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EFFECTS OF VARYING SAMPLE- AND CHOICE-STIMULUS DISPARITY ON SYMBOLIC MATCHING-TO-SAMPLE PERFORMANCE

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Six pigeons were trained on a typical two-stimulus two-response symbolic matching-to-sample task involving the discrimination of sample and choice stimuli that were signaled by different probabilities that pixels in an area were lit on a computer screen located behind the response keys. The disparities of the sample and choice stimuli were systematically varied across five experimental parts by manipulating the probabilities of pixel illumination. Across conditions within parts, the ratio of reinforcers obtainable for matching responses was varied over five levels. A recent model of detection based on the discriminability between the stimulus–response relations and between response–reinforcer relations provided an effective description of the data. Consistent with this model, changes in the disparity of the sample stimuli led to decreases in stimulus–response discriminability and left response–reinforcer discriminability unchanged. Equally, changes in choice-stimulus disparity caused a decrease in estimates of response–reinforcer discriminability and not in stimulus–response discriminability. Parameter invariance was thus obtained, and the variables expected to affect these parameters (sample and choice stimulus disparity) were correctly identified. The reasons for the failure of two recently reported studies to support parameter invariance under this model are discussed.

Key words: detection, parameter invariance, matching to sample, stimulus discriminability, response–reinforcer discriminability, key peck, pigeons

One major goal in the evolution of behavioral models of signal detection has been to develop a model in which one parameter measures the effects of conditional (antecedent) stimuli on behavior and another parameter measures the effects of consequential reinforcement (or payoff). For the model to be useful, the parameters must be independent in the sense that variables that affect one should have no effect on the other. One such model was suggested by Alsop (1991) and Davison (1991). However, two recent empirical studies of this model (Alsop & Davison, 1991; Nevin, Cate, & Alsop, 1993) indicated an interaction between measures of the discriminability of the sample stimuli–response relations and the discriminability of the response–reinforcer relations. The present study was a systematic replication of the research reported by Nevin et al. and investigated the effects of varying both the sample-stimulus and the choice-stimulus disparity on detection performance.

A typical two-stimulus two-response signal-detection procedure begins a trial with the presentation of a sample stimulus (S_1 or S_2). Following a sample presentation, two concurrently available response alternatives (B_1 and B_2) are presented, and depending on their relation to the sample stimulus, are often designated as "correct" or an "error." A response is called correct, and is occasionally followed by the reinforcer, if it is a B_1 response following S_1 or a B_2 response following S_2. The response is called an error (and may be followed by a punisher such as a period of blackout) if it is a B_2 response following S_1 or a B_1 response following S_2. Figure 1 shows the four stimulus–response outcomes of the detection task.

The results from detection tasks have often been analyzed using Davison and Tustin's (1978) detection model based on the generalized matching law. They suggested that behavior following S_1 presentations is described, in logarithmic form, by

$$\log\left(\frac{B_w}{B_x}\right) = a_{r_1}\log\left(\frac{R_W}{R_Z}\right) + \log c + \log d_s, \quad (1a)$$

and following S_2 presentations by

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Responses

Stimuli	B_1	B_2
S_1	W RFT	X EXT
S_2	Y EXT	Z RFT

Fig. 1. The stimulus–response events of the animal analogue of the yes-no signal-detection procedure. W, X, Y, and Z denote the cells of the matrix. RFT and EXT denote reinforcement and extinction, respectively.

$$\log\left(\frac{B_Y}{B_Z}\right) = a_{r_2}\log\left(\frac{R_W}{R_Z}\right) + \log c + \log d_s. \quad (1b)$$

In Equations 1a and 1b, B_i and R_i denote the numbers of responses emitted and reinforcers obtained, respectively, in the cells of the stimulus–response matrix (Figure 1). The parameters a_{r1} and a_{r2} are the sensitivities of behavior ratios to changes in the ratio of reinforcers on S_1 and S_2 presentations, respectively, and log c is inherent bias. Stimulus discriminability is measured by the parameter log d_s. As the two conditional stimuli become more different, a subject's performance on S_1 and S_2 presentations becomes more biased towards B_w and B_z, respectively. When the stimuli are indiscriminable (i.e., $d_s = 1.0$), Equations 1a and 1b reduce to Baum's (1974) generalized matching law.

Behavioral studies of detection performance have questioned the ability of the Davison and Tustin (1978) model to describe the obtained data (e.g., Alsop, 1991; Alsop & Davison, 1991; Davison, 1991; Davison & Jenkins, 1985; Davison & McCarthy, 1987, 1989; McCarthy & Davison, 1982, 1984; Nevin et al., 1993). Their model has been criticized on a number of empirical and theoretical grounds: (a) its inability to provide invariant parameters; (b) its inability to define clearly the operations that should, and should not, affect parameter estimates; (c) its inability to provide a good description of performances when errors are also reinforced; and (d) the impossibility of logically generalizing the model to more than two conditional stimuli and two responses.

One final problem with the Davison and Tustin (1978) model lies with its calculation of log d_s. The effect of increasing either the difference between the sample stimuli or the difference between the choice stimuli is to increase the relative number of B_w and B_z responses and therefore to increase log d_s. As a result, Davison and Tustin's log d_s measure is unable to separate the effects of varying the sample stimuli and varying the choice stimuli, and as such cannot give a measure of sample-stimulus discriminability independent of choice-stimulus discriminabilty.

Alsop (1991) and Davison (1991) individually suggested a model that was designed to correct some problems in previous detection models. For the two-stimulus two-response case, Alsop's and Davison's models are identical and will be regarded as one, the Alsop-Davison model. The difference between their models is that Davison's model was designed for the more general $n \times m$ case. This model expresses the assumption that subjects may make errors about which response they emitted following which conditional stimulus, and hence about what stimulus–behavior relations are reinforced. Thus, from the purview of the subject, the distribution of obtained reinforcers in the detection matrix may be quite different from the distribution recorded by the experimenter. Performance ratios in the two-stimulus two-response case are given by

Responses - d_r

Stimuli - d_s		**B_1**	**B_2**
	S_1	R_W	R_W/d_r
	S_2	R_W/d_s	$R_W/d_s/d_r$

Fig. 2. The effects that reinforcers delivered in Cell W (R_w) have on the other cells of the matrix according to the Alsop (1991) and Davison (1991) model. Responses are denoted as B_i and stimuli as S_i, where i represents the two alternatives (1 and 2). The parameters d_s and d_r are stimulus–response and response–reinforcer discriminability, respectively.

$$\frac{B_w}{B_x} = c\left(\frac{d_s d_r R_w + R_z}{d_r R_z + d_s R_w}\right) \tag{2a}$$

following an S_1 presentation and by

$$\frac{B_y}{B_z} = c\left(\frac{d_r R_w + d_s R_z}{d_s d_r R_z + R_w}\right) \tag{2b}$$

following an S_2 presentation, where all variables are as above. In this model, stimulus–response discriminability (d_s) is no longer an additive bias term as in the Davison and Tustin (1978) model, but functions in a similar manner and symmetrically to the response–reinforcer discriminability, d_r. The Alsop-Davison model implies that the ratio of responses emitted in each of the cells in the presence of each stimulus will exactly equal (i.e., strictly match) the "perceived" (or apparently obtained) reinforcers. In this model, behavior in each of the four cells of the matrix shown in Figure 1 is influenced by both the reinforcers R_w and R_z according to whether stimulus–response or response–reinforcer errors (or both) occurred (see Figure 2). The model's assumptions and the distribution of perceived reinforcers across the cells have been extensively reviewed by Alsop (1991) and Davison (1991).

