JOURNAL OF THE EXPERIMENTAL ANALYSIS OF BEHAVIOR 1992, 57, 249-266 NUMBER 3 (MAY)

BEHAVIOR DYNAMICS: ONE PERSPECTIVE

M. JACKSON MARR

GEORGIA INSTITUTE OF TECHNOLOGY

Behavior dynamics is a field devoted to analytic descriptions of behavior change. A principal source of both models and methods for these descriptions is found in physics. This approach is an extension of a long conceptual association between behavior analysis and physics. A theme common to both is the role of molar versus molecular events in description and prediction. Similarities and differences in how these events are treated are discussed. Two examples are presented that illustrate possible correspondence between mechanical and behavioral systems. The first demonstrates the use of a mechanical model to describe the molar properties of behavior under changing reinforcement conditions. The second, dealing with some features of concurrent schedules, focuses on the possible utility of nonlinear dynamical systems to the description of both molar and molecular behavioral events as the outcome of a deterministic, but chaotic, process.

Key words: dynamical systems, classical mechanics, mathematical models, chaotic dynamics, pigeons

Most of the papers appearing in this special issue are outgrowths of presentations at the Behavior Dynamics Conference held at Jacksonville State University, Jacksonville, Alabama, in June 1989. This conference was entirely supported by the Eminent Scholar Program at Jacksonville State University, the generous funding of which was a necessary, but not sufficient, condition for a successful meeting. Also required was the very creative and hard-working efforts of Bill and Betty Palya, Don Walter, Bud Gardner, and many other behind-the-scenes workers. Special thanks also go to Ed Fantino, Marc Branch, and Greg Galbicka for making this issue possible and encouraging other authors to participate with the call for papers on this topic.

The theme of the Jacksonville Conference was occasioned by my longstanding interest in fostering the development of mathematical approaches to behavior analysis (Marr, 1984, 1989) and in exploring the relationships, historical and conceptual, between behavior analysis and physics (Marr, 1985, 1986, 1990). As readers of this issue will quickly discern, behavior dynamics is a field with many different perspectives but with a common focus on descriptions of behavior change and the conditions bringing about that change. What follows is a rather general treatment of the concept of behavior dynamics as a metaphorical extension of the term as it is used in physics. In particular, operant behavior as it interacts with contingencies of consequences can be viewed as a dynamical system that involves the action of forces to maintain, displace, or dissipate behavior in various ways and controls possible states of equilibrium, stable or unstable.

BEHAVIOR AND DYNAMICS

The application of dynamic analyses to behavioral questions is a potentially pervasive exercise. Acquisition, extinction, periodic, aperiodic, and response-produced variations in reinforcement frequency under different contingencies, changes in quantity and quality of reinforcement, variations in delay of consequences, response patterns and their variation under simple schedules of reinforcement, sequential and distributional properties of molar as well as molecular features of behavior, and stimulus control all are areas familiar to behavior analysts that could conceivably benefit from consideration of dynamical systems. These are all old problems involving traditional procedures, largely studied under very restricted conditions, with the transitions between states almost totally ignored, both experimentally and theoretically.

Behavior analysts first sought to understand behavior change (Skinner, 1938), but somehow this goal became obscured by the aesthetics and seeming simplicity of steady-state

Please address correspondence and reprint requests to M. Jackson Marr, School of Psychology, Georgia Tech, Atlanta, Georgia 30332-0170.

performance, as exemplified in the description and exploration of behavior engendered by schedules of reinforcement. The early enchantment with schedules has waned, in part because the analyses of their properties turned out to be far more complex than first appeared. Traditional contingencies may be simple to describe, but the orderly patterns of responding they generate are substantial challenges to understanding. The stable or metastable responding engendered is the outcome of dynamic interactions between ongoing behavior and the consequences of that behavior. This was recognized early on by Skinner (1938), Morse (1966), Herrnstein and Morse (1958), Anger (1956), Blough (1966), and others, but major analytical treatments have only recently begun to emerge. In *The Behavior of Organisms* (1938), Skinner devoted much space to the treatment of "simple" acquisition and extinction. Despite his experimental and theoretical sophistication, his attempts at analytical description were, at best, crude. Interesting approaches to Skinner's results took some 50 years (e.g., Killeen's, 1988, formulation of the reflex reserve). The concept of the reflex reserve is not likely to take us very far today, but the kinds of results Skinner discussed still represent significant challenges to an adequate analysis of behavior dynamics.

In physics, dynamics is a field with a long and complex history, founded by Isaac Newton and subsequently developed by such giants as Euler, Lagrange, Laplace, Hamilton, and Poincaré (Dugas, 1988). By the late 19th century, dynamics entered a relatively dormant phase of creative activity. In recent years, however, the development of nonlinear dynamical systems theory has transformed the staid and venerable field of classical mechanics into the third great scientific revolution of the 20th century (after relativity and quantum mechanics). The principal phenomena now avidly studied by the modern nonlinear dynamicist were often studied by earlier investigators, but via closed, local, linear approximations. Virtually all instruction of students dealt with these kinds of procedures, not realizing that the assumption of linearity cast a thin but obfuscating veneer over the vast, intricate, and beautiful world of the everyday, the very phenomena the science had set out to understand in the first place.

