60.

Neef, N. A., Mace, F. C., & Shade, D. (1993). Impulsivity in students with serious emotional disturbance: The interactive effects of reinforcer rate, delay, and quality. *Journal of Applied Behavior Analysis, 26,* 37–52.

Nevin, J. A. (1998). Choice and behavior momentum. In W. O'Donohue (Ed.), *Learning and behavior therapy* (pp. 230–251). Needham Heights, MA: Allyn and Bacon.

Noll, J. E. (1995). The matching law as a theory of choice in behavior therapy. In W. O'Donohue & L. Krasner (Eds.), *Theories of behavior therapy: Exploring behavior change* (pp. 129–144). Washington, DC: American Psychological Association.

Northup, J., Wacker, D., Sasso, G., Steege, M., Cigrand, K., Cook, J., & DeRaad, A. (1991). A brief functional analysis of aggressive and alternative behavior in an outclinic setting. *Journal of Applied Behavior Analysis, 24,* 509–522.

Oliver, C., Hall, S., & Nixon, J. (1999). A molecular to molar analysis of communicative and problem behavior. *Research in Developmental Disabilities, 20,* 197–213.

Piazza, C. C., Fisher, W. W., Hanley, G. P., Remick, M. L., Contrucci, S. A., & Aitken, T. L. (1997). The use of positive and negative reinforcement in the treatment of escape-maintained destructive behavior. *Journal of Applied Behavior Analysis, 30,* 279–298.

Pierce, W. D., & Epling, W. F. (1983). Choice, matching, and human behavior: A review of the literature. *The Behavior Analyst, 6,* 57–76.

Pierce, W. D., Epling, W. F., & Greer, S. M. (1981). Human communication and the matching law. In C. M. Bradshaw, E. Szabadi, & C. F. Lowe (Eds.), *Quantification of steady-state operant behaviour* (pp. 345–352). Amsterdam: Elsevier/North Holland.

Roane, H. S., Vollmer, T. R., Ringdahl, J. E., & Marcus, B. A. (1998). Evaluation of a brief stimulus preference assessment. *Journal of Applied Behavior Analysis, 31,* 605–620.

Schroeder, S. R., & Holland, J. G. (1969). Reinforcement of eye movement with concurrent schedules. *Journal of the Experimental Analysis of Behavior, 12,* 897–903.

Smith, R. G., Iwata, B. A., Vollmer, T. R., & Zarcone, J. R. (1993). Experimental analysis and treatment of multiply controlled self-injury. *Journal of Applied Behavior Analysis, 26,* 183–196.

Thompson, R. H., & Iwata, B. A. (2001). A descriptive analysis of social consequences following problem behavior. *Journal of Applied Behavior Analysis, 34,* 169–178.

Vollmer, T. R., Borrero, J. C., Wright, C. S., Van Camp, C., & Lalli, J. S. (2001). Identifying possible contingencies during descriptive analyses of severe behavior disorders. *Journal of Applied Behavior Analysis, 34,* 269–287.

Vollmer, T. R., & Bourret, J. (2000). An application of the matching law to evaluate the allocation of two- and three-point shots by college basketball players. *Journal of Applied Behavior Analysis, 33,* 137–150.

Vollmer, T. R., Roane, H. S., Ringdahl, J. E., & Marcus, B. A. (1999). Evaluating treatment challenges with differential reinforcement of alternative behavior. *Journal of Applied Behavior Analysis, 32,* 9–23.

Worsdell, A. S., Iwata, B. A., Hanley, G. P., Thompson, R. H., & Kahng, S. (2000). Effects of continuous and intermittent reinforcement for problem behavior during functional communication training. *Journal of Applied Behavior Analysis, 33,* 167–179.

Received April 17, 2001
Final acceptance October 31, 2001
Action Editor, Nancy Neef

STUDY QUESTIONS

1. What is the general prediction made by the matching law, and how does the current study extend previous research on matching?

2. Briefly summarize the general experimental sequence.

3. What events were observed during the descriptive analyses, and how were they measured?

4. Summarize the results of the functional analysis for all participants.

5. How were proportional response rates and reinforcement rates for problem behavior calculated?

6. Summarize the results of the matching analysis, particularly those depicted in the middle right panel of Figure 2.

7. What are some implications of the results for the development of treatments for problem behavior?

8. Describe some ways, aside from extinction or the manipulation of reinforcement schedules, by which response allocation toward problem behavior and appropriate behavior (both concurrently reinforced) may be influenced.

Questions prepared by Stephen North and David Wilson, The University of Florida