The two main studies that examined the adequacy of the Alsop-Davison model were those of Alsop and Davison (1991) and Nevin et al. (1993). Alsop and Davison trained 6 pigeons on a number of discrete-trials matching-to-sample tasks in which the stimuli to be discriminated were two intensities of white light. Across conditions, the reinforcer rates for correct responses (R_w for B_w and R_z for B_z) were varied over at least three levels with the stimulus-presentation probability held constant at .5. This same procedure was repeated with seven different pairs of light intensities ranked in terms of physical disparity by measures taken from a photosensitive resistor. The main finding reported from these data was the existence of a U-shaped function between response–reinforcer and stimulus–response discriminability. Alsop and Davison concluded that, as this lack of parameter invariance violated their model's assumptions, the model was inadequate in describing the data from their study.

Nevin et al. (1993) arranged a discrete-trials conditional discrimination procedure designed to investigate whether the three terms of the discriminated operant (antecedent stimuli, behavior, and consequences) were functionally equivalent in their effect on performance. Specifically, they arranged a procedure like a matching-to-sample task in which food reinforcers were obtained by responding to a key with a short latency on trials signaled by one stimulus and by responding to the same key with a longer latency on trials signaled by another stimulus, thus generating two discriminated operants. They then took each of the terms defining two discriminative operants and varied their definition independently across experimental conditions. The physical disparity of the green sample stimuli (S_1 and S_2) was varied over two levels of intensity in an attempt to degrade stimulus–response discriminability. The disparity between the response latencies (B_1 and B_2) was also varied across two levels, and this was assumed to be a way of degrading response–reinforcer discriminability. Across the four experimental parts, the frequency of reinforcement (R_1 and R_2) was varied over three to five levels. Therefore, a matching-to-sample task was arranged in which food reinforcers were obtained by responding to a key with a short latency on trials signaled by one stimulus and by responding to the same key with a longer latency on trials signaled by another stimulus.

The analyses from the fits of the data to the Alsop-Davison model indicated that increases in the S_1-S_2 difference resulted in increases in stimulus–response discriminability, as expected. Similarly, increases in the B_1-B_2 difference led to increases in response–reinforcer discriminability. However, at a small S_1-S_2 disparity, an increase in the B_1-B_2 disparity decreased stimulus–response discriminability, and at the large S_1-S_2 disparity, an increase in the B_1-B_2 disparity increased stimulus–response discriminability. Estimates of response–reinforcer discriminability were also affected by changes in the sample disparity: A large S_1-S_2 difference gave larger estimates of d_r than did the small-sample disparity (see Nevin et al., 1993, Figure 5). There was, therefore, an interaction between stimulus–response and response–reinforcer discriminability, and Nevin et al. concluded that the Alsop-Davison model was inadequate in describing their data. Overall, they concluded that variations in antecedent stimuli, behavior, and consequences were not functionally equivalent.

We do have, however, a number of methodological concerns about Nevin et al.'s (1993) study. First, the lengths of the two types of trial were unequal, due to the requirement of short- and long-latency responses. Perhaps as a result, a large inherent bias towards B_1 was found, and up to 4.2 times as many responses were emitted to Response 1 as to Response 2. Large biases towards short-latency responses under such conditions are well known (e.g., Shimp & Hawkes, 1974, as reanalyzed by Nevin, 1982). However, the effect of such large inherent biases on the operation of the Alsop-Davison model fits is unknown. Equations 2a and 2b show inherent bias (c) as a multiplier (as in the generalized matching law), but the logic of the model and the way that it deals with biases caused by differential reinforcer frequencies suggest that an inherent bias may not be a simple multiplier. Another problem with the use of response latencies relates to scalar expectancy theory and the scalar property of timing (Gibbon, 1977, 1991; Zeiler & Hoyert, 1989). The scalar property of timing is the increase in the variance of estimates of duration with increasing duration, which has been extensively documented in animal timing (e.g., Gibbon & Church, 1981). In Nevin et al.'s study, differing variances for the two response latencies could lead to unpredictable effects on estimates of model parameter values depending on where the criterion for counting one latency versus the other was set. For instance, in conditions in which there was a large B_1-B_2 difference, B_1 was defined as a response latency between 0 and 1.0 s, and B_2 was defined as a latency greater than 2.0 s. Apparently, responses with latencies between 1 and 2 s were discarded.

Finally, the possible lack of stability in the research reported by Nevin et al. (1993) is of concern. Subjects were studied for a maximum of 14 sessions, and this number was decided upon from preliminary data. It is possible that 14 sessions may not be sufficient for stable behavior to be achieved. The increase in discriminability in replicated conditions, discussed as a learning effect by Nevin et al., is possibly the result of collecting unstable data.

The present experiment, therefore, was designed to investigate parameter invariance and the adequacy of the Alsop-Davison model without some of the methodological problems that were evident in Nevin et al.'s (1993) study. Both the sample and choice stimuli used were from the same dimension (pixel density), and trial lengths were equal for both sample-stimulus presentations. A discrete-trials symbolic matching-to-sample task was used in which the S_1-S_2 difference, the B_1-B_2 difference, and the probability of reinforcement for matching responses were varied. The disparities of the sample and choice stimuli were altered systematically by varying the probability of pixels that illuminated the defined stimulus areas on a computer monitor across experimental parts. This arrangement provided an effective experimental design in which the effects of sample and choice disparity on the parameters of the detection models were able to be independently investigated.

METHOD

Subjects

Six experimentally naive homing pigeons, numbered 81 to 86, were maintained at 85% ±15 g of their free-feeding body weights by postsession feeding of mixed grain. Water and grit were freely available in the home cages throughout the experiment.

Apparatus

The experimental chamber was 380 mm high, 350 mm wide, and 350 mm in depth. On the front wall of the chamber were three plastic keys (40 mm by 35 mm) with centers 80 mm apart and 225 mm above the grid floor. All three keys were used in the experiment and could be operated by a peck exceeding 0.1 N. Behind the keys was an opening 250 mm wide and 90 mm in length, where a monochrome (white pixels on a black background) computer monitor was situated, its face 5 mm away from the keys. When operative, the stimuli were presented on three areas of the computer monitor directly behind the response keys. The stimulus areas were 50 pixels in width and in length but appeared rectangular (as did the individual pixels) because of the aspect ratio of the screen. Responses to keys when the monitor area behind them was dark had no scheduled consequences and were not recorded.

