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# *CONDITIONAL DISCRIMINATION AND EQUIVALENCE RELATIONS: A THEORETICAL ANALYSIS OF CONTROL BY NEGATIVE STIMULI*

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A detailed analysis is presented of the ways in which control by the negative stimulus in two-comparison conditional discriminations may be expected to affect the outcome of tests for the properties of equivalence relations. Control by the negative stimulus should produce the following results: (a) no observable effect on symmetry tests; (b) reflexivity test results should look like "oddity" rather than "identity"; and (c) transitivity tests that involve an odd number of nodes should yield results that are 100% opposite to tests that involve an even number of nodes. The analysis also considers the effects of variation in the type of comparison-stimulus control between and within baseline conditional discriminations. Methods are suggested for experimentally regulating the type of control, and for verifying the predictions that the analysis generates. If suggested experiments continue to support the analysis, investigators who use two-comparison conditional discriminations to study equivalence relations will either have to control explicitly whether the positive or the negative comparison governs their subjects' choices, or they will have to abandon two comparisons and use three or more comparisons instead.

*Key words:* stimulus control, equivalence relations, matching to sample, conditional discrimination, theoretical analysis

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The controlling stimulus in a two-sample, two-comparison conditional discrimination may be either the positive or the negative comparison. The analysis presented here is concerned with the relevance of each kind of control to the experimental and theoretical analysis of equivalence relations. We provide a nearly exhaustive account of the test results to be expected when the baseline conditional discriminations involve one of the two kinds of control exclusively, in addition to results from tests based on mixtures of each kind of control. No new experiments are described here, although we cite findings (to be reported later in full) that help validate the utility of the analysis.

We have carried out this analysis for three major reasons: First, the analysis predicts many new findings, some important substantively and some methodologically. In other instances, the analysis forces the conclusion that certain combinations of control by positive and negative stimuli will preclude any predictability in experiments on equivalence relations.

Second, the analysis provides theoretical foundations for modifying some of the training and testing procedures that are currently used in research on equivalence relations. We show, for example, far more conclusively than Sidman (1987) indicated, how a failure to identify whether a two-comparison conditional discrimination procedure has generated control by the positive or negative comparison can limit the validity of conclusions about the formation of equivalence relations. More constructively, we indicate how to remove some of the uncertainties of interpretation by means of two-comparison procedures that permit the experimenter to determine whether control will rest with the positive or the negative comparison stimulus. We also show why certain of the tests for equivalence relations are preferable to the standard novel-stimulus tests for identifying control by the positive or negative comparison stimulus.

Third, our analysis provides a foundation on which to create experiments for the purpose of examining the very issues it raises. Our hope is that our presentation will encourage experimentation that might otherwise not be done.

## CONDITIONAL DISCRIMINATION AND EQUIVALENCE RELATIONS

### *Relations Between Samples and Positive Comparisons*

The uppermost section of Figure 1 schematically illustrates a set of four-term contin-

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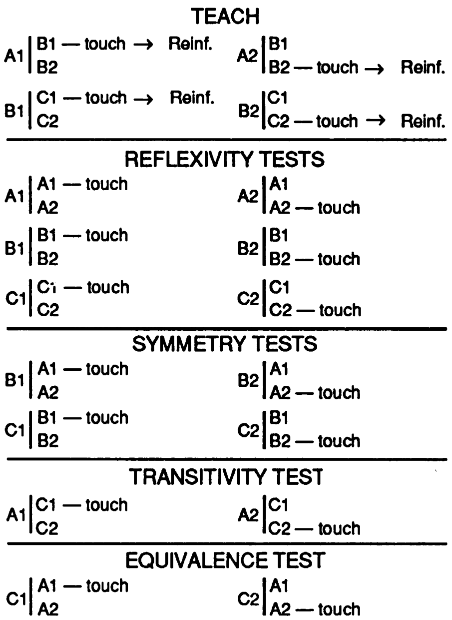


Fig. 1. The uppermost panel (TEACH) schematically illustrates a set of explicitly arranged (baseline) four-term contingencies (sample, comparison, act, consequence). Alphanumeric designations indicate stimuli. A vertical line separates each sample stimulus from the two comparisons that are presented along with it. The three stimuli constitute a trial type; each side-by-side pair of trial types represents a conditional discrimination. The four lower panels indicate the trial types presented to the subject during tests (without explicit reinforcement) for emergent relations after the baseline conditional discriminations had been taught (see text for details).

gencies (Sidman, 1986), with each side-by-side pair of trial types representing a conditional discrimination. One contingency may have taught a subject that in the presence of sample A1, touching comparison B1 will produce a reinforcer; in the presence of A2, touching B2 will produce the reinforcer. From the experimenter's point of view, one explicitly taught conditional discrimination gives rise to the conditional relations *if A1 then B1,* and *if A2 then B2*; the other explicitly taught conditional discrimination gives rise to the conditional relations *if B1 then C1,* and *if B2 then C2.* These conditional relations, the direct outcomes of reinforcement contingencies, will be termed *explicitly taught,* or *baseline,* relations.

Under some circumstances, the conditional relations will possess additional properties. We shall be concerned here with properties that are relevant to the definition of equivalence relations. (For detailed expositions of formal and behavioral requirements for the definition of equivalence relations, see Carrigan, 1986; Saunders & Green, 1992; Sidman, 1990; Sidman et al., 1982; Sidman & Tailby, 1982.)

One possibility is that each of the stimuli that make up the baseline relation might be found also to bear that relation to itself. For example, one might observe the subject choosing comparison A1 if the sample is also A1, and A2 if the sample is A2, suggesting that the baseline relation exhibits reflexivity. Another possibility is that each sample and its correct comparison might prove to be interchangeable with respect to their sample and comparison functions, suggesting that the baseline relation exhibits symmetry. After a subject has learned the A-B and B-C baselines, the baseline relation might exhibit a third property, transitivity, which would be demonstrated by choosing C1 in the presence of sample A1, and C2 in the presence of sample A2. Figure 1 summarizes these properties and their relevant tests.

Furthermore, if reflexivity, symmetry, and transitivity can be demonstrated, the baseline conditional relations can be described as being *equivalence* relations. In that event, one might expect the performances, *if C1 then A1* and *if C2 then A2* to emerge (Figure 1: equivalence test). This test does not identify a formal property of equivalence relations. It has, however, been used under many experimental circumstances as an abbreviated way to determine whether all three formal properties hold true (e.g., Sidman, 1990).

A stimulus relation consists of two components: a group of sample-comparison pairs, such as A1-B1 and B1-C1, and a behavioral component that defines the way the first stimulus in each pair relates to the second, such as *if A1 touch B1.* In this instance, *touch* defines the relation that stimulus A1 bears to stimulus B1. All stimulus pairs that make up a given relation share the same behavioral component.

The particular pairs that make up a given relation are specified by the defining properties of the relation. For example, if A-B and B-C denote an equivalence relation, reflexivity specifies that the A-A, B-B, and C-C pairs be included in the relation; symmetry specifies that the B-A and C-B pairs be included; and transitivity specifies that the A-C pairs be included. The *equivalence class* is the group of individual stimuli that make up the equivalence relation.

Thus, the transitivity test described above asks the subject, in effect, whether the sample-comparison pairs A1-C1 and A2-C2, which were not involved in any explicit contingency, are also members of the baseline relation. Given appropriate experimental controls (see below), a subject who touched comparison C1 in the presence of sample A1, and C2 in the presence of A2, would indicate that the baseline relation (its stimulus pairs *and* its behavioral definition) exhibits transitivity.

### *Relations Between Samples and Negative Comparisons*

The conditional discrimination procedure may generate conditional relations between samples and either positive or negative comparisons. We shall call the relation between samples and positive comparisons the *select* or *Type S* relation: In the presence of A1, select B1; in the presence of A2, select B2. Alternatively, the same performance could reflect a relation between samples and incorrect comparisons. We shall call this the *reject* or *Type R* relation: In the presence of A1, reject B2; in the presence of A2, reject B1 (Carter & Werner, 1978; Constantine, 1981; Cumming & Berryman, 1965).

We use *select* and *reject* here merely as shorthand labels to indicate which comparison stimulus, along with the sample, controls the subject's performance: the one designated by the experimenter as positive or the one designated as negative. *Type S* and *Type R control* represent sample-comparison relations in which the positive or negative comparison, respectively, controls the subject's performance.

The contrast between Type S and Type R is not to be equated with the contrast between relations that we call *sameness* and those that we call *difference, opposition,* or *negation* (e.g., Steele & Hayes, 1991). As we shall emphasize below, both Type S and Type R relations, like sameness, can also be equivalence relations. Difference, opposition, and negation, however, are never equivalence relations; they do not exhibit reflexivity and need not exhibit transitivity.

Because the procedure neither restricts nor encourages the incorporation of Type S or Type R relations into a subject's repertoire, one might expect a conditional discrimination to reflect either or both. Whether the subject selects the positive or rejects the negative comparison, touching the positive comparison brings a reinforcer. How can one tell whether the select or the reject relation is involved in a particular conditional discrimination?

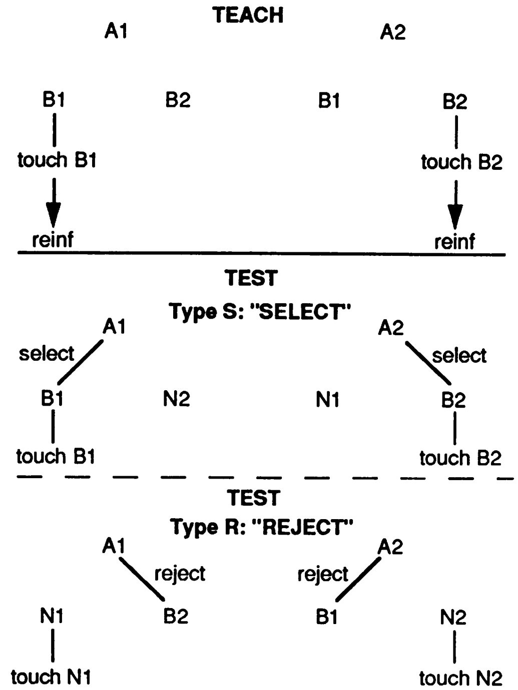


Fig. 2. Typical novel-stimulus tests that have been used to identify the type of relation established by conditional discrimination training.

### *Novel Stimulus Tests for Select and Reject Relations*

The question of whether Type S or Type R control accounts for a particular conditional discrimination has been answered in part by tests that substitute novel stimuli for either the positive or the negative comparisons (Berryman, Cumming, Cohen, & Johnson, 1965; Cumming & Berryman, 1965; Dixon & Dixon, 1978; Dixon, Dixon, & Spradlin, 1983; Farthing & Opuda, 1974; McIlvane et al., 1987; McIlvane, Withstandley, & Stoddard, 1984; Stromer & Osborne, 1982; Urcuioli & Nevin, 1975). Figure 2 shows examples of such test trials (appropriate controls for the effects of novelty itself must be assumed). The top panel shows the two types of training trials. During tests, these trials continue to be presented as a baseline, and test trials are inserted among the baseline trials.

