Text S1: The Axiomatic Basis of the Shapley Value

Let player i be a *null player* in v if $\Delta_i(S) = 0$ for every coalition S ($i \notin S$). Players i and j are *interchangeable* in v if $\Delta_i(S) = \Delta_j(S)$ for every coalition S that contains neither i nor j. The Shapley value is the only efficient value that satisfies the three following axioms:

Axiom 1 *(Symmetry) If i and j are interchangeable in game v then $\gamma_i(v) = \gamma_j(v)$.*

Intuitively, this axiom states that the value should not be affected by a mere change in the players' "names".

Axiom 2 *(Null player property) If i is a null player in game v then $\gamma_i(v) = 0$.*

This axiom sets the baseline of the value to be zero for a player whose marginal importance is always zero.

Axiom 3 *(Additivity) For any two games v and w on a set N of players, $\gamma_i(v+w) = \gamma_i(v)+\gamma_i(w)$ for all $i \in N$, where $v+w$ is the game defined by $(v+w)(S) = v(S) + w(S)$.*

This last axiom constrains the value to be consistent in the space of all games.