The accelerated interest in behavior change and transition parallels the exciting developments in the general study of dynamical systems and the spread of its application to problems of enormous diversity, including behavior analysis. Here are some general but interactive issues raised by consideration of behavior-dynamic processes:

Molar versus molecular approaches. How are they best defined and what connections are there between them? What constitute the behavioral units upon which, for example, reinforcement acts? We know that during the acquisition of any complex performance, behavior present at the beginning may disappear, and new types of behavior may emerge. What variables control which types of change and in what directions?

Discrete versus continuous models. Assumptions about molar versus molecular issues may lead to the question of under what conditions is behavior more effectively treated as a continuous, as opposed to a discrete, variable? This has clear implications for choices of analytical methods.

Stochastic versus deterministic models. Can the putative stochastic character of much behavioral data be generated by a deterministic dynamical description characteristic of a chaotic process? What criteria should be applied to assess the feasibility of such a description? Is it worth it?

Behavioral variation. Behavioral variation is the foundation for the generation of new behavior. As such, to understand behavior change at all, we need good general theories of behavior variation. But what should they be like?

Constants of motion. Are there dimensions of behavior that are conserved when behavior changes? What, if any, extremum principles can we invoke in the manner, say, of Hamiltonian mechanics? Certainly, operant behavior is highly dissipative, requiring the relatively frequent impulse of consequence to keep it going. But the organism does not necessarily come to rest upon extinction of a given class of behavior; it just does something else. How does the acquisition or elimination of one class of behavior affect others?

Meta-theoretical structure. To what extent must hypothetical constructs and intervening variables play a role? Can an analogy with physical dynamical systems reduce our reliance on those problematical features of psychological theory? Alternatively, are models derived from analogues of physical systems appropriate to deal with biological systems (see Zeiler, 1992)?

Methods. What new formalisms and experimental procedures can be brought to bear on both the empirical and theoretical analysis of behavior dynamics? For example, unlike classical mechanics where time has no inherent direction, most, if not all, behavioral phenomena can be shown to possess irreversible features. What methods can we use to deal with this pervasive effect?

Most of these topics and questions have a long research history in behavior analysis, and some are directly or indirectly addressed by papers in this issue. In the course of the following discussion some of these issues will be considered in more detail. They are like entwined motives blending into a single composition.

THE PHYSICS–BEHAVIOR AXIS

Throughout its history, psychology has been nourished by and sought inspiration from that most prolific and fundamental of sciences—physics. From Descartes to La Mettrie, to Fechner and Helmholtz, to Loeb and Freud, to Köhler and Wertheimer, to Hull and Skinner, models and descriptions of action have been constructed whose inspiration came directly from physics. The experimental analysis of behavior in its philosophical foundations and its practice is a fairly straightforward extension and transmogrification of methods and concepts of the likes of Galileo, Newton, Faraday, Mach, and Gibbs. Skinner's well-known allusion to Russell's remark comparing force and reflex emphasizes a special relationship and is taken to be more than a casual and comfortable metaphor.

Newton's achievement was to describe the action of forces as causes of changes in motion; Skinner's achievement was to describe the action of reinforcement as a cause of changes in behavior. Just as *force* was a concept inferred to understand changes in motion, *reinforcement* was a concept inferred to understand changes in behavior. Reinforcement in its interaction with ongoing behavior can thus be considered the equivalent of force in physics, because it is a principal agent of change in the rate of behavior. Newton founded the field of dynamics, concerned with the composition of forces as they affected the motion of bodies; Skinner founded behavioral dynamics, concerned with contingencies of reinforcement as they affected the behavior of organisms.

Look to a fundamental concept in the experimental analysis of behavior—contingency of reinforcement. Here we find a dynamical system driven by the interactions between behavior and consequence, allowing infinite variation, and engendering, along with a bewildering fine structure, molar temporal patterns of responding that rival the elegant beauty of planetary motion.

To be sure, there are some important differences between the concept of force in physics and reinforcement in behavior analysis. Among the most salient is the fact that forces act through space, but reinforcers act through time. In both cases, however, the exact quantitative relation holding between magnitude and distance determines the effects seen. Considerable theory and research have been devoted to ascertaining the relations holding between effects on responding and the time between a response and a reinforcer (see, e.g., Commons, Mazur, Nevin, & Rachlin, 1987).

An even more important difference, perhaps, is that action through time implies history, and history implies irreversibility. In Newtonian mechanics, time has no inherent direction and predictability depends only on imposing initial conditions on general solutions of the proper differential equations of motion. The origin of the initial conditions is irrelevant. Further, if the equations are linear, small changes in initial conditions lead to small changes in outcomes. In contrast, the application of a contingency of reinforcement yields effects that may depend not only on a host of initial conditions (such as the particular behavior occurring, its ongoing rate, prevailing stimulus conditions, deprivation level, etc.) but also on a long history with other contingencies under other conditions (see, e.g., Barrett, 1985; Morse & Kelleher, 1977).