The center panel shows test trials in which novel stimuli, N1 and N2, replace the incorrect comparisons. If the subject continues to touch B1 or B2 in such test trials, one might infer that Type S control predominated in the baseline. Indeed, the results illustrated here suggest that the subject might always have chosen correctly without ever having identified the negative stimulus on any baseline or test trial.

If, however, novel stimuli replace the correct comparisons, as in the bottom panel of Figure 2, and the subject consistently chooses a novel stimulus, one might infer that Type R control predominated. In this instance, the subject need never have identified the stimuli that were actually touched.

But what if a subject who is given the test for Type S control (center section of Figure 2) had learned a Type R relation? In test trials, new stimuli replace the incorrect comparisons that the subject had learned to reject. What is this subject to do when faced with trials that eliminate the familiar sample-comparison pairs?

In the absence of familiar comparisons to reject, the subject may vacillate from trial to trial between the comparison stimuli. Such vacillation, along with selections of the novel stimuli in the other kind of test (bottom panel), would justify the inference that Type R control characterized the subject's baseline performance. A less decisive outcome would have the subject, deprived of previously learned bases for choice, nevertheless acting consistently. Given a history in which each kind of baseline trial always had its own correct choice, a subject may "arbitrarily assign" one comparison to each sample (Saunders, Saunders, Kirby, & Spradlin, 1988).

Thus, on test trials with novel stimuli substituted for the negative comparisons, a subject might always touch B1 in the presence of sample A1 and B2 in the presence of A2 even though the baseline relation was Type R. In spite of such test results, an inference that the relation during training and baseline was Type S would be invalid. The experimenter, however, would have no way of knowing this.

Test trials with novel stimuli may actually teach the subject an alternate relation. For example, in the test for the Type S relation, a subject whose baseline performance had been Type R might recall that whenever comparison B2 had been there to be rejected, B1 had been available to be touched. The subject might then deal with the test trial by selecting B1; on test trials with A2 as the sample, the subject might learn to select B2. Then, having learned a Type S relation on test trials, the subject might find that selecting also "worked" on baseline trials. Being the most consistently available basis for choice (Devany, Hayes, & Nelson, 1986; Sidman, Kirk, & Willson-Morris, 1985; Sidman, 1992), Type S control might then replace Type R not just in the test but in the baseline also. Similarly, in tests for Type R control, a subject who had learned the select relation might shift to Type R control both in test and in baseline trials.

The point being made here is not that novel stimulus tests are incapable of identifying Type S and Type R control, but rather that such tests may themselves bias a learner toward a particular type of relation. Novel-stimulus tests may not accurately reflect the type of control that existed before the test.

Novel-stimulus tests possess this potential defect because a particular kind of probe can identify either Type S or Type R control, but not both. By disallowing a previously learned relation, such probes may permit a weaker relation to emerge in test trials, and even to replace the original relation in baseline trials. The next section will show that reflexivity or transitivity tests (Figure 1), unlike novel-stimulus tests, are each capable of identifying both Type S and Type R control. We shall develop the point that Type S and Type R relations should yield different yet predictable results when tested for reflexivity and transitivity.

## TYPE S VERSUS TYPE R CONTROL AND THE PROPERTIES OF EQUIVALENCE RELATIONS

An issue that has not been addressed in the literature is how a subject's learning to select correct comparisons or to reject incorrect comparisons might differentially affect the outcome of tests that are designed to assess equivalence relations. A fundamental question is whether the Type R relation can exhibit the properties of equivalence relations. In dealing with this question, we shall show that one cannot predict the outcome of reflexivity, transitivity, and equivalence tests unless one can tell whether conditional discrimination training has established Type S or Type R control.

Several assumptions underlie our analysis. First, we assume throughout that the baseline conditional relations we are evaluating for the properties of equivalence relations are indeed equivalence relations. Given the validity of this assumption, a stimulus-control analysis makes it possible to predict how Type S and Type R control will affect a subject's performance in tests (Figure 1) that are relevant to the formation of equivalence classes.

Second, we must assume that the type of control prevailing during baseline trials will continue during test trials. Faulty experimental control of various kinds can easily cause this assumption to be violated. For example, the following have generated violations: failing to prepare subjects appropriately for unreinforced trials (Sidman et al., 1985, p. 30, Subject E.H.); failing to maintain overall reinforcement density (unreported observations); presenting test trials that permit the emergence of consistent sample-comparison relations other than the ones the test is intended to evaluate (Stikeleather & Sidman, 1990); and explicit or implicit instructions that lead to changes in controlling variables (Green, Sigurdardottir, & Saunders, 1991; Sigurdardottir, Green, & Saunders, 1990).

A third assumption, closely related to the second, is that the tests do not by themselves teach a subject the conditional discriminations that are supposed to emerge as a function of the subject's training history. Improper design of the series of test trials can cause this assumption to be violated (Harrison & Green, 1990).

Finally, our analysis is directed at conditional discriminations in which two comparisons are presented per trial. As we point out in the final paragraphs, the use of three or more comparisons per trial is likely to prevent Type R control, thereby removing the need to evaluate its effects on the formation of equivalence relations.

*Reinforcement Contingencies and Sample-Comparison Relations*

Figure 3 outlines a set of explicitly taught conditional discriminations, A-B and B-C, and illustrates the sample-comparison relations and

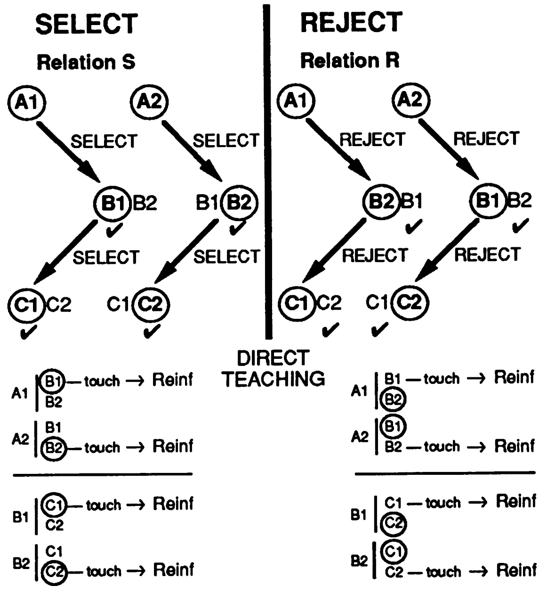


Fig. 3. Two relations, S and R, that two-comparison A-B and B-C conditional discrimination training may establish. In the upper sections, arrows point from samples to related comparison stimuli, circles indicate the controlling stimuli, and checkmarks indicate the comparison stimulus that the subject actually touches. Diagrams in the lower section illustrate the reinforcement contingency and the controlling comparison (circled) for each trial type under Relations S and R.

the subject's recorded choices that are to be expected with Relations S and R. Arrows in the upper section point from sample to comparison stimuli. Circles around the alpha-numeric stimulus identifiers indicate controlling stimuli, and a checkmark indicates the comparison that the subject actually touches. For example, given sample A1 under Relation S, the subject both selects and touches comparison B1. The related pair is A1-B1; B1 is the controlling comparison. Given sample A2, the subject selects and touches comparison B2. The related pair is A2-B2; B2 is the controlling comparison. Similarly, with related pairs B1-C1 and B2-C2, the subject selects and touches the controlling comparison.

The same training procedure can produce a different outcome, shown under Relation R. Here, given sample A1, the subject rejects comparison B2 and touches B1. The related pair is A1-B2; the controlling comparison is B2, even though the subject touches B1. Also, given sample A2, the related pair is A2-B1; the controlling comparison is B1, even though the

Table 1

Examples of trial types intended to bias subjects toward selecting the correct comparison (Relation S) or rejecting the incorrect comparison (Relation R). Touching the left-hand member of each comparison pair produces a reinforcer. Roman numerals identify groups of trial types.

| Relation S | | Relation R | |
|---|---|---|---|
| I | II | V | VI |
| A1 → B1B2 | B1 → C1C2 | A1 → B1B2 | B1 → C1C2 |
| A1 → B1X1 | B1 → C1Y1 | A1 → X1B2 | B1 → Y1C2 |
| A1 → B1X2 | B1 → C1Y2 | A1 → X2B2 | B1 → Y2C2 |
| A1 → B1X3 | B1 → C1Y3 | A1 → X3B2 | B1 → Y3C2 |
| III | IV | VII | VIII |
| A2 → B2B1 | B2 → C2C1 | A2 → B2B1 | B2 → C2C1 |
| A2 → B2X1 | B2 → C2Y1 | A2 → X1B1 | B2 → Y1C1 |
| A2 → B2X2 | B2 → C2Y2 | A2 → X2B1 | B2 → Y2C1 |
| A2 → B2X3 | B2 → C2Y3 | A2 → X3B1 | B2 → Y3C1 |

subject touches B2. Similarly, with related pairs B2-C1 and B1-C2, the subject does not touch the controlling comparison. At this point, however, the experimenter has no way of knowing that the comparisons the subject is seen to touch are not the controlling stimuli.

Diagrams in the lower section of Figure 3 illustrate the reinforcement contingencies. Each sample is at the left of its pair of comparisons, with the controlling comparison circled. Side by side, these Relation S and Relation R diagrams make it easier to see that even with no change in the programmed contingencies, the comparisons that are related to a given sample depend on the prevailing type of control. With Type S control, the subject touches the comparison that is related to the current sample; with Type R, the subject touches the comparison that is not related to the current sample.

### *Experimental Control of Type S and Type R Relations*

The analysis to follow will indicate how tests for some of the properties of equivalence relations can identify Type S and Type R control. Experimental confirmation of that analysis, however, will require techniques for explicitly controlling the nature of the A-B and B-C conditional discriminations. One technique for controlling the nature of the sample-comparison relation was used successfully by Carrigan (1986), and was replicated by Johnson and Sidman (1991)[1]. The technique is based on the assumption that subjects who are given two means of performing a particular conditional discrimination will do the task in a way that requires them to learn fewer discriminations.

Table 1 shows how to make use of this assumption to bias a subject's conditional discrimination performance toward Type S or Type R control. The first trial type in each group is a baseline trial (see Figure 3). The remaining three trial types in each of the left-hand groups are intended to bias control toward Type S, and those in each of the right-hand groups will bias control toward Type R. For example, with Group V, a subject could produce a reinforcer on every trial either by learning always to reject comparison B2 or to select B1, X1, X2, or X3; Type R control is highly likely. Similarly, the trial types in Groups VI through VIII are also likely to generate A-B and B-C conditional discriminations that are governed by Type R control.

Trial types in Groups I through IV, however, are likely to create the select relation; a subject could produce reinforcement on every trial either by always selecting one comparison or by rejecting the four different comparisons in each group. The entire left-hand baseline in Table 1 could be learned, minimally, as four Type S related pairs or, maximally, as 16 Type R related pairs; the right-hand baseline would be learned as four Type R or 16 Type S related pairs.