Situated beneath the center key and 20 mm above the grid floor was a food tray (50 mm wide, 35 mm long, and 20 mm deep) that provided access to wheat. On top of the chamber was a container filled with wheat that was attached to a stepper motor tuned to deliver an average of two to three grains of wheat. The wheat fell through a tube (10 mm diameter) running down the side of the chamber into the food tray. Food reinforcement consisted of access to the delivered wheat, during which time the computer monitor was darkened and the food tray was illuminated by a yellow light-emitting diode (LED) for 8 s. When the reinforcement period ended, the magazine light was extinguished, and the center stimulus area was illuminated.

A yellow LED situated in the center of the ceiling acted as a houselight and provided a constant source of illumination in the chamber. An exhaust fan attached to the outside of the chamber provided ventilation and helped to mask external noise. All experimental events and data recording were controlled by an IBM®-compatible personal computer, programmed in Pascal, that was situated in a room remote from the experimental chamber.

Procedure

Initial training. The subjects were initially trained in a standard three-key experimental chamber with similar dimensions to those used for the experiment proper. They were trained to eat from the food magazine and then to peck all three keys using an autoshaping procedure (Brown & Jenkins, 1968). Subjects were then placed in the chamber described above with one difference—a touch screen was used in place of the response keys. Once the subjects had been trained to emit pecks directly on the screen, single and concurrent variable-interval schedules were arranged, and the size and location of the stimuli signaling the alternatives were varied along with variations in the relative reinforcer rate (totaling 67 sessions).

A discrete-trials matching-to-sample task was then arranged for the remainder of the preliminary training. The events of a trial are discussed in detail below, but for pretraining there were two differences. First, a correction procedure was used initially such that, following an incorrect response, the same sample stimulus was repeated until a correct response had been emitted. Second, sample and choice stimuli were identical, whereas in the experiment proper the choice stimuli symbolically related to the sample stimuli. The matching-to-sample task pretraining consisted of 44 sessions in which the disparity of the sample and the choice stimuli and the frequency of reinforcement were varied. Condition 1 proper began, and then the touch screen was replaced as critical areas of the screen became insensitive to pecks. The plastic response keys were placed over the three areas on which the sample and choice stimuli were presented. Data collected for Condition 1 that used the touch screen were discarded.

The experiment consisted of five parts. The sequence of experimental parts and con-

Table 1

Sequence of experimental conditions, probabilities of the sample- and choice-stimulus areas being filled with pixels, probability of reinforcement for a correct S_1 response, and the number of training sessions in each condition. Note that Conditions 1 to 26 and 47 to 51 do not appear in this table because they were designed for another, but similar, experiment.

Part	Condition	Sample-stimuli pixel probabilities	Choice-stimuli pixel probabilities	Probability of S_1 reinforcement	Number of sessions
1	27	.35, .65	.35, .65	.5	25
	28			.1	22
	29			.9	27
	30			.3	21
	31			.7	23
2	32	.35, .65	.45, .55	.5	20
	33			.3	20
	34			.9	35
	35			.1	23
	36			.7	27
3	37	.45, .55	.35, .65	.5	23
	38			.1	28
	39			.7	23
	40			.3	21
	41			.9	20
4	42	.50, .50	.35, .65	.3	22
	43			.9	27
	44			.1	22
	45			.7	21
	46			.5	33
	52			.5	26
5	53	.45, .55	.45, .55	.5	22
	54			.1	21
	55			.9	27
	56			.3	23
	57			.7	25

Table 2

Logarithmic estimates of stimulus–response discriminability, response–reinforcer discriminability, and inherent bias for each subject and the group data from fits to Equations 2a and 2b for each part. The percentage of data variance accounted for by the model (%VAC) and the mean square error (MSE) are indicators of the goodness of fit.

Bird	Part	log d_s	log d_r	log c	%VAC	MSE
81	1	0.763	1.072	−0.170	93	0.011
	2	0.612	0.320	−0.019	92	0.002
	3	0.101	0.765	−0.100	96	0.003
	4	0.003	0.482	−0.021	95	0.002
	5	0.000[a]	0.051	−0.011	15	0.003
82	1	1.371	0.989	−0.033	99	0.002
	2	1.168	0.321	−0.015	89	0.005
	3	0.544	1.131	−0.203	97	0.004
	4	0.000[a]	1.130	−0.026	99	0.001
	5	0.401	0.396	0.023	97	0.001
83	1	1.669	1.370	−0.064	99	0.002
	2	1.134	0.491	0.044	97	0.003
	3	0.672	1.214	−0.084	97	0.005
	4	0.018	0.963	−0.012	99	0.001
	5	0.423	0.404	0.053	94	0.002
84	1	1.247	1.110	0.004	99	0.003
	2	0.912	0.366	0.083	86	0.007
	3	0.543	1.453	−0.048	99	0.001
	4	0.021	0.852	−0.052	98	0.001
	5	0.323	0.198	0.111	59	0.006
85	1	1.018	0.969	−0.100	94	0.012
	2	1.381	0.203	0.081	66	0.009
	3	0.241	0.827	0.058	97	0.003
	4	0.000[a]	0.651	0.038	96	0.002
	5	0.183	0.021	0.001	10	0.001
86	1	1.360	0.654	−0.006	91	0.015
	2	1.074	0.195	−0.024	92	0.001
	3	0.374	0.744	−0.020	99	0.001
	4	0.000[a]	0.507	−0.018	98	0.001
	5	0.311	0.212	0.004	98	0.001

[a] Parameter values that were constrained to a logarithmic value of zero.

ditions, and the number of training sessions conducted in each, are shown in Table 1. The general procedure common to all parts of the experiment will be described first, and the specific manner in which the parts differed will then be described.

A discrete-trials procedure was employed throughout, and the sequence of events within a trial was as follows: A trial began with the illumination of the center area with a certain density of pixels. The two sample stimuli presented had probabilities $p(S_1)$ and $p(S_2) = 1 - p(S_1)$ of individual pixels being turned on (Table 1). The presentation of the two samples was randomized across trials with a probability of .5. A single peck on the center key resulted in the illumination of the two side (choice) areas, without extinguishing the sample. The location of the two choice stimuli on the left and right areas was randomized across trials ($p = .5$). The pixel-probability densities of the sample and choice stimuli are shown in Table 1. Because the densities of the stimuli were determined by probabilities, the stimuli differed from trial to trial, preventing the subjects from identifying a stimulus based on patterns of pixels. The first response emitted on either side key extinguished all pixels on the screen. If the peck had been to the choice key that symbolically matched the sample (defined as the least dense sample