Continued exposure to a contingency results in what is commonly called steady-state performance. Characteristics of steady-state performance under various schedules of reinforcement have been extensively studied since Skinner's early work (Ferster & Skinner, 1957; Morse, 1966; Nevin, 1973; Schoenfeld, 1970; Skinner, 1938; Zeiler, 1977; Zeiler & Harzem, 1979). This program of research has focused on carefully describing the patterns of responding under different schedules and specifying the variables controlling those patterns, interdependent tasks of far greater difficulty than initially anticipated. "Steady state" need not, of course, mean "static state," but rather can include conditions of dynamic equilibrium, subject to shifts under the influence of sometimes subtle variables (e.g., Staddon, 1988). An interesting example is the fixed-interval schedule that engenders a pattern typically described as a pause after reinforcement lasting about a third to a half of the value of the interval parameter, followed by an increasing rate of responding until the moment of reinforcer delivery. This description conceals as much as it reveals. As Skinner pointed out many years ago, there are inherent variations in the pattern—response rate from session to session, response rate from interval to interval, response rate within the interval, and response grouping or tempo within an episode of responding (Skinner, 1938). In addition to these variations, one could focus on changes in pause times from interval to interval, or the conditions prevailing at the moment responding begins, or the sequential properties of all the possible performance measures associated with the schedule (see, e.g., Gentry, Weiss, & Laties, 1983). Sequential properties are of special dynamical interest because they may provide the most revealing clues as to the "forces" controlling responding moment to moment. In the same way, knowing the position and velocity of a moving body as a function of time will define an orbit from which certain dynamical relations may be determined.

BEHAVIOR IN THE MACHINE

The Molar Approach

The fundamental question of dynamics in physics is how forces act on bodies to change their motion. An answer to this question involves Newton's three laws of motion, along with such abstract concepts as mass, inertia, momentum, first and second moments, force–distance functions, potential and kinetic energy, and so forth. A fundamental question in behavior dynamics is how reinforcement acts on behavior to change it. An answer to this question depends, in part, upon knowing what features of behavior, under what conditions, are most sensitive to reinforcing effects. This leads naturally to the question of what dimensions or measures of behavior are appropriate to the analysis of contingency dynamics. The problem is usually framed in terms of molar versus molecular approaches, an old but still very contentious issue in behavior analysis (e.g., Baum, 1989; Bickel & Etzel, 1985; Galbicka & Platt, 1986; McDowell & Wixted, 1986; Nevin, 1982; Shimp, 1979, 1982).

Some lessons from physics may also be helpful here in establishing a rapprochement between molar and molecular perspectives. Both classical mechanics and classical thermodynamics had no need for molecular theories. The laws of mechanics do not depend on knowing the ultimate composition of bodies. Likewise, powerful laws relating variables such as pressure, volume, and temperature could be derived and used without having to ask, for example, what the "real" nature of temperature was. Behavior analysts now have a repertoire of molar laws: Herrnstein's hyperbola, the matching law with response rate or time allocation, the multivariate rate equation, behavior momentum relations, various feedback functions, global maximizing functions, hyperbolic discounting, and so forth, all of which have effective descriptive and predictive value in at least some situations. The functional relations were not derived on the basis of an understanding of the fine structure of responding; indeed, in the case of time allocation, there is just barely a need to specify a behavior at all. However, despite a modicum of success here and there, no one is likely to assert that we are on the threshold of a general molar theory of schedule performance.

Molar approaches to behavior analysis, in common with classical thermodynamics, have tended to focus on steady-state conditions. This does not mean that such approaches have no interest in dealing with dynamical conditions. For example, melioration can be considered a dynamical theory (Herrnstein, 1982). Melioration, however, focuses on an equilibrium state congruent with a steady reinforcement rate; in other words, it may talk about dynamics, but in reality only predicts end states. Nevertheless, molar approaches can treat dynamical systems for which the problem is to predict functions characterizing how behavior will change with time, especially under transient or continuously varying conditions. Such a system is typically described by differential or difference equations. The work of Myerson and his colleagues (Myerson & Hale, 1988; Myerson & Miezin, 1980) exemplifies the effectiveness of this approach.

A given differential equation can model a multitude of systems. A particular system is specified by the interpretation of, and values assigned to, variables and parameters, as well as specification of initial and boundary conditions to yield particular solutions. Any mechanical system describable by a set of differential equations, for example, can be modeled by an electrical circuit; this is the basis of an analog computer. In the same way, any behavioral system describable by a set of differential equations has a mechanical (and electrical) analog. Of course, if one already has a useful set of equations describing some behavior system, there may be little value in looking for a mechanical analog. However, a mechanical system may provide the inspiration for a useful behavioral description. Herein is a foundation or strategy for developing dynamical theories of behavior. It is not the only one, of course, as papers in this issue will attest. The method is also fraught with difficulties, especially in characterizing pertinent behavioral variables and their mechanical analogs, but certainly the limits are yet to be tested.