As our analysis proceeds, we shall indicate where it would be subject to experimental test by explicitly controlling the baseline conditional discriminations in ways that Table 1 illustrates. Clever experimenters will un-

[1] Johnson, C., & Sidman, M. (1991, May). *Stimulus classes established by sample-and-S− conditional-discrimination performance.* Paper presented at the meeting of the Association for Behavior Analysis, Atlanta, Georgia.

doubtedly devise additional, and perhaps more effective, techniques for gaining experimental control over the occurrence of select and reject relations.

*The Reflexivity Test*

A conditional relation is reflexive if each sample and comparison exhibits the same conditional relation with respect to itself. If the relation is reflexive, it will include several sample-comparison pairs (such as A1-A1, B1-B1, etc.) that were not involved in the baseline contingencies. Applying the behavioral definition of Relation S to these stimulus pairs yields the following outcomes: *If A1 select A1, if B1 select B1,* and so forth. Applying the behavioral definition of Relation R, however, yields *If A1 reject A1, if B1 reject B1,* and so forth.

Figure 4 illustrates the expected outcomes if the A-B and B-C conditional discriminations that a subject has learned under Type S or Type R control are reflexive. Solid arrows in the upper sections represent the directly taught A-B and B-C conditional discriminations; broken arrows represent relations that should emerge during testing. This analysis generates the striking prediction that reflexivity tests will yield opposite results for Relations S and R. For example, in the upper left corner (sample A1 and comparisons A1 and A2) the circled stimuli and the checkmark show that when the A1-A1 pair belongs to a select relation, the subject will touch comparison A1. When the same A1-A1 pair belongs to Relation R, however (at the right of the divider), the subject rejects the controlling comparison and touches A2.

All of the broken arrows in Figure 4 show the same contrast between Type S and Type R control in reflexivity tests. Relation S has the subject touching the comparison that is the same as the sample; Relation R has the subject rejecting the comparison that is the same as the sample and touching the other. In the bottom diagrams, the samples, the controlling comparisons, and the comparisons that are actually touched in all reflexivity tests are shown side by side for each type of relation. All tests show the controlling comparisons to be the same in both relations, but the comparison actually touched is the controlling comparison in Relation S and the other comparison in Relation R. It appears, then, that both Relations S and R can be reflexive. Test results indicating the reflexivity of Relation R will, however, differ from results that indicate the reflexivity of Relation S.

Although the subject's performance under Relation S looks like identity matching, and under Relation R like oddity, appearances here can deceive. In Relation R, touching the odd stimulus is simply a by-product of the subject's rejection of the identical stimulus. It is conceivable that some subjects might know nothing about the odd stimulus except where it is; only its location—so that the subject can touch it—need enter into the reinforcement contingency.

Reflexivity tests, then, can show whether the positive or the negative comparison in a conditional discrimination is the controlling stimulus. Unlike novel-stimulus tests (Figure 2), each of which permits only one controlling relation to be demonstrated, the reflexivity test allows a subject to continue either to select or to reject, without having to learn a new relation because of the test itself.

Johnson and Sidman (1991) confirmed the contrasting outcomes of reflexivity tests for select and reject relations. Dube, Green and Serna (in press) also presented data that are consistent with our analysis. The results of reflexivity tests in most two-comparison studies, however, have been consistent with Type S control (e.g., Saunders, Wachter, & Spradlin, 1988; Sigurdardottir et al., 1990). The rarity of data that indicate Type R control may be a consequence of the practice of giving reflexivity tests before teaching subjects the arbitrary conditional relations in which the tested stimuli are to participate. Such prior testing can reveal nothing about the type of control in the critical conditional discriminations, because those are as yet untaught. Tests for Type S or Type R control must be given after the subject has learned the baseline conditional discriminations.

Yet, one may ask why such prior tests of identity matching usually give evidence of Type S control, with subjects seeming to match on the basis of sameness rather than difference or oddity. A likely source of the low frequency of observed Type R control is the subject's preexperimental history. It is reasonable to assume that people have to perform many conditional discriminations daily. If only two choices were available and all other conditions were equal, the likelihood of either type of control would be the same. But all other conditions are never

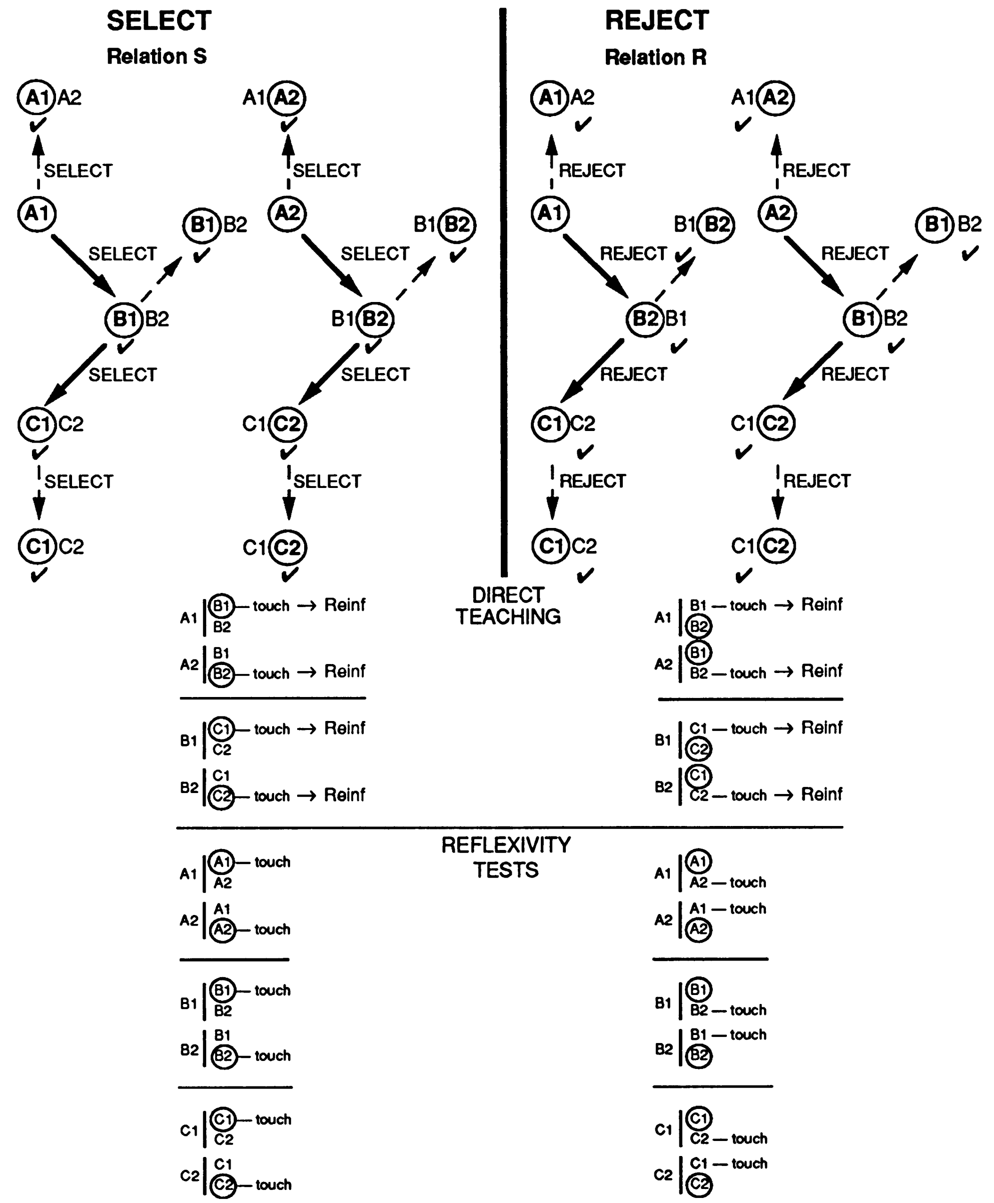


Fig. 4. Outcomes to be expected if the A-B and B-C conditional discriminations that a subject has learned under Type S control (left) or Type R control (right) are reflexive. Arrows in the upper sections point from sample stimuli to controlling (circled) comparisons. Solid arrows represent baseline A-B and B-C conditional discriminations; broken arrows represent conditional discriminations that should emerge during reflexivity tests. Diagrams in the lower sections illustrate, for each trial type, related sample-comparison pairs and the comparisons actually touched under Relations S and R during direct teaching and during reflexivity tests.

equal. To have only two choices available is probably itself relatively rare. As the number of available choices increases, Relation S still requires a person to learn only one discrimination per sample, but Relation R requires many.

Consider a four-choice trial with A1 as the sample, B1 the correct comparison, and B2, B3, and B4 the incorrect comparisons. To do this trial correctly under Type S control, a person need learn only one discrimination: *If A1, select B1.* Under Type R control, however, a person would have to learn three discriminations: *If A1, reject B2, B3, and B4*. Thus, an $N$-comparison trial requires one Type S discrimination or $N - 1$ Type R discriminations. If a learner tends to favor the form of control that requires fewer discriminations, one should expect a person's history of Type R control to be minimal. One might therefore expect a low likelihood of Type R control in an experimental situation also. By giving subjects appropriate experimental histories (see, for example, Table 1), it should be possible to produce either Relation S or Relation R in reflexivity tests.

### The Symmetry Test

A conditional relation is symmetric if the same relation holds when the former samples become comparisons and the comparisons become samples. What is to be expected in symmetry tests if a conditional discrimination is characterized by Type S or by Type R control? If the relation is symmetric, it will include several sample-comparison pairs other than those in the baseline. Applying the behavioral definition of Relation S to stimulus pairs in which former comparisons are samples and former samples are comparisons yields the following outcomes: *If B1, select A1, if B2, select A2,* and so forth. Applying the behavioral definition of Relation R, however, yields different stimulus pairs: *If B1, reject A2, if B2, reject A1,* and so forth.

Even though the controlling comparisons differ, Type S and Type R relations can be expected to yield the same recorded data in symmetry tests. The upper left corner of Figure 5 shows that a subject who has learned to select and touch comparison B1 when the sample is A1 (solid arrow) will, when given the symmetry test, select and touch comparison A1 when B1 is the sample (broken arrow). The upper right corner shows the same symmetry test given to a subject who has learned Relation R. This subject, although rejecting A2 when the sample is B1, will, like the subject who has learned Relation S, touch comparison A1. The broken arrows in Figure 5 show the subject touching the same comparison in a given symmetry test, whether the relation is Type S or Type R. In the bottom diagrams, the controlling comparison in each test can be seen to differ under Relations S and R, but the comparison actually touched is the same. The recorded results of symmetry tests, therefore, unlike the results of reflexivity tests, give the experimenter no clue about the nature of the controlling relation in the directly taught conditional discriminations. For confirmatory data, see Johnson and Sidman (1991) and Dube et al. (in press).