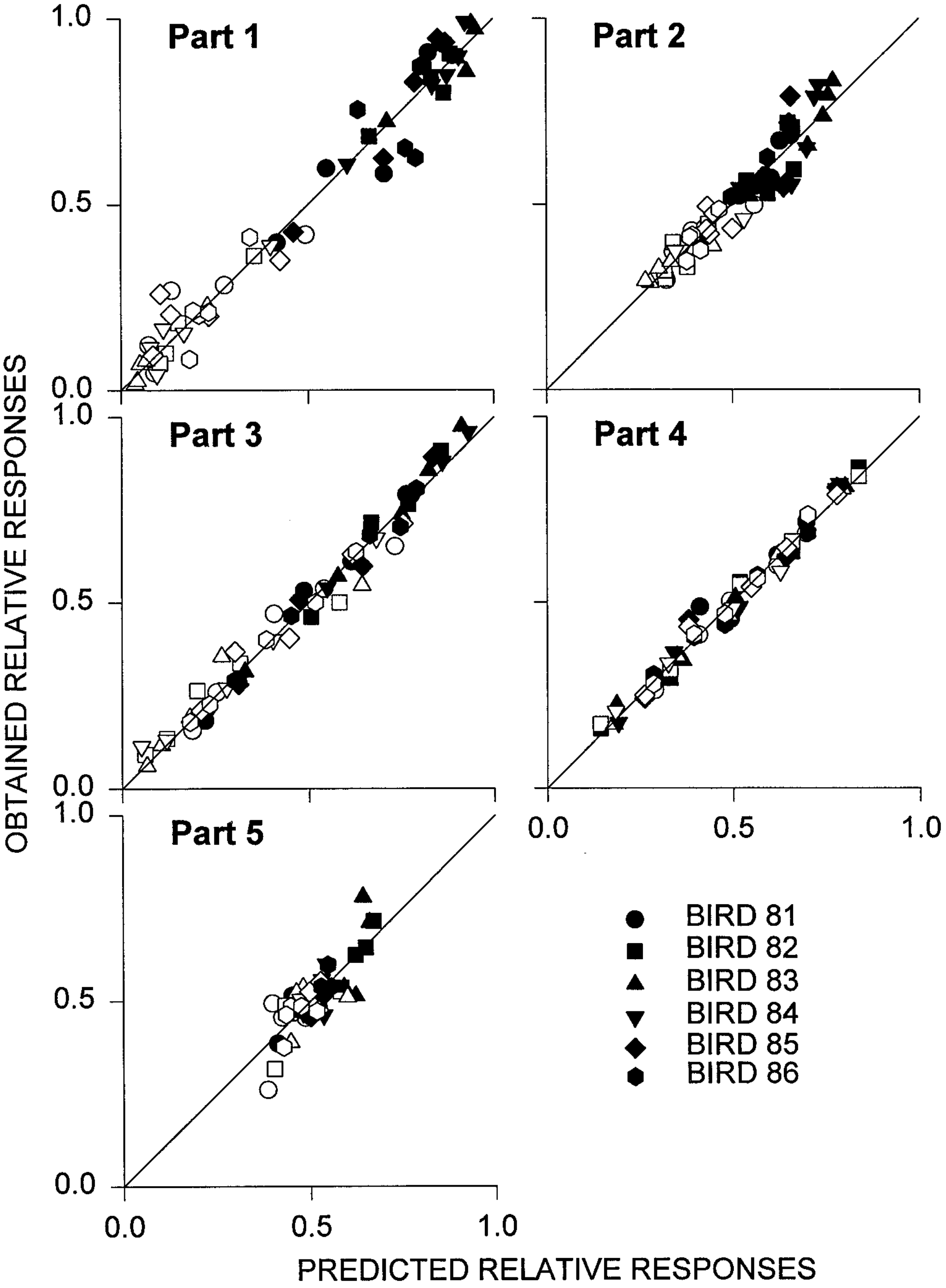

Fig. 3. Obtained relative response rates plotted as a function of predicted relative response rates from the Alsop (1991) and Davison (1991) model for Stimulus 1 (closed circles) and Stimulus 2 (open circles). The data from all subjects in Parts 1 to 5 are shown. The diagonal line represents perfect prediction of the obtained behavior ratios.

	Pt 1 .3 d_r	Pt 2 .3 d_s	Pt 2 .1 d_r	Pt 3 .1 d_s	Pt 3 .3 d_r	Pt 4 0 d_s	Pt 4 .3 d_r	Pt 5 .1 d_s	Pt 5 .1 d_r
Pt 1 .3 d_s			*	*		*	*X	*	*
Pt 1 .3 d_r			*	*		*		*	*
Pt 2 .3 d_s			*	*		*	*X	*	*
Pt 2 .1 d_r					*	*	*		
Pt 3 .3 d_s					*X	*	*X	*	*
Pt 3 .3 d_r						*	*X	*	*
Pt 4 0 d_s							*	X	*
Pt 4 .3 d_r								*	*
Pt 5 .1 d_s									

Fig. 4. Significant pairwise differences between parameter estimates for all parts of the experiment. Each column or row gives the part of the experiment (Pt), the pixel-probability difference, and whether the parameter in the comparison is d_s or d_r. Asterisks show significant differences on a sign test. The finding of a difference, or a lack of difference, that is incompatible with the model of Equations 2a and 2b is indicated by an X.

matching the least dense choice stimulus) and a food reinforcer was available, the food tray was illuminated for 8 s and two or three grains of wheat fell into it. When a food reinforcer was not scheduled, a matching response led to the start of a new trial and was not reinforced. Nonmatching responses were followed by the darkening of the stimulus areas for 5 s.

Food reinforcers were arranged as follows: At the start of each session and after every reinforcer delivery, a food reinforcer was allocated to the next correct S_1 or S_2 response with a fixed probability (see Table 1). This reinforcer remained available, and no more were arranged, until that reinforcer was delivered. This is a controlled reinforcer-ratio procedure (McCarthy & Davison, 1984, 1991). Within each part of the experiment, the relative probability of S_1 and S_2 reinforcers was varied across conditions (Table 1).

A new trial (i.e., the illumination of the center-key area) began 5 s after food presentation or after an error response had been emitted, or immediately after a nonreinforced correct response. During the 5-s blackouts, all stimulus areas were extinguished and responses were ineffective and were not recorded. A noncorrection procedure was in effect throughout the experiment, with presentations of the samples on the center-key area on any given trial independent of both the sample and the accuracy of choice on the preceding trial. The houselight remained on throughout the session.