As an example, consider a modified random-interval schedule in which the interval parameter (and hence the reinforcement frequency) varied with time according to the function

$$r = r_m \sin^2(2\pi t/T), \tag{1}$$

where r is the reinforcement rate, r_m is the maximum reinforcement rate, t is time, and T is the period. Thus, scheduled reinforcement rate will cycle from zero to a maximum and back to zero over the period T. How will response rate vary under this condition? Studies of contingencies of this sort with a time-dependent reinforcement frequency have been made (e.g., Hunter & Davison, 1985; Johnson & Wheeler, 1982; McDowell & Sulzen, 1981; Staddon, 1964). Of this group, only Hunter and Davison provided an analytic treatment of response-rate variation comparing predicted with obtained performance.

The present approach might begin with these intuitions: Responding may be considered to be driven by the current reinforcement rate. It may be expected that momentary response rate will be a function not only of momentary reinforcement rate but also the change in that rate. Ongoing responding may be considered to have a certain resistance to change (i.e., inertia or momentum; see, e.g., Nevin, 1992). Thus, there must be a certain threshold of sensitivity to changes in reinforcement frequency so that momentary responding may not track exactly the momentary reinforcement frequency. This may also produce a phase lag between the response-rate function and the reinforcement rate function.

A mechanical system with analogous properties consists of a bucket with a hole in the bottom pushed down into a tank of water (see Staddon & Ettinger, 1989, pp. 67–70, for a similar analysis of a different behavior system). As the bucket is pushed down, the water begins to flow into the bucket through the hole. The rate of filling will depend jointly on (a) the difference between the depth of the bucket in the tank and the current level of water in the bucket (equivalent to the distance between the surface of the water in the tank and the surface of the water in the bucket) and (b) the size of the hole. If the bucket is pushed to a given depth and held there, the bucket will fill to match the depth of the bucket in the tank. If the hole is as big as the bottom of the bucket, then the bucket will fill instantly and completely "in phase" with the change in depth. A smaller hole, however, will have the effect of "distorting" the filling function and altering the phase relation. Now, imagine that the bucket is not simply pushed into the water and held there at a fixed depth, but is pushed up and down according to a squared sinusoidal function of time (i.e., Equation 1). The level of the water will cycle up and down in some relation to the pushing function. The equation describing the bucket system is

$$dR_b/dt = h(R_d \sin^2(2\pi t/T) - R_b), \tag{2}$$

where R_b is the depth of the water in the bucket, h represents the hole size, and R_d is the maximum depth of the bucket. What are the corresponding variables in the behavior system? The tendency to respond (call it "response strength" as manifested here by response rate) is analogous to the depth of water in the bucket. The changing reinforcement frequency that drives responding is instantiated in the force driving water in and out of the bucket, namely the joint effect of the momentary depth of the bucket and the hole size. To deal with the behavior system using Equation 2, we need to have an expression for response rate as a function of reinforcement rate. For this we can use Herrnstein's hyperbolic function:

$$R = k\ r/(r + r_0). \tag{3}$$

Substituting this function in Equation 2 above will yield finally:

$$dR_b/dt = h\ [(kr_m \sin^2(2\pi t/T) \div r_m \sin^2(2\pi t/T) + r_0) - R_b]. \tag{4}$$

A solution to this equation is shown in Figure 1, superimposed on the associated reinforcement rate function (Equation 1). Figure 2 shows some data collected with a pigeon under a similar squared sinusoidal random-interval schedule. (I am indebted to Bill Palya and Don Walter for their roles in collecting and displaying these data.) The theoretical and experimental functions show at least a superficial similarity. An adequate test, however, of a model of the kind described would, at a minimum, require a number of parametric manipulations (e.g., changing the period or amplitude of the reinforcement rate function).

The method of mechanical analog, though potentially useful in the analytic description of dynamical behavioral systems, has special challenges in its application. The primary problem is to characterize and isolate corresponding variables in the two domains. This, of course, requires a theory that specifies what variables are important in each domain, as well as their conceptual status and interrelation both within and across the two domains. Moreover, it is one thing to generate metaphors linking two fields, for example, to speak of "behavioral momentum" or "reinforcement" as "force." But without a corresponding mathematical account, such metaphors are of little, if any, value. A mathematical model, properly constructed, demands careful specification of pertinent variables and their quantitative interrelations. As such, the model should then not only capture the originally selected features of the phenomenon of interest but also possess a generality to encompass other pertinent conditions and effects. For example, a proper model of how responding might change concurrently with changes in reinforcement frequency should deal with numerous kinds of reinforcement frequency functions, not simply sinusoidal ones like Equation 1. Of course, any given mechanical system serving as an analog to model a behavioral system can, at best, capture only a limited set of features. Refinement in modeling depends on the selection of more appropriate (and, thus, usually more complex) models coupled with better assessment of the behavioral variables.

Selection of a model depends, at least in part, on identifying corresponding functions in the mechanical and behavioral domains (e.g., exponential, sinusoidal, hyperbolic, etc.). These functions, regardless of their origin, are solutions to some class of differential equations. Thus, generality is inherent in the properties of these equations. In other words, a given differential equation may describe an infinite variety of systems, but all these systems share common properties.