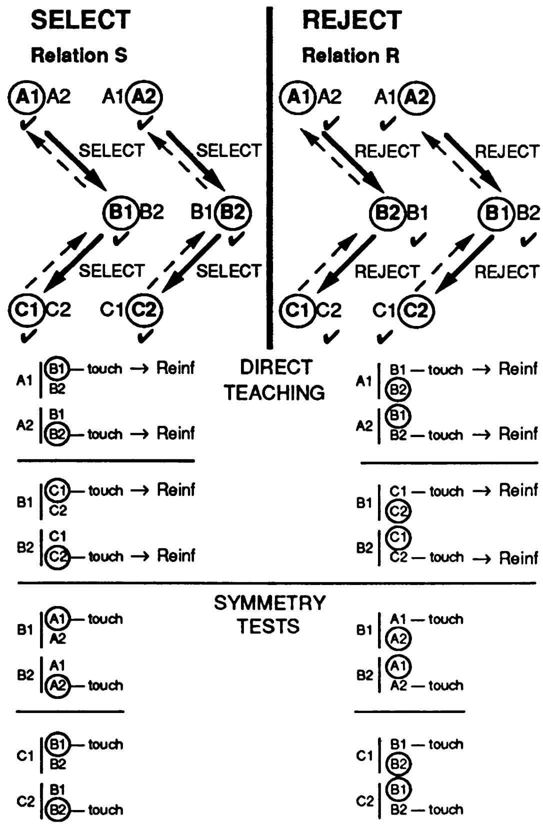


Fig. 5. Outcomes to be expected if two baseline conditional discriminations (A-B and B-C), learned under Type S or Type R control, are symmetric. Solid arrows represent baseline conditional discriminations; broken arrows represent conditional discriminations that should emerge during symmetry tests. The lower diagrams illustrate, for each trial type, the related sample-comparison pairs and the comparisons actually touched under Relations S and R during direct teaching and during symmetry tests.

### The Transitivity and Equivalence Tests

If A-B and B-C performances are based on the same relation, and if that relation is transitive, it will include two sample-comparison pairs not involved in the baselines: A1-C1 and A2-C2. Transitivity, therefore, is evaluated by testing the A-C conditional discriminations. A relation that possesses all the properties nec-

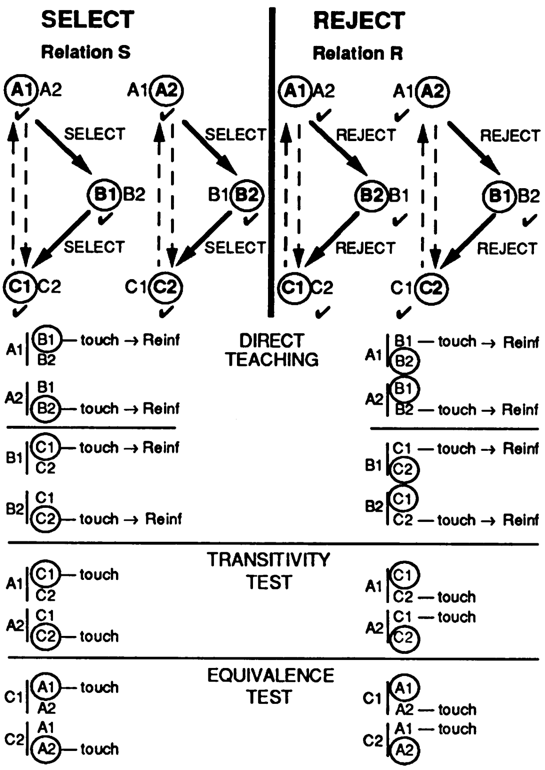


Fig. 6. Outcomes to be expected (broken arrows) if the two baseline conditional discriminations (solid arrows) learned under Type S or Type R control are transitive, and if the related sample-comparison pairs are members of an equivalence relation. The lower diagrams illustrate, for each trial type, the related sample-comparison pairs and the comparisons actually touched under Relations S and R during direct teaching and during transitivity and equivalence tests.

essary to define equivalence will also include the sample-comparison pairs C1-A1 and C2-A2. Equivalence can therefore be evaluated by testing the C-A conditional discrimination. (See Sidman, 1990, for a fuller exposition of the requirements and implications of this abbreviated test for equivalence.)

Our analysis of controlling relations generates the prediction that transitivity or equivalence tests, like reflexivity tests, will yield opposite results for Relations S and R. Broken arrows and circles in the upper sections of Figure 6 show that the controlling stimuli in the A-C and C-A tests will be the same in both relations, but the checkmarks show that the subject's recorded choices will differ. If the subject's A-B and B-C training established Type S control, then one can expect the transitivity test trials also to reflect Type S control:

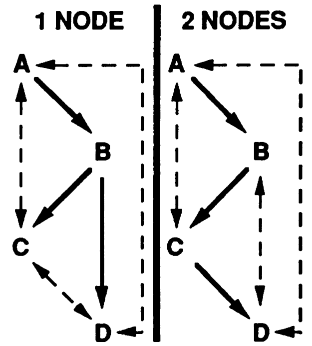


Fig. 7. Expansion of three-member ABC classes to four-member ABCD classes by maintaining a single node (B) or by adding a second node (C). Solid arrows indicate explicitly taught and broken arrows indicate emergent conditional discriminations.

*If A1, select B1; if B1, select C1; therefore (given transitivity) if A1, select (and touch) C1.* If, however, the subject's A-B and B-C training established Type R control, then one can expect the opposite transitivity test results: *If A1, reject B2; if B2, reject C1; therefore (given transitivity), if A1, reject C1 (and touch C2).* Similar predictions hold for the equivalence tests. With sample C1 and comparisons A1 and A2, the subject will either select and touch A1 (Relation S) or will reject A1 and touch A2 (Relation R).

In the bottom sections of Figure 6, all tests show the expected controlling comparisons to be the same in both relations, but the comparison actually touched will be the controlling comparison in a select relation and the other comparison in a reject relation. For confirmatory data see Carrigan (1986) and Johnson and Sidman (1991).

*Tests in expanded classes.* By teaching the two-comparison A-B and B-C conditional discriminations, one can establish two three-member classes of equivalent stimuli, A1B1C1 and A2B2C2. One might then attempt to add a fourth member, D1 or D2, to each class. Figure 7 illustrates two general ways to expand the classes: one is explicitly to teach the subject B-D conditional discriminations, and the other is to teach C-D (A-D would serve as well). Either method can bring the D stimuli into the classes, thereby establishing four-member ABCD classes.

These two ways of enlarging the classes differ with respect to the number of nodes in the expanded network of baseline conditional discriminations. The left section of Figure 7 shows that B remains the only node, the only stimulus related to two or more stimuli by explicit contingencies. In the right section, B and C both constitute nodes. (Nodality is closely related to the *stages* concept, used in formulations of equivalence that arose largely from paired-associates research, e.g., Jenkins, 1963. For theoretical discussions of nodality in equivalence relations, see Fields & Verhave, 1987; Fields, Verhave, & Fath, 1984. For relevant data, see Fields, Adams, Verhave, & Newman, 1990; Lazar, Davis-Lang, & Sanchez, 1984; Saunders, Wachter, & Spradlin, 1988; Sidman et al., 1985.)

A-D and D-A tests generate the most interesting difference between the one- and two-node class-expansion paradigms. In the one-node paradigm, the results of A-D and D-A tests should depend on the type of control in the same way that was shown to be expected for A-C and C-A tests (Figure 6). (Only if other variables impose an upper limit on class size itself might an increase in the number of stimulus pairs clustered about a single node influence the outcome of transitivity or equivalence tests; at some class size, new stimulus pairs would then fail to exhibit the relational properties.) The results of A-D and D-A tests in the two-node paradigm, however, can be expected not to depend on the type (S vs. R) of control. Figure 8 illustrates the related components of four-member classes that are established by the two-node paradigm. (Two-headed arrows indicate that stimuli they point to can function either as samples or comparisons.)

Like the original A-C and C-A tests, B-D and D-B tests evaluate transitivity and equivalence of two directly taught conditional discriminations that cluster about a single node, now C rather than B. The recorded results of B-D and D-B tests will therefore depend on the type of control, replicating the results of A-C and C-A tests. Unlike A-C, C-A, B-D, and D-B tests, however, the recorded results of A-D and D-A tests will not indicate the type of control. For example, if the sample is A1 and the relation is select, the subject will both select and touch D1; if the relation is reject, the subject will reject D2 but will still touch D1. With any given sample, the same recorded results are to be expected in A-D and D-A tests regardless of whether the baseline is under Type S or Type R control. Thus, when the baseline conditional discriminations being evaluated contain two nodes, transitivity and equivalence tests will fail to distinguish between Type S and Type R control. It is not possible to infer the type of relational control solely from the results of A-D or D-A tests in four-member classes that contain two nodes.

The dependence of transitivity and equivalence tests on the type of the sample-comparison relation for single-node three-member classes, and the independence of such tests from the type of the relation in two-node four-member classes, have been confirmed by Carrigan (1986) and Johnson and Sidman (1991). What has not yet been confirmed, however, is the inference that successive enlargement of the classes by the addition of a node will cause transitivity and equivalence tests to "flip-flop" or "toggle" with respect to their dependence on, or independence from, the type of control in the baseline.

Figure 9 illustrates the results to be expected if a potential fifth member is added to each class by teaching a subject D-E conditional discriminations. Establishing a third node makes three new transitivity tests possible, C-E, B-E, and A-E, and three new equivalence tests, E-C, E-B, and E-A. As examples, let us analyze only the transitivity tests; equivalence tests can be expected to yield the same conclusions. In conformity with A-C and B-D tests, the emergent C-E conditional discriminations can be expected to differ under Relation S and Relation R. B-E tests evaluate transitivity in a potential new four-member class, BCDE. In conformity with similar A-D tests, emergent B-E conditional discriminations should show the expected constancy of the recorded results under Relations S and R.

A-E tests evaluate transitivity in the potential new five-member class, ABCDE, which the addition of a third node made possible. The A-E tests for transitivity in the three-node paradigm should yield results like those for classes established by the single-node rather than by the two-node paradigm. Emergent A-E conditional discriminations, like C-E, B-D, and A-C, can be expected to yield different recorded results under Relations S and R. For example, with A1 as the sample, the subject will select and touch comparison E1 if the relation is Type S, but will reject E1 and touch E2 if the relation is Type R.