Experimental sessions were conducted 7 days a week. Each training session began in blackout and ended in blackout either after the delivery of 40 food reinforcers or after 40 min had elapsed, whichever event occurred first. The data collected were the numbers of correct and error responses to each sample stimulus and the numbers of food reinforcers obtained from matching responses. The data recorded from a trial were given one of 32 codes that was determined by which sample stimulus was presented, which stimulus appeared on the left or right choice area, whether or not a reinforcer was arranged, and, if a reinforcer was arranged, whether it

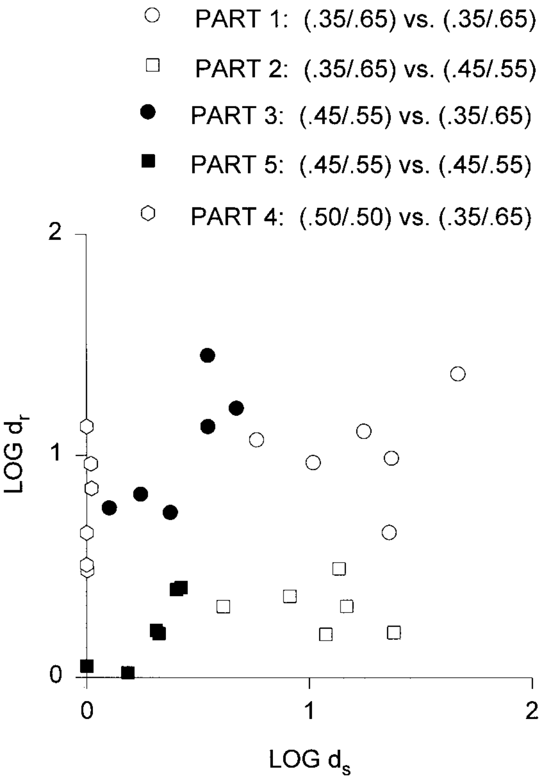

Fig. 5. Group response–reinforcer discriminability values plotted as a function of group stimulus–response discriminability for all conditions of the experiment.

was obtained by the subject. The times of each event were also recorded, but they were not used in the present paper.

Experimental conditions were changed once each of the 6 birds had met the following stability criteria five, not necessarily consecutive, times: Following 10 training sessions, the proportion of correct responses to Stimulus 1 was calculated for each session across successive blocks of three sessions. The criterion required that there were no monotonic increasing or decreasing trends in proportion correct. As a result, the minimum number of training sessions in each condition was 17. The number of training sessions in each condition ranged from 20 to 35, with a mean of 24 averaged across all conditions.

Across the five parts of the experiment, the stimulus disparity of both the sample and choice stimuli was varied by altering the probabilities of illuminating pixels. Within each part, five different relative-reinforcer frequencies were arranged, ranging from .1 to .9. Experimental conditions were replicated once in Part 4 to check on the stability of the data collected. In Part 1, the pixel probabilities of both the S_1 sample stimuli and the choice stimuli were .35 and .65. In Part 2, the sample pixel probabilities were .35 and .65, and the choice pixel probabilities were .45 and .55. The Part 2 sample and choice probabilities were reversed in Part 3. In Part 4, identical probabilities (.50 and .50) were arranged for the sample stimuli while the choice probabilities were .35 and .65. Finally, in Part 5, both sample and choice pixel probabilities were .45 and .55.

RESULTS

Unless stated otherwise, the data used in the analyses to follow were the numbers of correct (B_w and B_z) and incorrect (B_x and B_y) responses to the two sample stimuli and the numbers of reinforcers obtained (R_w and R_z) for correct choice responses. The data were summed over the last five sessions of each condition and are shown in full in the Appendix. To correct for any zero cells, 0.5 was added to the summed behavior (B_w, B_x, B_y, and B_z) for all conditions, as suggested by Hautus (1995).

For each subject, the data from each experimental part were analyzed using the Alsop-Davison signal-detection model as shown in Equations 2a and 2b. Logarithmic behavior ratio measures (log B_w/B_x and log B_y/B_z) were fitted to these equations using the Microsoft Excel Solver®, a nonlinear iterative fitting program. Estimates of d_s, d_r, and c were calculated such that the sum of deviations between obtained and predicted logarithmic behavior ratios was minimized simultaneously using both Equations 2a and 2b. The range of normal values of d_s and d_r is one to infinity (no discrimination to perfect discrimination), but occasionally values of less than one were obtained. A systematic occurrence of such estimates would suggest that the model was wrong, but in only 4 cases (of 72) was a parameter value less than one obtained. All these estimates were of d_s, and they were for Bird 81 in Part 5 and for Birds 82, 85, and 86 in Part 4. For these fits only, the final fit reported here constrained the value of d_s to 1.0 (or greater) in order to provide the best estimate of d_r and c. The Solver was set to run until it found a suitable solution using linear extrapolation from a tangent vector to obtain initial estimates of the parameters, forward differencing for estimates of the partial derivatives, and a quasi-Newton method for computing the search direction. Default settings of tolerance (0.05) and precision (0.000001) were used for the solution process.

The results of these analyses are shown in Table 2. Log discriminability and inherent bias values are shown in this table, and will be reported hereafter, to facilitate plotting. The percentage of variance accounted for by the model was greater than 90% in all but seven fits, and four of these occurred in Part 5. A better measure of the adequacy of the fits (one that is independent of the size of the d_s and d_r parameters) is given by the mean square error between the log data and predictions, and these values were less than 0.02 in all cases. In summary, the fits of the data to the model were very good. This is confirmed by the plots, in Figure 3, of the obtained relative responses (B_1 responses divided by total responses) as a function of predicted relative responses for performances in S_1 and S_2. If the fitted model predicted perfectly, the data would fall on the major diagonals. Figure 3 shows that there were no systematic deviations between the obtained and predicted relative behavior measures, and that performance in the presence of S_1 and S_2 was described equally well. There appears to be a possible systematic deviation from the main diagonal for Part 5, but a linear regression between the predicted and obtained data had a slope of 1.003 with a standard error of 0.1, so there was no systematic deviation. As Table 2 shows, across all parts there was no preference for one response over another, with inherent bias (c) being small and unsystematic.

In Part 1, the differences between the pixel probabilities were .3 for both the stimuli and the response choices (both .35 vs. .65). Sign tests showed that there was no significant difference (p = .05) between these discriminability estimates, with mean values of log d_s and log d_r of 1.24 and 1.03. Equally, in Part 5, both pixel-probability differences were .1 (.45 vs. .55), and again on the same test, there was no significant difference between the estimates (log d_s and d_r, respectively, averaged 0.27 and 0.21). However, on sign tests, both the d_s and d_r values between Parts 1 and 5 were significantly different at p = .05. In Part 2, d_s values (pixel difference .3) were significantly greater than d_r values (pixel difference .1), with mean values of 1.05 and 0.31. Similarly, in Part 3, d_s values (pixel difference .1, mean discriminability 0.41) were significantly smaller than d_r values (pixel difference .3, mean discriminability 1.02).

There are many other comparisons that can be made with the present data to check whether parameter estimates for the same physical disparity (whether stimulus or choice disparity) were not significantly different, and to check whether estimates for different physical disparities were significantly different. The full set of binary comparisons is shown in Figure 4. In this figure, significant differences on a sign test (which requires the six comparisons for each part to be all be in the same direction) are shown. Of the 45 possible binary comparisons, only six were incompatible with the Alsop (1991) and Davison (1991) model. Five of these contrary examples were significant differences where there should have been none, and four of these involved the .3 pixel difference for choices arranged in Part 4 when the pixel difference for stimuli was zero. With seven possible comparisons in this particular data set, four contrary examples do not reach statistical significance, although the pattern may be indicative. Overall, 39 binary comparisons were consistent with the model out of a total of 45, showing significant support for the model.