Molecular Approaches

The notions of atoms and molecules were in some disrepute in the 19th and early 20th centuries, but theoreticians like Maxwell and Boltzmann gave credence to these hypothetical constructs by demonstrating that it was possible to derive classical thermodynamical laws and to generate and solve other interesting problems by application of what came to be known as kinetic theory and statistical mechanics. For example, temperature could be equated with the average kinetic energy of a huge collection of speeding, caroming, and colliding molecules. The irony was that the molar laws of classical mechanics, applied to large ensembles of constituent units, provided the primary method; thus, a link was forged between classical mechanics and classical thermodynamics, two fields that had previously been considered, with some perplexity, as independent.

To distinguish between "molar" and "molecular" in a behavioral context is not a trivial exercise (T. Thompson & Lubinski, 1986; T. Thompson & Zeiler, 1986). We do not have "ready-made" behavioral units (e.g., hydrogen atoms); rather, our units depend on our procedures. They are dynamic in that they can change during the course of exposure to a set of contingencies. Once it is possible to identify or establish a class of behavior putatively entering into some consistent relations with variables of interest, inevitably the *temporal* pattern of behavior becomes an essential part of

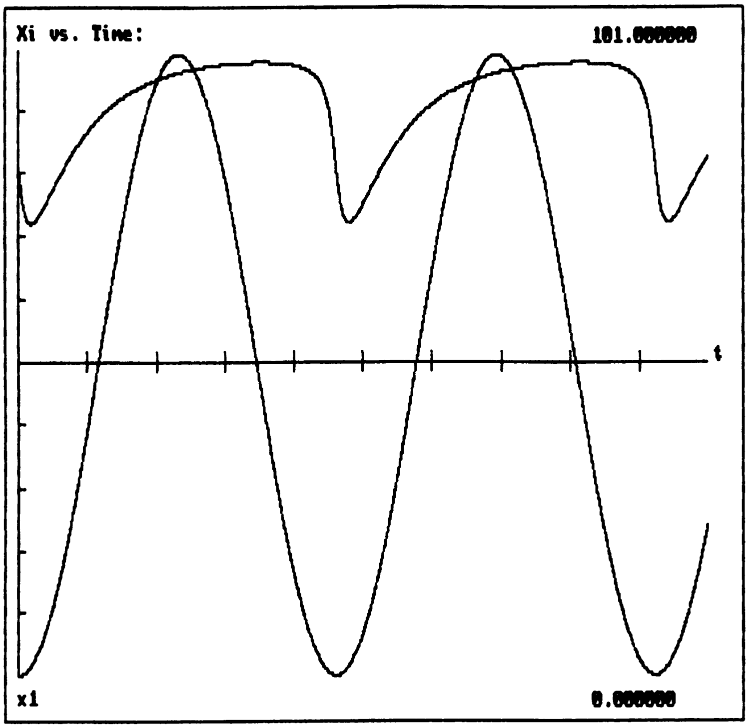

Fig. 1. A solution to Equation 4 describing a bucket with a hole in the bottom being pushed up and down into a tank of water. The solution shows the depth of water in the bucket as a function of time, plotted along with the squared sinusoidal pushing function. Notice the rounded top and phase lag of the depth function.

any analysis. This pattern, typically, both controls and is controlled by the temporal pattern of events impinging on the behavior. The molar–molecular issue deals primarily with the nature and extent of that dynamic interaction. Is a specified class of behavior primarily under the influence of local, momentary, and short-term events? If so, then perhaps local, momentary, and short-term features (e.g., interresponse times: IRTs) of the behavior should be emphasized in an account. Alternatively, behavior–event interactions may involve integration over significant intervals, in which case summary or distributional features of behavior (e.g., response rate) may form the variables of an account. In principle, both local and global forms of dynamic interaction may occur (or neither).

In the face of a degree of success of molar theories of behavior (e.g., Baum, 1989), molecular approaches have the special onus of showing not only that molecular events must be taken into account, but also that selected properties of those events must be examined. The primary challenge, however, is to formulate a correspondence principle showing how molar properties emerge from molecular events. Alternatively, a molar theory may consider molecular events, but only in a general way. For example, to generate a molar theory one might assume that IRTs are independent events described by an exponential distribution (e.g., Rachlin, 1978).

A molecular unit of responding, the IRT, was appreciated very early, and its selective reinforcement was invoked as a major factor in understanding overall rate differences in schedules (Anger, 1956; Morse, 1966; Skinner, 1938). Subsequently, numerous studies have demonstrated that IRTs can be sensitive to selective reinforcement and punishment (e.g., Anger, 1956; Blough, 1966; Galbicka & Branch, 1981; Galbicka & Platt, 1984, 1986; Morse, 1966; Shimp, 1968; Staddon, 1968), yet it is still debated to what extent that sensitivity contributes to an understanding of performance under contingencies of reinforcement (e.g., Angle, 1970; Baum, 1973; Catania, 1962; Galbicka & Platt, 1986; Kintsch, 1965; McDowell & Wixted, 1986; Nevin, 1982; Prelec,