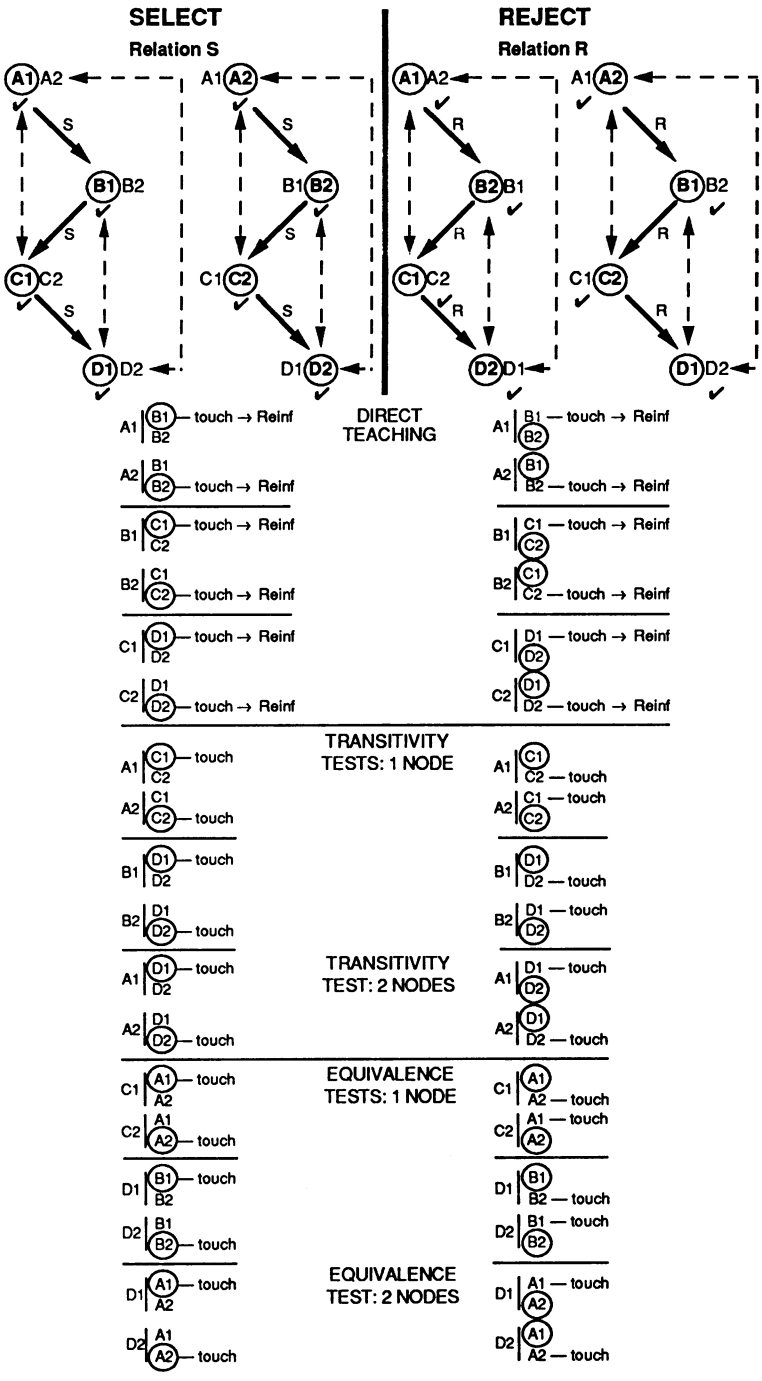


Fig. 8. Class expansion from three to four members via a two-node paradigm. Illustrations of the toggling effect as transitivity and equivalence are tested in the context of one-node and two-node baselines.

Our stimulus-control analysis indicates that transitivity (and equivalence) tests can be expected to (a) distinguish between Type S and Type R control in single-node classes, (b) fail to distinguish between the two kinds of control in two-node classes, and (c) again distinguish between Type S and Type R in three-node classes. Given explicitly taught conditional relations that possess the properties of an equivalence relation, the analysis, if it were carried

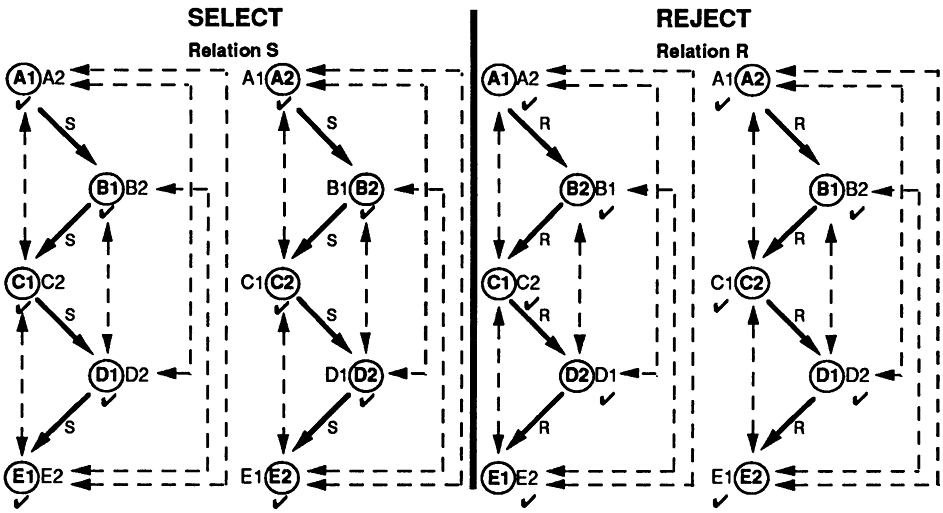


Fig. 9. Class expansion to five members via a three-node paradigm, illustrating the expected continuation of the toggling effect.

beyond the three-node network illustrated in Figure 9, would permit the following generalization: When conditionally related stimuli in the baseline cluster about an odd number of nodes, the recorded results of transitivity and equivalence tests will depend on whether the baseline control is Type S or Type R; when the related baseline pairs cluster about an even number of nodes, transitivity and equivalence tests will yield the same results regardless of whether the baseline control is Type S or Type R. This general prediction still lacks experimental confirmation.

*Variations in the Type of Controlling Relation*

Our analysis so far has assumed the same type of control in all of the explicitly taught conditional discriminations. Such constancy, however, need not prevail. A subject might, for example, select correct comparisons in A-B and reject incorrect comparisons in B-C. The type of control might even vary from one sample to another within a particular conditional discrimination. For example, with A1 as the sample, a subject might select and touch comparison B1, but with A2 as the sample, the subject might reject B1 and touch B2. How would each of these kinds of variation affect the predicted outcomes of tests for equivalence, transitivity, symmetry, and reflexivity?

*Transitivity and equivalence.* Given two conditional discriminations, A-B and B-C, Type S control in one and Type R control in the other would make A-C meaningless as a test for transitivity. To take a more obvious example, suppose A-B were to represent the relation *larger than,* and B-C *smaller than.* Each relation by itself is transitive, but this could not be demonstrated by testing A-C. If A1 were larger than B1, and B1 smaller than C1, how would one characterize the relation between A1 and C1? The results of an A-C test would have no bearing on the transitivity of A-B or B-C. Any consistency in such tests would have to be accounted for in some other way. Similarly, in the A-C test diagrammed in Figure 10, would Type S or Type R control determine the subject's choice? Furthermore, which comparison, C1 or C2, would be selected or rejected?

Variation in the type of control shown in Figure 10 could be produced experimentally by using appropriate groups of trial types from Table 1 to teach a subject the baseline conditional discriminations: Groups I and III to teach A-B and Groups VI and VIII to teach B-C. Consistent test results, however, would have to be attributed to something other than transitivity or equivalence. One cannot evaluate transitivity or, by extension, equivalence, when the A-B and B-C relations differ.

What predictions for transitivity and equivalence tests does a stimulus-control analysis generate if the type of relation remains constant from A-B to B-C but varies within each of those conditional discriminations? Suppose, for example, Type S control prevails when A1

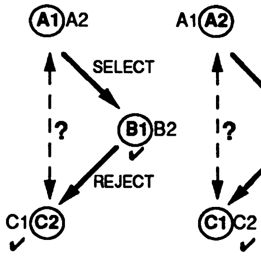


Fig. 10. An example of variation in the type of control from one baseline conditional discrimination to the other. With Type S control prevailing in A-B and Type R in B-C, the results of A-C and C-A tests are unpredictable.

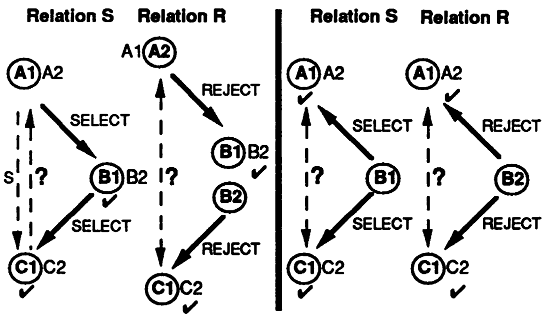


Fig. 11. Examples of variation in the type of control within each baseline conditional discrimination: expected outcomes of A-C and C-A tests. In the two left-hand diagrams, the baseline conditional discriminations are A-B and B-C; in the two right-hand diagrams, the baselines are B-A and B-C.

or B1 is the sample, and Type R prevails when A2 or B2 is the sample. Figure 11 illustrates two examples. Let us first examine the left-hand diagrams. In baseline trials with A1 as the sample, the subject selects and touches comparison B1; with A2 as the sample, the subject rejects B1 and touches B2. B-C trials show similar shifts in the type of control with B1 and B2 as samples. Such variation could be produced experimentally by using appropriate groups of trial types from Table 1 to teach a subject the baseline conditional discriminations: Groups I and VII to teach A-B and Groups II and VIII to teach B-C.

Variations of this nature in the type of control produce an anomaly with respect to the predictability of transitivity and equivalence tests. Although the A1-C1 relation is predictable, the results of test trials with A2, C1, or C2 as the sample cannot be predicted even if the baseline relations are equivalence relations.

This indeterminacy arises from three sources. (*Indeterminacy* refers here only to the absence of test predictability on the basis of the type of control in the baseline.) First, given the illustrated shifts from Relation S to Relation R, a subject can be expected to touch comparison C2 only if C1 has been rejected; C2 never functions as a controlling comparison in either type of relation. What is a subject to do, then, when faced with a test trial in which C2 is the sample?

Second, C1 is a controlling comparison in both Relation S and Relation R. In a C-A trial with C1 as the sample, can we expect a subject to select or to reject the comparison? And then, which comparison will be selected or rejected? When either C1 or C2 is the sample in test trials, therefore, the nature of its relation to A1 or A2 is predictable, if at all, only on grounds other than equivalence.

Finally, the related pairs A2-B1 and B2-C1 possess no element in common. A-C test trials with A2 as the sample can therefore tell us nothing about the transitivity of those relations, nor can a C-A test say anything about equivalence.

The right-hand diagrams in Figure 11 show what is to be expected in another common arrangement for studying equivalence relations. Here, the directly taught conditional discriminations are B-A and B-C. In these baseline conditional discriminations, the controlling comparison is always either A1 or C1 (never A2 or C2) independently of whether the controlling relation is Type S or Type R. Thus, faced with an A-C test trial in which A1 is the sample, will a subject select or reject C1? With C1 as the sample, will a subject select or reject A1? On such trials, the subject's training history places Relation S and Relation R in conflict; no prediction can be based solely on transitivity or equivalence.

Tests with A2 or C2 as the sample will also yield indeterminate results. Because A2 and C2 have no history as controlling stimuli, either in Relation S or R, neither transitivity nor equivalence provides a basis for predicting whether their use as samples in test trials will cause subjects to select or to reject any given comparison.