These results are shown in Figure 5, where obtained log d_r values are plotted as a function of log d_s values. There is an apparent correlation between these two parameter estimates, but a linear regression between them gave a slope of 0.13 with a standard error of 0.15. Thus, the relation between them was not significant.

Overall, the analyses showed that estimates of log d_s were ordered appropriately with mean estimates of 0.01 (zero pixel difference), 0.34 (.1 difference), and 1.14 (.3 difference). Similarly, mean estimates of d_r were 0.26 (.1 difference) and 0.94 (.3 difference).

DISCUSSION

The present experiment systematically varied both stimulus–response and response–reinforcer discriminability by changing the arranged physical disparity between the sample stimuli and between the choice stimuli, respectively. Fits of the present data to the Alsop-Davison model showed that the percentage of data variance accounted for by these fits was high, indicating that the data were well described by the model (see Table 2). Such a result adds further evidence that the Alsop-Davison model is an effective descriptor of 2 × 2 detection performance, supporting the past literature (e.g., Alsop, 1991; Alsop & Davison, 1991; Davison, 1991; Nevin et al., 1993). However, although the model may fit data well, the model is an effective descriptor only inasmuch as two further conditions are met. First, measures of discriminability should be directly related to the physical disparity of the conditional and choice stimuli. This was demonstrated by both Alsop and Davison and by Nevin et al. Second, discriminability measures should be invariant: For a particular disparity, each stimulus–response or response–reinforcer measure should not be affected by the disparity of, respectively, the response–reinforcer or the stimulus–response disparity. The Alsop-Davison model was found lacking on this second criterion by both Alsop and Davison and Nevin et al.

The present research found a clear and significant relation between stimulus disparity and measures of discriminability, be they stimulus–response or response–reinforcer discriminability. This result supports the finding by Alsop and Davison (1991) and Nevin et al. (1993) that discriminability measures from the Alsop-Davison model behave in an appropriate way. Further, the present data also showed overall that measures of stimulus–response and response–reinforcer discriminability did not differ significantly whether a particular stimulus disparity was arranged as a conditional stimulus pair or as a choice-stimulus pair (Figure 4). This finding is an important validation of the symmetrical way in which the Alsop-Davison model deals with stimulus–response and response–reinforcer discriminability.

Although measures of stimulus disparity differed appropriately according to the physical stimulus disparity, they did not differ significantly (except in four comparisons, see Figure 4) as a function of the alternate (conditional stimulus or choice) disparity in the situation. Thus, measures of discriminability did, in the present research, show the invariance required by the model.

Parameter invariance is assumed theoretically by all behavioral models of detection but has not always been demonstrated empirically. Alsop and Davison (1991), for example, reported a U-shaped function between stimulus–response discriminability and response–reinforcer discriminability when their data were analyzed using Equations 2a and 2b. We reanalyzed these data, with the Hautus (1995) correction added, and with constraints that did not allow discriminability estimates to be less than one. The need for constraints was not mentioned in Alsop and Davison's paper. A transformation of these estimates into log d_s and log d_r was made in order to compare these values directly with those presented by Alsop and Davison. This reanalysis failed to show the U-shaped function reported by Alsop and Davison, and indeed parameter invariance was quite well supported. A post hoc Friedman test (Marascuilo & McSweeney, 1977) indicated that response–reinforcer discriminability between the 21 pairwise comparisons of sample stimuli sets differed statistically in only one case, between Sets D and G in the Alsop and Davison study ($p < .05$; $\Delta = 1.61$). Response–reinforcer discriminability was *not,* therefore, a clear function of stimulus–response discriminability. The data we obtained here (Figure 5) showed no evidence at all that stimulus–response and response–reinforcer discriminabilities are related.

The remaining data set that did not show parameter invariance was that of Nevin et al. (1993). Our major concern with these data is the clearly increasing discriminabilities throughout the experiment, indicating that performance might not have been stable. However, we do not wish to argue these results away because Nevin et al.'s findings may indicate limitations on the Alsop-Davison model, either (a) when conditional and choice stimuli are varied on different dimensions, or (b) when response–reinforcer discriminability is varied using response latency, or (c) when strong inherent biases are present and are unable to be correctly assimilated using the current model. It behooves us, then, to investigate the present data in the same way as analyzed by Nevin et al.