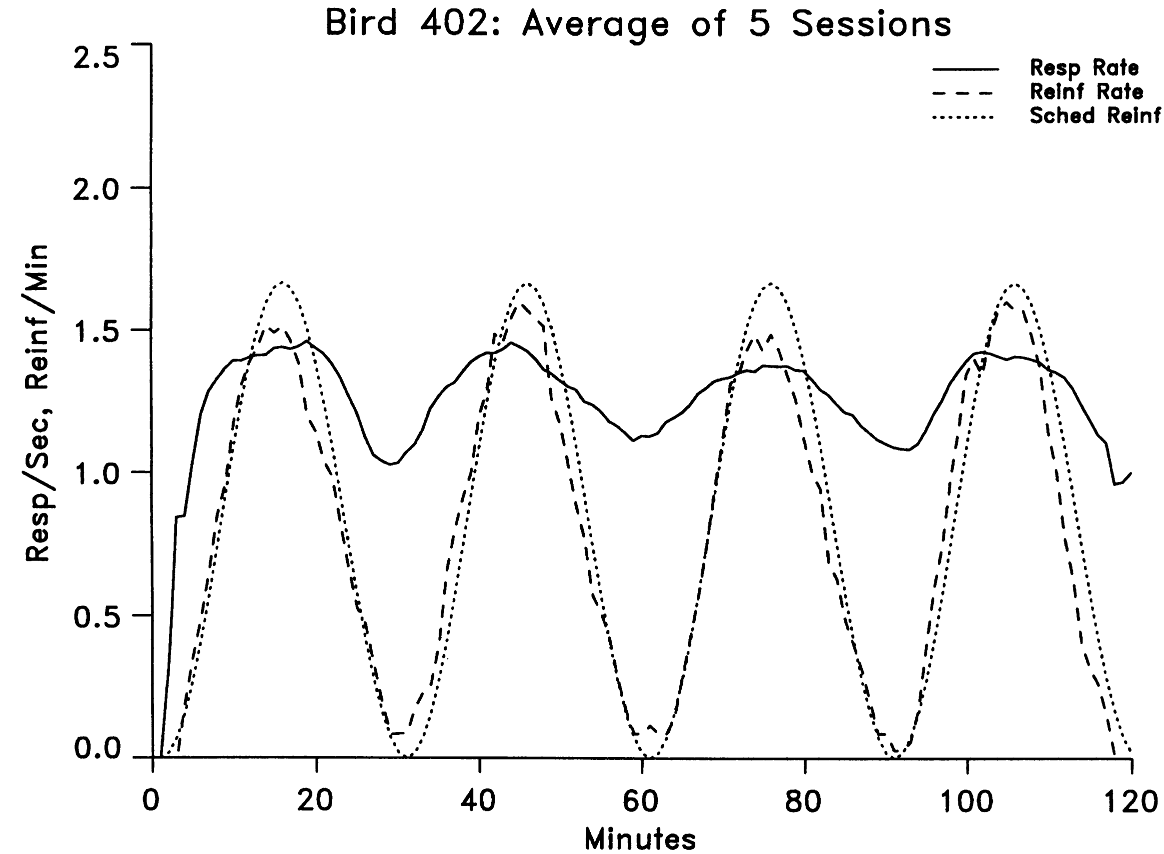

Fig. 2. The performance of a pigeon under a squared sinusoidal reinforcement schedule. Over a period of 30 min, a random-interval schedule arranged reinforcement frequencies that varied from 0 to about 1.7 per minute. A session comprised four such cycles. The theoretical and programmed reinforcement rates are shown along with the corresponding response rate. Response rate shows the rounded top characteristic of the solution shown in Figure 1, indicating that, like the bucket, response rate does not track reinforcement exactly; it is differentially sensitive to rates of change of reinforcement frequency, as Herrnstein's hyperbola implies. Unlike the bucket, responding in this case is closely in phase with reinforcement frequency and shows less variation with changes in reinforcement frequency.

1982; Shimp, 1979, 1982; Silberberg, Warren-Boulton, & Asano, 1988; Vaughan, 1987; Vaughan & Miller, 1984).

Selective reinforcement of IRTs represents a kind of Maxwell's Demon, a fantastic creature that could, in apparent violation of entropy, separate high-speed from low-speed molecules and thus be able to heat (and cool) selected regions of a gas-filled chamber. Response rate is analogous to temperature in that it is a statistical property of an ensemble of IRTs. The behavior analyst has the advantage over the physicist because IRTs can be directly observed, measured, and selected. Thus, the behavior analyst can become a Maxwell's Demon (without violating entropy principles, of course). Questions remain, however, as to the detailed structure of response rate, and those structural features upon which contingencies act. The behavior analyst is at a disadvantage compared to the physicist in that, to begin to answer questions of this sort, one has to specify not only the distributional structure but also the sequential structure of IRTs. An additional disadvantage, of course, is that there exist no clear principles of behavior dynamics, molar or molecular, to bridge the two domains. To a large extent, the separation between the two remains because of an inadequate methodology rooted in the traditional "simple" schedules. As Galbicka and Platt (1986) point out:

> Standard reinforcement schedules were developed because of the ease with which they could

> be programmed, not because of what they could tell us about behavior. To the extent ease of programming is correlated with the number of variables left uncontrolled, a continued reliance on simple schedules as analytic devices ensures continued ignorance of the variables controlling behavior. (p. 379)

One of the major criticisms of molecular approaches to behavior analysis is that events like IRTs have no coherent or consistent sequential structure germane to a molar account (Baum, 1989; Blough, 1963; Nevin, 1982). For example, IRTs engendered by variable-interval (VI) schedules are said to be described by a Poisson distribution (Blough, 1963; Mueller, 1950). There is evidence to the contrary, however, not only with respect to VI schedules, but others as well (Wertheim, 1965). The structures revealed so far are certainly not simple. But even if the fine structure of responding indicated only what appeared to be stochastic properties, this would not necessarily militate against a molecular theory of the molar properties of responding. Distributions described as stochastic may be generated by deterministic functions (e.g., Glass & Mackey, 1988). This is a highly significant feature of some nonlinear dynamical systems, in particular, those deemed chaotic.

CHAOTIC DYNAMICS

To provide a perspective on the subsequent discussion, a brief treatment of nonlinear dynamics will follow. (Two good elementary surveys of the area are those of Gleick, 1987, and Stewart, 1989.) A consideration of modern nonlinear dynamical systems raises the question of the meaning of a distribution, putatively described by a stochastic process. As is already well known, deterministic models of dynamical systems, as embodied in nonlinear difference or differential equations, can yield solutions having a stochastic character. This property is known as *chaos,* and one speaks of chaotic dynamical systems (Baker & Gollub, 1990; Glass & Mackey, 1988; Jackson, 1989; Moon, 1987; Rasband, 1990; Ruelle, 1989; Shaw, 1984; J. Thompson & Stewart, 1986). To determine whether or not an actual physical system demonstrates chaos is not easy in practice (see Moon, 1987, for a clear and detailed discussion of this problem). A simplified list of criteria might include:

1. Sensitivity to initial conditions. Small changes in starting conditions may quickly result in totally different outcomes. Because initial conditions can never be known exactly, the end result is unpredictable. As applied to a behavior dynamical system, a number of variables could be relevant (e.g., behavioral history, states of deprivation, moment-to-moment contingency changes, stimulus control variations, context, etc.). The metastable character of behavior–consequence interactions has been stressed, for example, by Morse and Kelleher (1977). The events controlling exactly when the first response occurs, say, in a fixed-interval schedule, must be subtle indeed. Although one can certainly give anecdotal lip service to the sometimes delicate nature of behavioral outcomes ("For lack of a nail, a kingdom was lost"), the criterion is difficult to apply.

It should be emphasized that sensitivity to initial conditions in a nonlinear deterministic system arises not from the random character of impinging events, nor from the possibilities generic to a large number of degrees of freedom, but from the inherent dynamic properties of the system. Explicit assumptions of nonstochastic influences and a small number of significant degrees of freedom are made in the derivation of equations describing the time evolution of a system. In a behavioral system, neither of these assumptions may be tenable (especially the latter); in any case, without very precise control of relevant variables and a suitable analytic account, unambiguous assessment of sensitivity remains elusive.

2. A broad spectrum of output frequencies with a simple periodic or constant input. Power spectra of such systems are "noisy." Very little work has been done on power spectra of behavioral data. There is an inherent problem in applying standard procedures to IRTs as well as to other molecular events because they are not typically recorded at equal time intervals. Nevertheless, Weiss and his colleagues have generated expectation density plots that have some similarity to power spectra (Weiss, 1970; Weiss, Laties, Siegel, & Goldstein, 1966; Weiss, Ziriax, & Newland, 1989). The results with monkeys responding under IRT $> t$ schedules indicate that during acquisition IRTs occur at random (i.e., the spectrum is broad). Once stable performance is attained, however, there are clear dominant frequencies appearing in harmonic relation. As expected, one of the frequencies occurs near the reinforced IRT. There are no sharp bands, however, but rather considerable spread around the peaks. Thus, there is a good deal of noise in the system. It is difficult to interpret IRT spectra in relation to the criterion above because the behavioral system is not being "driven" periodically. To complicate matters further, under the IRT > t schedule, the driving frequency depends on the output frequency of the system; that is, the frequency of reinforcement depends on the frequency of responding.

3. Display of the time evolution of the system reveals a strange attractor. The concept of an attractor is fundamental to a description of any dynamical system. Basically it represents, using suitable coordinates, the long-term states of the system after transients have decayed. A system may have multiple attractors, some stable, some not, depending on the initial conditions and values of certain parameters of the system (e.g., damping, etc.). Consider a pendulum comprised of a weight attached to a rigid rod, which is in turn attached to a pivot that allows the weight to swing freely through 360° in a plane. If we displace the weight, it will swing back and forth. We can plot the motion, or orbit, of this system in a phase space with the position along the x axis and velocity along the y axis. This is called the phase portrait of the motion. If we specify that both position on the right and velocity to the right is positive and leftward position and velocity are negative, then a frictionless pendulum with a small displacement will have a circular orbit on phase space with the origin at the center. The minimum velocity will be at the maximum displacement and vice versa. Now let us introduce a bit of reality by adding friction to the system. Regardless of the initial displacement or velocity we give to the pendulum, it will ultimately settle or be attracted to its vertical position at rest. In phase space, all orbits will spiral inward to the fixed point attractor at the center.