Variation in the type of control, then, whether between or within conditional discriminations, would negate or cloud the significance of presumed transitivity and equivalence tests, especially if those tests were for some other reason to yield consistent results. It is vital, therefore, to be able to identify the type of control in each baseline conditional discrimination. Are there ways to identify Type S and Type R control even when that control varies?

One way to appraise the type of control is to determine whether the outcomes of seeming transitivity and equivalence tests flip-flop as the potential size of the classes they encompass is enlarged by the addition of nodes (Figures 8 and 9). There is no reason to expect such regular changes if the type of control varies. Additional tests are also possible: Analyses to be outlined below will show that the type of control can often be identified by means of reflexivity and class-membership tests.

*Symmetry.* In Figure 5, we saw that symmetry tests cannot be expected to distinguish between Type S and Type R control. Variability in the type of control does not alter this prediction.

*Reflexivity.* Some reflexivity tests can retain their ability to discriminate between Type S and Type R control even when the type of control varies. In Figure 12, which shows Type S control in A-B and Type R control in B-C, the predictions in reflexivity tests that involve A and C stimuli are relatively straightforward: A-B is an instance of Relation S; in A-A reflexivity tests, subjects can be expected to select and touch the comparison that is identical to the sample. On the other hand, in C-C reflexivity tests, subjects can be expected to touch the comparison that differs from the sample. This combination of test results—"identity matching" with one pair of samples and "oddity matching" with the other pair—would inform us that the type of relation varies from one directly taught conditional discrimination to the other.

Tests of this prediction, with select and reject relations controlled experimentally, have yet to be carried out. The baseline that is to be used during such tests must, however, be carefully selected. For example, a baseline that includes both A-B and B-C trials will provide a context that contains both Relation S and Relation R. This combination introduces an element of uncertainty into the prediction of which of the two baseline relations will prevail in A-A and C-C test trials, and other factors may enter the picture to resolve this uncertainty (see the later discussion of contingency reversals). A more cautious course to follow would be to present A-A trials in the context of the A-B baseline (Table 1, Groups I and III) and C-C trials in the context of the B-C baseline (Table 1, Groups VI and VIII).

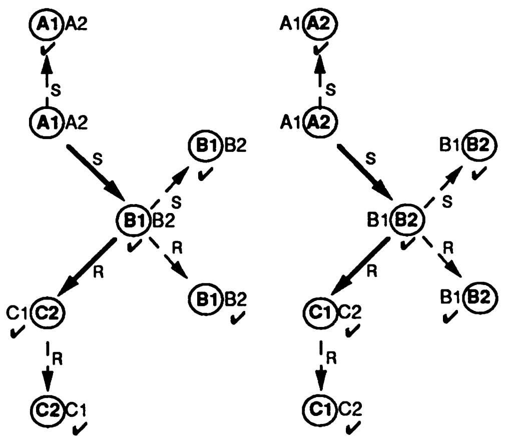


Fig. 12. The expected outcomes of reflexivity tests when the A-B conditional discrimination is under Type S control and B-C is under Type R control.

Reflexivity tests in which B1 or B2 serves as the sample will also require restricted baseline contexts. The B stimuli, unlike the A and C stimuli, are included both in Type S (A-B) and Type R (B-C) relations. B-B test trials that are embedded in a baseline of both A-B and B-C trials will therefore be subject to conflicting sources of control. Presenting B-B trials in a baseline context that consists only of A-B trials (Relation S) can, however, be expected to yield "identity matching"; in a baseline consisting only of B-C trials (Relation R), the reflexivity test should yield "oddity."

What results are to be expected in reflexivity tests when the type of control remains constant from one conditional discrimination to the other but varies within each? In conjunction with Figure 11, we saw that this kind of variation clouds the evaluation of transitivity or equivalence. Still, some reflexivity tests can provide information about Type S and Type R control. For example, in the two left-hand diagrams in Figure 13, A-A reflexivity tests provide a clear indication of variability in the nature of the relation that controls the A-B conditional discrimination: With A1 included in Relation S and A2 in Relation R, a subject can be expected always to touch comparison A1 in the test, regardless of whether A1 or A2 is the

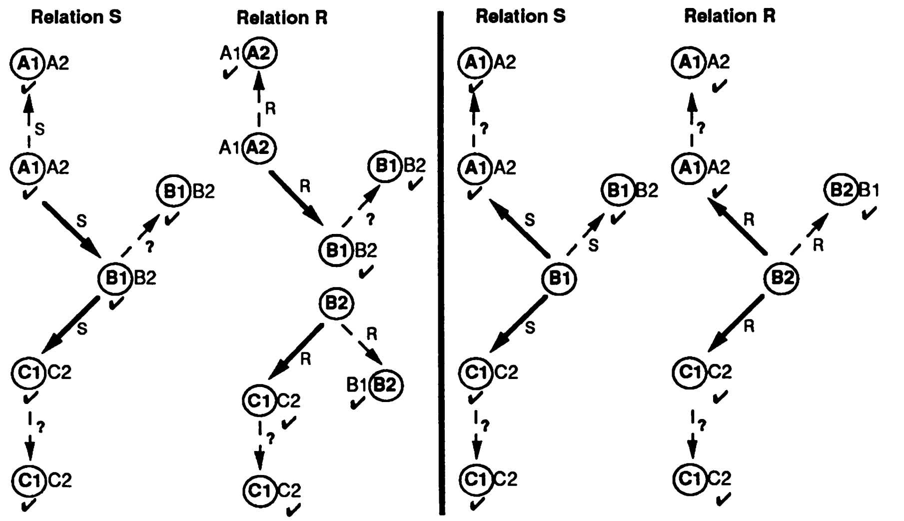


Fig. 13. The expected outcomes of transitivity, equivalence, and reflexivity tests when the type of control varies as a function of the sample within each conditional discrimination. In the two left-hand diagrams, the baseline conditional discriminations are A-B and B-C; in the two right-hand diagrams, the baselines are B-A and B-C.

sample. Also, because B2 is involved only in a reject relation (B2-C1), B-B test trials with B2 as the sample should lead a subject to touch B1.

The remaining B-B and C-C test results are unpredictable. B1 and C1 are controlling comparisons in both Relations S and R, thereby setting up conflicts in test trials with B1 or C1 as the sample. And because C2 is never a controlling comparison, the type of control in B-C has no bearing on the outcome of trials in which C2 is the sample.

The arrangement in the two right-hand diagrams in Figure 13 has the advantage of maintaining the B stimuli as common elements in the directly taught B-A and B-C conditional discriminations. Here, B-B reflexivity tests, like A-A tests in the left-hand diagrams, clearly document variability in the type of control within each conditional discrimination: If B1 is included in Relation S and B2 in Relation R, a subject can be expected always to touch comparison B1 in the test, regardless of whether B1 or B2 is the sample.

The results of A-A and C-C tests, however, are unpredictable: With A1 and C1 involved in both Relations S and R, a trial with A1 or C1 as a sample sets up a conflict; and because neither A2 nor C2 is ever a controlling comparison in the baseline conditional discriminations, the type of control in those discriminations has no bearing on the outcome of test trials in which A2 or B2 is the sample.

Selected reflexivity tests can therefore reveal variations in the type of controlling relation from one conditional discrimination to another, or within a particular conditional discrimination. By using these tests to identify the variations, one can determine the validity of transitivity and equivalence tests. Again, these predictions can be verified by experimentally controlling whether a Type S or Type R relation governs the baseline conditional discriminations.

### *Class-Membership Tests*

If conditional relations are also equivalence relations, the outcome will be at least two equivalence classes. The composition of each class will depend on the type of control. In the case of two-member classes, Type S control will give rise to the related pairs A1-B1 and A2-B2; Type R control will give rise to the pairs A1-B2 and A2-B1. With three-member classes of equivalent stimuli, as in Figure 3, Relation S can be expected to lead to the two classes A1B1C1 and A2B2C2. If, however, the baseline conditional discriminations are instances of equivalence Relation R, then the outcome can be expected to be two other classes, A1B2C1 and A2B1C2.

This prediction that Relations S and R will yield different equivalence classes has not been directly tested. Figures 6, 8, and 9 illustrate indirect tests, but independent identification of the stimuli in each class is desirable. A tactic for doing so would be to create new kinds of classes, one containing A1 and C1, and the other, A2 and C2 (e.g., Lazar, 1977; Sidman, Wynne, Maguire, & Barnes, 1989; Vaughan, 1988). Then, determine which of the new classes also includes B1 and which includes B2.

For example, a subject might be taught sequence classes, with A1 and C1 being *firsts* and A2 and C2 being *seconds* (e.g., Lazar, 1977; Sigurdardottir et al., 1990; Wulfert & Hayes, 1988). Then, if the subject touched B1 first and B2 second, without having been directly taught a B1-B2 sequence, one could conclude that B1 was in the same class as A1 and C1, and that B2 was in the same class as A2 and C2. This would also indicate that A-B and B-C were instances of Relation S. On the other hand, if, in a test, the subject touched B2 first and B1 second, one would have to conclude that the classes were A1B2C1 and A2B1C2, and that A-B and B-C were instances of Relation R. If such tests were carried out in contexts like those illustrated in Figures 8 and 9, they would be even more convincing; they would involve larger classes and would contain internal replications.

Experimental control over the type of relation in the baseline conditional discriminations permits the kind of class-membership test outlined above. Uncontrolled variation from one baseline conditional discrimination to another would, however, preclude such a simple test. If A-B and B-C were not members of the same relation (Figure 10), any class membership test that depended on assumed A-C or C-A relations would be of doubtful validity. A test for class membership would have to be free of such dependence. Such a test is, in fact, possible. Without giving a detailed description, the general tactic for testing class membership would again be to create new kinds of stimulus classes, but this time four rather than two: one containing A1, another containing A2, a third containing C1, and a fourth containing C2. Then, determine which of the new classes also included B1 and which included B2.

Would class membership tests still be useful if the type of relation varied within the conditional discriminations? The answer must be a highly qualified "yes." In the two examples of variation illustrated in Figure 11, the results of class-membership tests are not predictable on the basis of the type of control in the baseline conditional discriminations. The causes of this unpredictability are the same as those that were shown in conjunction with Figure 11 to reduce the predictability of transitivity and equivalence tests.

This indeterminacy when the nature of the relation varies within a conditional discrimination may, however, still be useful. Its possible utility arises from the contrast with the high level of predictability of class-membership tests when the controlling relation is constant, or when the variation occurs from one baseline conditional discrimination to another. If a class-membership test produced results that were indecipherable (either variable or consistent in ways not predicted by our analysis), this would be a strong indication of the kind of variation that Figure 11 illustrates.

*Reversing the Conditional Discriminations*

A potentially useful technique for analyzing equivalence classes is to reverse the baseline contingencies. For example, teach a subject first to touch B1 when the sample is A1 and B2 when the sample is A2. Then, after showing that the A-B relations are equivalence relations, teach the subject to do the opposite: Touch B2 when the sample is A1 and B1 when the sample is A2. What effects will such reversals have on the outcome of tests for the properties of equivalence relations? The predictions vary, depending on whether, among other factors, Relation S or Relation R is involved.