Figures 6 and 7 show the estimated log d_s

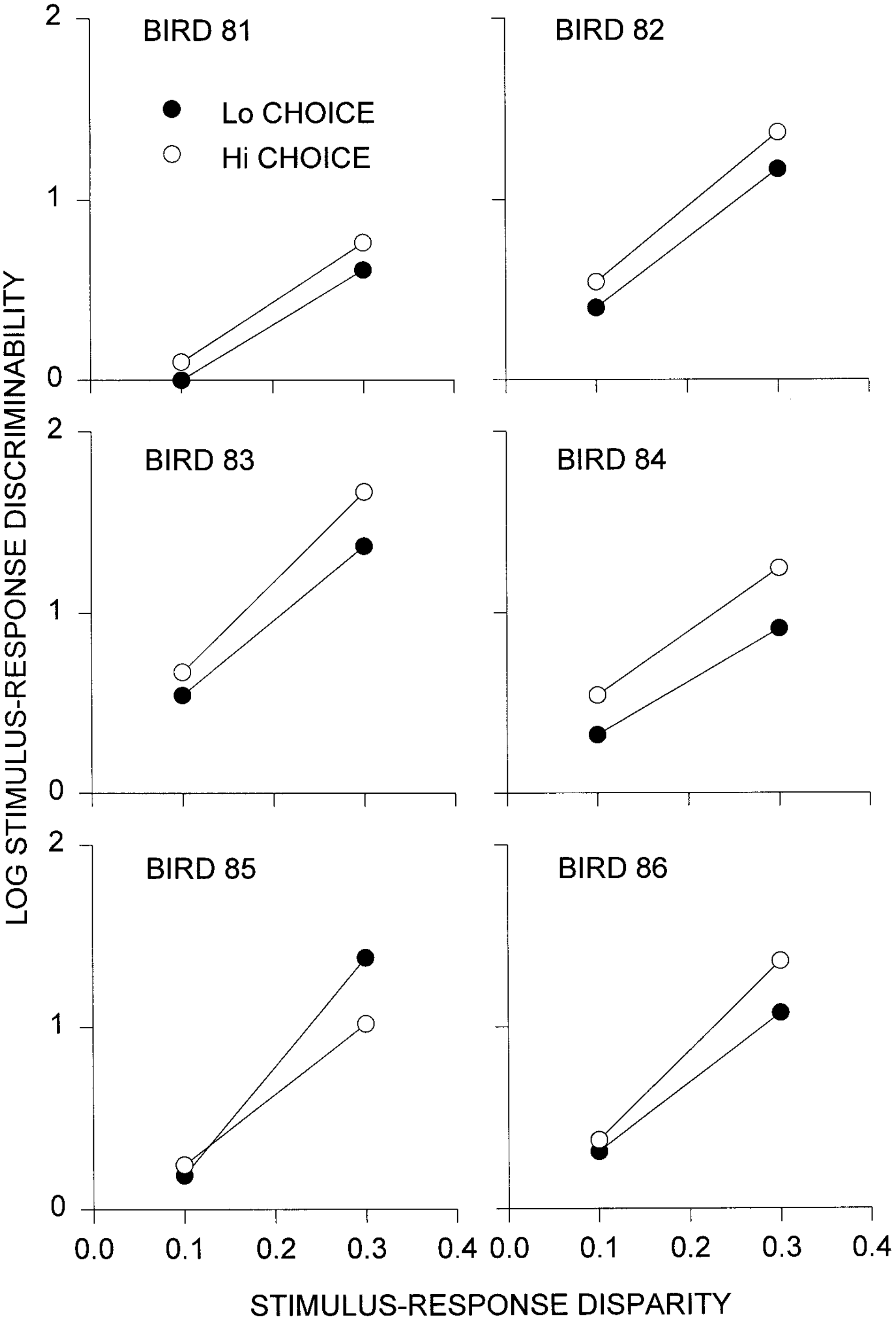

Fig. 6. Log stimulus–response discriminability values as a function of the physical stimulus disparity for high and low choice-stimulus disparity for Parts 1 to 3 and 5.

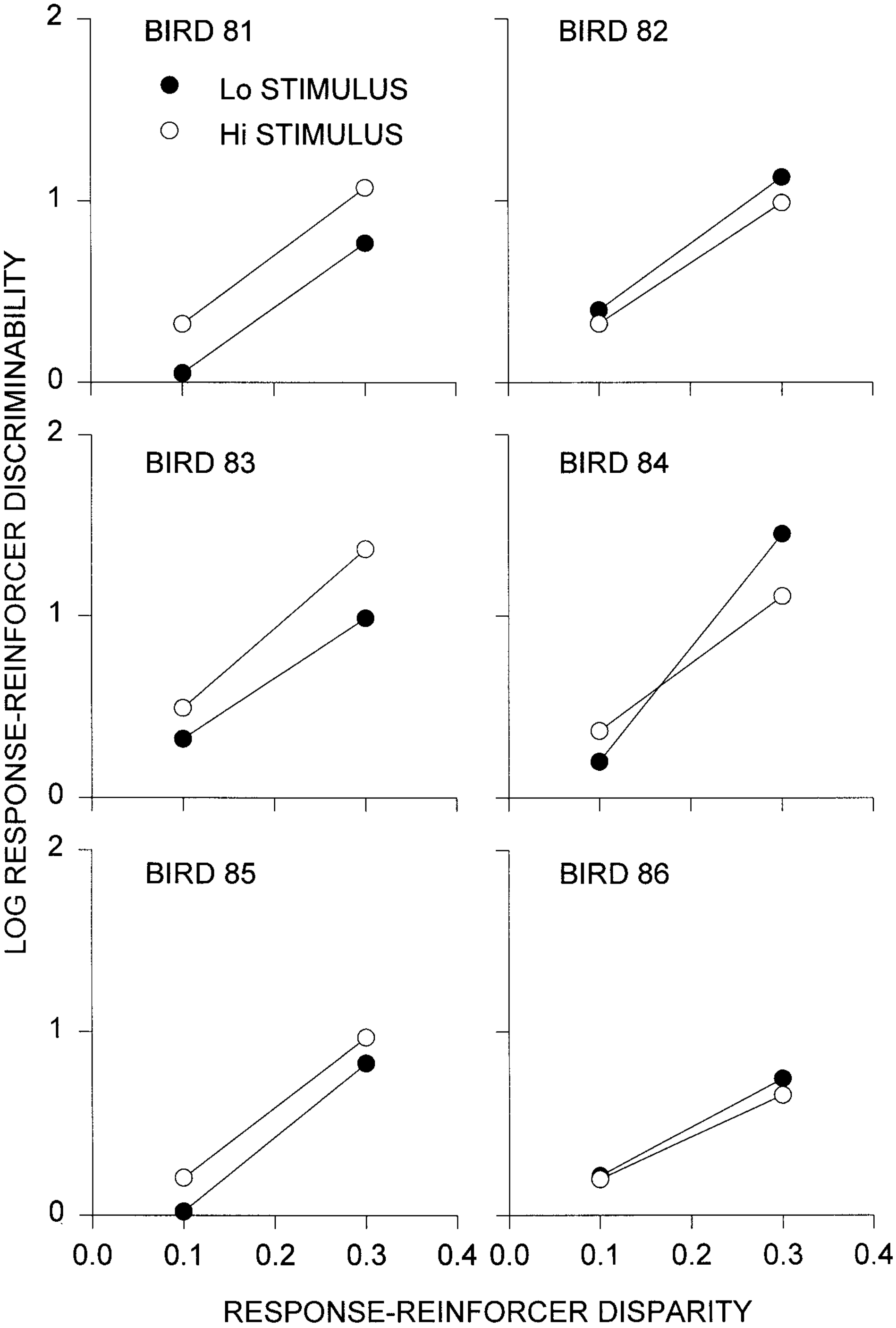

Fig. 7. Log response–reinforcer discriminability values as a function of the physical stimulus disparity for high and low sample-stimulus disparity for Parts 1 to 3 and 5.

and d_r values obtained in the present experiment plotted in the same way as in Nevin et al. (1993) (their Figure 5). Nevin et al. found that estimates of log d_s changed more sharply with changes in stimulus discriminability when there were large differences in the disparity of the responses. Figure 6 shows no such effect in the present data (i.e., the open-circle plots were not steeper than the filled-circle plots). Nevin et al. also found that log d_r values were larger for large stimulus disparities than they were for small disparities. Although Birds 81, 83, and 85 showed a similar effect here (Figure 7), the other subjects showed quite different effects. Overall, then, it is clear that the data from the present procedure were very different from those collected by Nevin et al., and that the present data support the Alsop (1991) and Davison (1991) model.

The difference in results between the present experiment and those of Nevin et al. (1993) thus remains unresolved. However, it is our view that the difference may have resulted from the third suggestion made above—the strong inherent bias toward the shorter latency alternative that occurred in Nevin et al.'s procedure. Although Alsop's (1991) and Davison's (1991) model seems to describe correctly the effects of bias caused by reinforcement-frequency differences, it deals with inherent bias in a way that is quite different from the way it deals with reinforcement bias. However, inherent bias can be caused by constant reinforcer differences (e.g., reinforcer magnitude differing between the alternatives). Equations 2a and 2b would have to deal with this as a nonunit value of c, whereas the logic of the model would require it to appear in the same parts of the equations as does R, the reinforcer frequencies. The model, therefore, requires further testing in conditions in which a constant bias is produced and maintained, and in which two sources of bias are manipulated.