Contingency reversals, by definition, require consideration of the subject's experience with the stimuli and relations being analyzed, and with the tests. If a controlling stimulus–response relation is no longer reinforced, and therefore stops occurring (as happens when a simple discrimination is reversed), one cannot conclude that the controlling relation no longer exists (Ray, 1969). The same may be said of stimulus–stimulus relations. Even though a relation between a sample and a particular comparison is no longer correlated with reinforcement (as happens when a conditional discrimination is reversed), the original relation may later reappear, particularly during tests in which the contingencies and/or the context exert less than optimal control.

This kind of historical variable must always be considered in analyzing the results of discrimination reversals. If relations established before and after a reversal become simultaneously and equally possible in a test trial, other variables will resolve the conflict between the two relations. It is up to the investigator to ensure that those other variables are relevant and not extraneous to the purpose of the experiment.

Saunders, Saunders, Kirby, and Spradlin (1988) and Pilgrim and Galizio (1990) described experiments in which the membership of equivalence classes was apparently not rearranged after one or more baseline conditional discriminations had been reversed. Saunders et al. attributed their results to the prepotency of the relations that had emerged in the original tests, and suggested several experiments to check this possibility. Pilgrim and Galizio, who tested all three properties of equivalence relations, found seeming inconsistencies among the tests. For this reason, they considered it unlikely that their subjects were simply maintaining stimulus classes that had been established before the reversal. Because the baseline conditional discriminations in these studies corresponded to our B-A, B-C paradigm (Saunders et al. actually used A-B, C-B), we will use that arrangement in the following discussion. The upper panel of Figure 14 illustrates the expected outcomes of tests for symmetry (A-B and C-B) and transitivity/equivalence (A-C and C-A) when the baseline consists of B-A and B-C trials.

As in our analysis of the A-B, B-C baseline, Relations S and R can be expected to yield the same recorded results in symmetry tests but opposite results in transitivity or equivalence tests (Figures 6 and 7). The results that both Saunders, Saunders, Kirby, and Spradlin (1988) and Pilgrim and Galizio (1990) described after their subjects were exposed to the initial contingencies were consistent with Type S control. Then, with A-B remaining the same, both groups of investigators reversed the B-C conditional discriminations, as illustrated in the center panel of Figure 14. With B1 as the sample, subjects now received reinforcers when they touched C2 instead of C1, and with B2 as the sample, when they touched C1 instead of C2. (Pilgrim and Galizio required their subjects to displace a comparison card.) With the new baseline, C-B symmetry tests showed the expected reversals; given C2 as a sample, subjects touched B1, and given C1, they touched B2 (Pilgrim & Galizio, 1990). Both groups of experimenters found, however, that in spite of the reversal of the B-C contingencies, the subjects' performance on A-C and C-A tests did not change.

If, after a discrimination reversal, the baseline relation remains Type S (Figure 14, left-hand diagrams in the center panel), then a reversal of the subject's symmetry test (C-B) performance, accompanied by maintenance of the original A-C and C-A test performances, would indeed provoke a reappraisal of our formulation of equivalence relations. The seeming difference between the symmetry and transitivity (or equivalence) tests might be reconciled, however, by considering the possibility that a reversal may change the baseline relation from Type S to Type R. Such a change would not be unreasonable. Given a particular sample, a reversal of the contingencies might lead a subject not to select a new comparison but rather to reject the old one.

A change from Type S to Type R control might take place only in the B-C conditional discrimination, or it might spread also to B-A. If a discrimination reversal causes the control in both B-C and B-A to change to Type R (Figure 14, right-hand diagrams in the center panel), then one would predict the test results that have been reported; the recorded C-B performances would reverse, and the recorded A-C and C-A performances would remain the same as they had been before the contingency reversal.

It is not likely that the control in both B-C and B-A changed from Type S to Type R in the Pilgrim and Galizio (1990) study. These investigators did not do class-membership tests, but they did test for reflexivity. As shown in Figure 4, Relation S and Relation R should yield different recorded results in reflexivity tests. Pilgrim and Galizio, however, reported no changes in those tests after the contingency reversal.

A more likely possibility is that a contingency reversal in the B-C conditional discrimination will change only the B-C relation from Type S to Type R. Then, one would still expect changes in symmetry tests like those reported, but A-C and C-A tests would no longer be valid indicators of transitivity/equivalence. As we discussed in conjunction with Figure 10, if such tests are to be relevant to the evaluation of equivalence, A-B and B-C must be members of the same relation. In this instance, because the baseline performance might reflect two distinct relations, a test spanning both

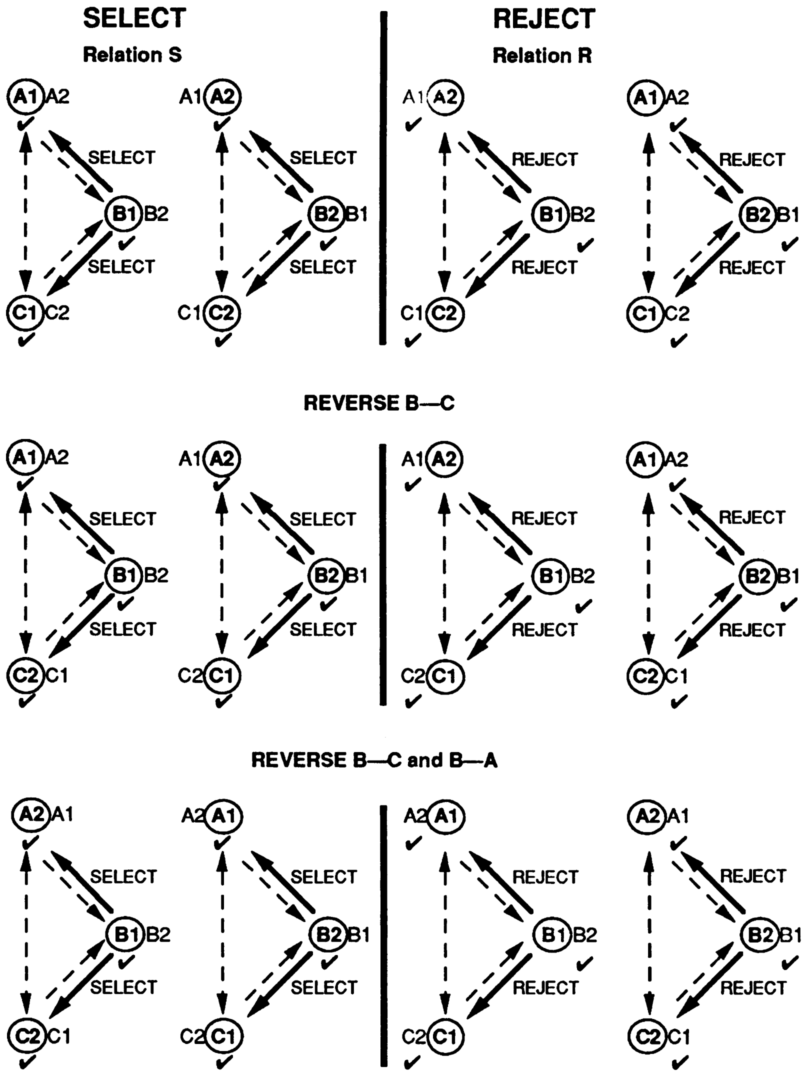


Fig. 14. Upper panel: the expected outcomes of tests for symmetry (A-B and C-B) and transitivity/equivalence (A-C and C-A) when the baseline consists of B-A and B-C conditional discriminations under Type S and Type R control. Center panel: the expected effects of reversing the B-C conditional discriminations. Bottom panel: the expected effects of reversing both the B-C and B-A conditional discriminations.

baseline relations cannot be expected to yield results that are based on transitivity.

What can subjects be expected to do when given A-C or C-A tests in which transitivity of the current baseline relations is irrelevant to their choice of a comparison? At least two predictions are reasonable. One is that subjects, like 1 reported by Pilgrim and Galizio (1990) and 1 by Saunders, Saunders, Kirby, and Spradlin (1988), will behave inconsistently. Another is that subjects, like 3 reported by Pilgrim and Galizio and 2 by Saunders et al., will do the same as they did in the first test, before the reversal.

Our earlier analysis of the situation in which the type of relation differed between A-B and B-C suggested that A-A and C-C reflexivity tests should reflect this difference (Figure 12). They did not do so in the Pilgrim and Galizio (1990) study. We also noted, however, that reflexivity tests given in the context of a mixed Type S and Type R baseline might produce a conflict; in such an event, subjects might simply continue to do the same as they had in the first reflexivity tests. If only the B-A baseline were used for A-A tests and only the B-C baseline were used for C-C tests, conclusions about reflexivity might be more valid. The baseline was shown to be especially important for B-B tests.

Pilgrim and Galizio (1990) went on to reverse the contingencies in the B-A conditional discriminations, while maintaining the reversed B-C contingencies. They were again puzzled by a seeming discrepancy between the results of symmetry and transitivity tests: Once more, the symmetry tests reflected the reversed contingencies, but the transitivity tests remained unchanged. The left-hand diagrams in the bottom section of Figure 14 show, however, that these results are to be expected if the complete reversal causes the baseline control to shift back to Type S. Such a shift is entirely reasonable; as Pilgrim and Galizio pointed out, the performances they observed were consistent not just with the original contingencies but also with the completely reversed contingencies. Given that the reversed relations are Type S, there is no reason to consider the performance in the two kinds of tests to be contradictory. To determine whether this assumption is not merely reasonable but is also correct will require not only tests for the properties of equivalence relations but also explicit class-membership tests.

Our analysis suggests that when interpreting the effects of discrimination reversals on tests for the properties of equivalence relations, it is critical to determine whether the same relation is being tested before and after the reversals. Experimental control over the nature of the baseline relations, evaluation of the properties of equivalence in the context of compatible baselines, and explicit class-membership tests are needed to determine whether changes in the nature of the baseline relation can account for data obtained without such control, and whether postulated shifts from Type S to Type R control, or vice versa, are accompanied by the predicted alterations in the composition of the classes.

## CONCLUSIONS

We have provided a detailed analysis of ways in which control by the negative stimulus in two-comparison conditional discriminations may affect the outcome of tests for the properties that define equivalence relations. Some of the experiments that are needed to evaluate the adequacy of the analysis have been done, and the results so far lend considerable plausibility to the analysis. Many experiments remain to be done, both to test the predictions arising from the analysis and to determine whether the analysis can account for existing experimental data. We believe that the suggested experiments are worth doing. They emphasize the potential power of a stimulus-control analysis that does not require the postulation of any new theoretical concepts, constructs, models, or postulates to integrate data from a variety of experiments on equivalence relations.