From the above discussion, we suggest that no previous study has convincingly demonstrated a general lack of parameter invariance between the estimates of stimulus–response and response–reinforcer discriminability derived from fits to the Alsop-Davison model, and the present study supported parameter invariance. Particularly good support for the model are the results from the experimental arrangement of Part 4 in which physically identical sample-stimulus probabilities were arranged and therefore estimates of stimulus–response discriminability should have theoretically been at their minimum (1.0). The findings from this part (Table 2) show that this indeed occurred, with log stimulus–response discriminability measures close to zero for 3 subjects and constrained at zero for the 3 other subjects.

In sum, the model of Alsop (1991) and Davison (1991) provided excellent fits to the data obtained from this experiment. The theoretical assumptions of the model were also met empirically. This experiment therefore demonstrated that manipulations in the disparity of the sample stimuli only affected measures of stimulus–response discriminability and, similarly, manipulations in the disparity of the choice stimuli only affected measures of response–reinforcer discriminability.

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APPENDIX

The numbers of responses emitted and reinforcers obtained for each subject in all conditions of the experiment.

Bird	Condition	w	x	y	z	R_1	R_2
81	27	204	146	82	224	92	108
	28	103	156	29	212	31	169
	29	213	15	90	125	180	20
	30	152	102	11	228	52	148
	31	205	20	65	165	149	51
	32	175	132	130	174	108	92
	33	158	144	126	213	51	149
	34	191	87	143	144	181	19
	35	128	191	92	219	20	180
	36	202	99	149	193	133	67
	37	200	176	173	195	101	99
	38	62	279	49	266	27	173
	39	217	138	226	195	138	62
	40	92	227	93	267	51	149
	41	213	56	190	101	182	18
	42	187	197	177	254	69	131
	43	215	85	179	82	182	18
	44	94	238	88	247	21	179
	45	227	135	237	159	152	48
	46	183	220	217	215	102	98
	52	220	245	226	204	107	93
	53	205	186	213	219	94	106
	54	151	202	152	218	24	176
	55	191	204	204	212	178	22
	56	233	212	230	192	59	141
	57	201	197	216	188	146	54
82	27	189	47	21	194	102	98
	28	176	81	18	204	20	180
	29	208	23	85	150	185	15
	30	200	37	16	212	67	133
	31	260	27	38	172	137	63
	32	229	89	106	161	108	92
	33	147	131	92	216	41	159
	34	201	137	140	174	177	23
	35	169	131	93	197	21	179
	36	222	92	106	215	153	47
	37	177	70	73	205	108	92
	38	52	223	24	240	19	181
	39	196	60	94	186	142	58
	40	125	146	37	240	67	133
	41	213	21	137	137	180	20
	42	139	333	129	277	64	136
	43	262	41	268	51	180	20
	44	48	255	57	277	19	181
	45	256	146	272	138	140	60
	46	197	159	190	158	97	83
	52	214	196	209	199	112	88
	53	198	159	137	201	95	105
	54	115	168	86	208	18	182
	55	229	88	173	106	176	24
	56	185	127	124	202	75	125
	57	227	127	150	134	144	56

APPENDIX

(Continued)

Bird	Condi-tion	w	x	y	z	R_1	R_2
83	27	195	32	17	227	94	106
	28	131	50	3	202	13	187
	29	192	5	42	144	187	13
	30	173	20	5	207	59	141
	31	189	2	17	200	130	70
	32	219	78	98	201	105	95
	33	189	98	86	212	60	140
	34	218	44	95	150	176	24
	35	143	131	82	198	16	184
	36	224	58	100	190	136	64
	37	212	75	49	209	99	101
	38	71	156	14	222	21	179
	39	221	37	89	162	129	71
	40	174	131	28	215	55	145
	41	205	5	123	102	184	16
	42	142	274	142	259	66	134
	43	272	63	250	60	175	25
	44	70	244	49	239	22	178
	45	241	141	234	138	134	66
	46	199	190	177	205	101	99
	52	212	204	214	210	103	97
	53	210	131	138	191	96	104
	54	145	204	117	233	17	183
	55	217	56	155	108	175	25
	56	185	187	146	218	62	138
	57	217	101	168	165	140	60
84	27	210	37	37	188	91	109
	28	131	84	26	202	17	183
	29	196	1	86	135	183	17
	30	168	36	8	188	63	137
	31	208	23	34	187	137	63
	32	189	101	109	182	103	97
	33	176	142	121	202	63	137
	34	213	46	121	142	175	25
	35	172	144	123	206	17	183
	36	253	67	126	134	144	56
	37	217	54	69	186	117	83
	38	48	184	28	222	16	184
	39	231	31	109	164	144	56
	40	134	114	33	221	57	143
	41	225	9	184	89	180	20
	42	131	228	129	257	63	137
	43	274	60	266	64	180	20
	44	65	309	79	305	22	178
	45	240	150	209	150	143	57
	46	180	190	184	197	110	90
	52	216	212	209	217	102	98
	53	188	156	179	182	100	100
	54	199	222	224	200	15	185
	55	225	68	188	113	165	35
	56	225	205	190	183	62	138
	57	209	122	178	173	148	52

APPENDIX

(Continued)

Bird	Condi-tion	w	x	y	z	R_1	R_2
85	27	205	42	51	199	94	106
	28	89	120	23	225	19	181
	29	242	16	85	158	181	19
	30	171	103	71	203	58	142
	31	200	11	48	193	150	50
	32	195	153	153	212	93	107
	33	196	163	198	203	59	141
	34	205	54	114	149	182	18
	35	173	129	154	203	15	185
	36	193	75	136	151	140	60
	37	195	131	130	192	104	96
	38	84	217	59	225	26	174
	39	247	68	210	123	147	53
	40	165	160	123	212	59	141
	41	251	30	187	75	176	24
	42	184	223	174	228	56	144
	43	226	53	230	61	183	17
	44	66	209	77	233	20	180
	45	241	146	261	143	139	61
	46	192	163	198	169	94	79
	52	190	221	170	193	95	105
	53	202	172	176	173	101	99
	54	198	202	196	198	12	188
	55	205	209	197	190	181	19
	56	182	224	217	219	59	141
	57	205	182	187	194	153	47
86	27	186	111	57	225	102	98
	28	143	46	17	191	20	180
	29	215	31	93	133	178	22
	30	171	91	59	218	64	136
	31	201	29	48	181	136	64
	32	190	152	133	187	98	102
	33	187	153	142	204	68	132
	34	193	115	177	188	178	22
	35	145	134	109	206	19	181
	36	210	154	129	214	143	57
	37	239	111	134	201	112	88
	38	86	211	49	227	22	178
	39	227	95	155	155	147	53
	40	141	163	69	237	51	149
	41	213	51	183	104	169	31
	42	135	198	155	221	64	136
	43	252	115	278	100	180	20
	44	85	195	86	223	22	178
	45	213	160	200	155	129	71
	46	177	226	163	186	89	97
	52	206	218	181	201	97	103
	53	190	168	164	212	104	96
	54	129	170	119	213	11	189
	55	196	121	203	140	179	21
	56	184	204	172	188	56	144
	57	220	156	166	182	132	68