Of perhaps greater importance is the relevance of the analysis not to the theory of equivalence relations, but to the conduct of experiments. For example, carry out reflexivity tests after the baseline has been established, not before; take into account the number of baseline nodes when evaluating a relation for transitivity, or when using the abbreviated test for equivalence; embed test trials in baseline and historical contexts that do not contain or generate conflicting relations; give explicit tests for Type S and Type R control whenever the nature of the baseline relation is in doubt; and, because baseline variations in the type of control will cloud and may even invalidate the interpretation of equivalence tests, use procedures that explicitly control the nature of the baseline relation. If the analysis continues to receive experimental support, one of two options with respect to the use of two-comparison procedures in the study of equivalence relations will become mandatory: Either explicitly control whether the teaching procedures generate Relation S or Relation R, or abandon the two-comparison procedure.

Experimental guarantees of Type S or Type R control in two-comparison conditional discriminations require considerable labor. The extra time and effort that such guarantees entail may cause the two-comparison procedure to be abandoned in studies of equivalence relations. The use of four comparisons should ensure the occurrence of Relation S, because Relation R would require a subject to learn many more related stimulus pairs. The use of three comparisons is likely to achieve the same outcome more economically; Figure 15 illus-

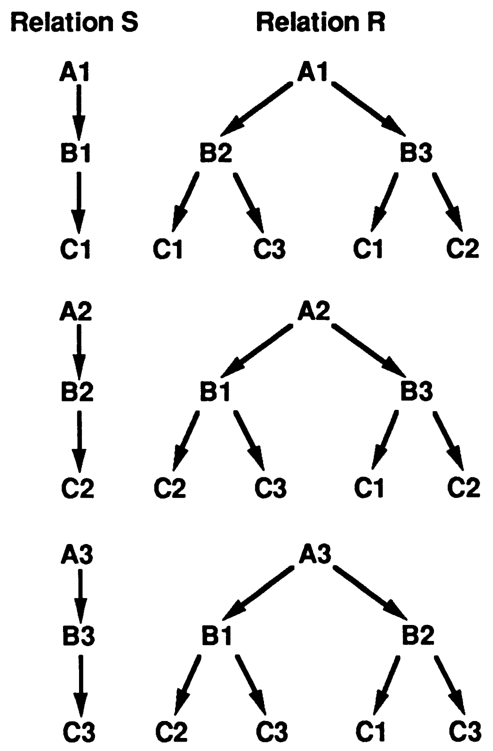


Fig. 15. The A-B and B-C conditional discriminations a subject would have to learn under Relations S and R in the context of a three-comparison procedure.

trates the difference in the context of a three-comparison procedure.

Under Relation S, a subject who is given A1 as the sample need learn only to select comparison B1; given B1 as the sample, the subject need learn only to select C1; and so forth. With three samples in A-B (A1, A2, A3) and three in B-C (B1, B2, B3), and with only one comparison related to each sample, a subject would have to learn the six illustrated sample-comparison pairs. Under Relation R, however, a subject would have to learn to reject two of the comparisons for each of the six samples, for a total of 12 sample-comparison pairs. When three rather than two comparisons are used, the economy of effort for a subject who has to learn only six pairs under Type S control, rather than 12 pairs under Type R control, should almost guarantee the occurrence of Relation S. Such an outcome will permit experimenters to avoid errors of interpretation that inevitably arise from their inability to distinguish between Relation S and Relation R in the two-comparison situation.

## REFERENCES


Berryman, R., Cumming, W., Cohen, L. R., & Johnson, D. F. (1965). Acquisition and transfer of simultaneous oddity. *Psychological Reports,* **17,** 767–775.

Carrigan, P. F. (1986). *Conditional discrimination and transitive relations: A theoretical and experimental analysis.* Unpublished doctoral dissertation, Northeastern University, Boston.

Carter, D. E., & Werner, T. J. (1978). Complex learning and information processing by pigeons: A critical analysis. *Journal of the Experimental Analysis of Behavior,* **29,** 565–601.

Constantine, B. (1981). *An experimental analysis of stimulus control in simple conditional discriminations.* Unpublished doctoral dissertation, Northeastern University, Boston.

Cumming, W. W., & Berryman, R. (1965). The complex discriminated operant: Studies of matching-to-sample and related problems. In D. I. Mostofsky (Ed.), *Stimulus generalization* (pp. 284–330). Stanford, CA: Stanford University Press.

Devany, J. M., Hayes, S. C., & Nelson, R. O. (1986). Equivalence class formation in language-able and language-disabled children. *Journal of the Experimental Analysis of Behavior,* **46,** 243–257.

Dixon, M. H., & Dixon, L. S. (1978). The nature of standard control in children's matching to sample. *Journal of the Experimental Analysis of Behavior,* **30,** 205–212.

Dixon, M., Dixon, L., & Spradlin, J. (1983). Analysis of individual differences of stimulus control among developmentally disabled children. In K. Gadow & I. Bialer (Eds.), *Advances in learning and behavior disabilities* (Vol. 1, pp. 85–110). New York: JAI Press.

Dube, W. V., Green, G., & Serna, R. W. (in press). Auditory successive conditional discrimination and auditory stimulus equivalence classes. *Journal of the Experimental Analysis of Behavior.*

Farthing, G. W., & Opuda, M. J. (1974). Transfer of matching-to-sample in pigeons. *Journal of the Experimental Analysis of Behavior,* **21,** 199–213.

Fields, L., Adams, B. J., Verhave, T., & Newman, S. (1990). The effects of nodality on the formation of equivalence classes. *Journal of the Experimental Analysis of Behavior,* **53,** 345–358.

Fields, L., & Verhave, T. (1987). The structure of equivalence classes. *Journal of the Experimental Analysis of Behavior,* **48,** 317–332.

Fields, L., Verhave, T., & Fath, S. (1984). Stimulus equivalence and transitive associations: A methodological analysis. *Journal of the Experimental Analysis of Behavior,* **42,** 143–157.

Green, G., Sigurdardottir, Z. G., & Saunders, R. R. (1991). The role of instructions in the transfer of ordinal functions through equivalence classes. *Journal of the Experimental Analysis of Behavior,* **55,** 287–304.

Harrison, R. J., & Green, G. (1990). Development of conditional and equivalence relations without differential consequences. *Journal of the Experimental Analysis of Behavior,* **54,** 225–237.

Jenkins, J. J. (1963). Mediated associations: Paradigms and situations. In C. N. Cofer & B. S. Musgrave (Eds.), *Verbal behavior and learning: Problems and processes* (pp. 210-245). New York: McGraw-Hill.

Lazar, R. (1977). Extending sequence-class membership with matching to sample. *Journal of the Experimental Analysis of Behavior,* **27,** 381–392.

Lazar, R. M., Davis-Lang, D., & Sanchez, L. (1984). The formation of visual stimulus equivalences in children. *Journal of the Experimental Analysis of Behavior,* **41,** 251–266.

McIlvane, W. J., Kledaras, J. B., Munson, L. C., King, K. A. J., de Rose, J. C., & Stoddard, L. T. (1987). Controlling relations in conditional discrimination and matching by exclusion. *Journal of the Experimental Analysis of Behavior,* **48,** 187–208.

McIlvane, W. J., Withstandley, J. K., & Stoddard, L. T. (1984). Positive and negative stimulus relations in severely retarded individuals' conditional discrimination. *Analysis and Intervention in Developmental Disabilities,* **4,** 235–251.

Pilgrim, C., & Galizio, M. (1990). Relations between baseline contingencies and equivalence probe performances. *Journal of the Experimental Analysis of Behavior,* **54,** 213–224.

Ray, B. A. (1969). Selective attention: The effects of combining stimuli which control incompatible behavior. *Journal of the Experimental Analysis of Behavior,* **12,** 539–550.

Saunders, R. R., & Green, G. (1992). The nonequivalence of behavioral and mathematical equivalence. *Journal of the Experimental Analysis of Behavior,* **57,** 227–241.

Saunders, R. R., Saunders, K. J., Kirby, K. C., & Spradlin, J. E. (1988). The merger and development of equivalence classes by unreinforced conditional selection of comparison stimuli. *Journal of the Experimental Analysis of Behavior,* **29,** 145–162.

Saunders, R. R., Wachter, J., & Spradlin, J. E. (1988). Establishing auditory stimulus control over an eight-member equivalence class via conditional discrimination procedures. *Journal of the Experimental Analysis of Behavior,* **49,** 95–115.

Sidman, M. (1986). Functional analysis of emergent verbal classes. In T. Thompson & M. D. Zeiler (Eds.), *Analysis and integration of behavioral units* (pp. 213–245). Hillsdale, NJ: Erlbaum.

Sidman, M. (1987). Two choices are not enough. *Behavior Analysis,* **22,** 11–18.

Sidman, M. (1990). Equivalence relations: Where do they come from? In D. E. Blackman & H. Lejeune (Eds.), *Behavior analysis in theory and practice: Contributions and controversies* (pp. 93–114). Hillsdale, NJ: Erlbaum.

Sidman, M. (1992). Equivalence relations: Some basic considerations. In S. C. Hayes & L. J. Hayes (Eds.), *Understanding verbal relations* (pp. 15–27). Reno, NV: Context Press.

Sidman, M., Kirk, B., & Willson-Morris, M. (1985). Six-member stimulus classes generated by conditional-discrimination procedures. *Journal of the Experimental Analysis of Behavior,* **43,** 21–42.

Sidman, M., Rauzin, R., Lazar, R., Cunningham, S., Tailby, W., & Carrigan, P. (1982). A search for symmetry in the conditional discriminations of rhesus monkeys, baboons, and children. *Journal of the Experimental Analysis of Behavior,* **37,** 23–44.

Sidman, M., & Tailby, W. (1982). Conditional discrimination vs. matching to sample: An expansion of the testing paradigm. *Journal of the Experimental Analysis of Behavior,* **37,** 5–22.

Sidman, M., Wynne, C. K., Maguire, R. W., & Barnes, T. (1989). Functional classes and equivalence relations. *Journal of the Experimental Analysis of Behavior,* **52,** 261–274.

Sigurdardottir, Z. G., Green, G., & Saunders, R. R. (1990). Equivalence classes generated by sequence training. *Journal of the Experimental Analysis of Behavior,* **53,** 47–63.

Steele, D., & Hayes, S. C. (1991). Stimulus equivalence and arbitrarily applicable relational responding. *Journal of the Experimental Analysis of Behavior,* **56,** 519–555.

Stikeleather, G., & Sidman, M. (1990). An instance of spurious equivalence relations. *The Analysis of Verbal Behavior,* **8,** 1–11.

Stromer, R., & Osborne, J. G. (1982). Control of adolescents' arbitrary matching-to-sample by positive and negative stimulus relations. *Journal of the Experimental Analysis of Behavior,* **37,** 329–348.

Urcuioli, P. J., & Nevin, J. A. (1975). Transfer of hue matching in pigeons. *Journal of the Experimental Analysis of Behavior,* **24,** 149–155.

Vaughan, W., Jr. (1988). Formation of equivalence sets in pigeons. *Journal of Experimental Psychology: Animal Behavior Processes,* **14,** 36–42.

Wulfert, E., & Hayes, S. C. (1988). Transfer of a conditional ordering response through conditional equivalence classes. *Journal of the Experimental Analysis of Behavior,* **50,** 125–144.

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