For more complex systems such as a forced but damped pendulum, attractors may be periodic orbits, or limit cycles, such that nearby orbits are attracted to them. They may also be multiply periodic; that is, the trajectory loops two or more times before returning to the starting point. One can visualize such orbits as flowing over the closed surface of a torus or donut. A simple periodic attractor courses once around the torus before joining itself, a two-cycle orbit goes around twice, and so on. Even more complex are quasi-periodic orbits that never return to the same point but sweep out the entire closed surface of the torus, like a wire wrapped around and completely covering a toroidal conductor to make a solenoid.

Finally, there is a chaotic or strange attractor, which, in the case of the forced and damped pendulum, not only never returns to the same point, but stretches and contracts and folds the surface of the torus into a delicate and infinitely layered composition like some cosmic chef's puff pastry.

Such a geometry defies visualization in three dimensions; but, through a procedure developed by Poincaré around the turn of the century, slices may be made through the surface to yield a cross-section of the attractor in two dimensions. Such a slice is called a Poincaré section. Every time the orbit cuts through the section, it marks a point on that plane. A simple limit cycle, for example, shows up as a single point, called a fixed point; a two-cycle makes two fixed points; a quasi-periodic attractor that covered the surface of the torus makes a circle (or ellipse). These examples are also attractors because they represent dynamical end states following the decay of any transients. Thus, a Poincaré section through an attractor is also an attractor.

A section through a chaotic attractor in three-space would reveal in two-space the folding and stretching of layers mentioned above, each few thousand points yielding more and more detail. As the motion proceeds, the points appear seemingly at random within a bounded region, like stars at dusk. In the abstract, the detail is infinite; no matter how much a region is magnified, more structure will be present. This is a rough definition of a fractal, a geometrical entity with noninteger dimension. Think of a cumulus cloud. It possesses a complex structure at many orders of magnification on down to a single water drop. From a great distance, it appears to have bulk or solidity; of course, it is not solid, but neither is it some kind of surface. Dimensionally, it is in between, more than two but less than three.

The Poincaré plane displays a discrete mapping, in that each point (x_{n+1}, y_{n+1}) can be said to be the output of a function operating on the previous point (x_n, y_n). That is, $(x_{n+1}) = F(x_n, y_n)$ and $(y_{n+1}) = G(x_n, y_n)$. Thus, the plot is a picture of the time evolution of the system, displaying the sequence of states of the system.

If the system is periodically driven, then a Poincaré section can be made that, ideally, shows all the possible states of the system at equal time intervals.

Phase-plane portraits and associated Poincaré sections can be generated by computer routines, given a set of differential equations describing the system along with parameter values and initial conditions. Except in the simplest cases, the solutions *are* the portraits and sections. In other words, the solutions may not be expressible as closed-form equations or even tables of numbers. Thus, modern dynamics relies heavily on geometry and topology and is absolutely dependent on extensive computer facilities involving sophisticated graphics routines.

The study of real dynamical systems may begin, however, not with differential equations but with experimental data. What can be done in this more common case in which data are gathered on a dynamical system, but no set of equations has or perhaps could be written to describe the system? More specifically, what pictorial procedures are available to reveal more clearly dynamical properties of behavioral data? Behavior analysts are very much used to forms of geometric description of data with little or no analytic treatment. We show cumulative records (or, at least, we used to) and plot graphs. What we have had to say about behavior has largely come from graphical analysis of data *because* we do not have a satisfactory analytical account. Fundamental to a dynamical description is the display of a behavioral variable as a function of time. The cumulative record is an ingenious method for viewing how response rate changes as a function of time. It is not, however, suited to detailed analysis of the sequential properties of responding or its local structure (Shimp, 1979).

There are presently two related display procedures that hold promise for a dynamical perspective on IRTs as well as other molecular data. One is the band plot, in which each IRT is plotted as a function of time or serial position. Some boundary points have to be specified, such as the beginning and end of an experimental session, the maximum interreinforcement interval, or the position of an IRT within an interreinforcement interval (e.g., within a ratio schedule run). The second display procedure is a return plot (sometimes called a joint interval plot) that in two-dimensional form consists of plotting IRT_{n+1} versus IRT_n. Thus, a return plot represents a crude form of a Poincaré plot—crude, because the state of the system is not sampled periodically.

Among the first to present band plots was Blough (1963), who developed a procedure for graphically recording sequences of IRTs of pigeons' key pecks under variable-interval, fixed-ratio (FR), and IRT > *t* schedules, extinction, and transitions between particular pairs. The IRTs appeared on a vertical scale as points of light on an oscilloscope while the beam swept out the time (or response number in the ratio schedule). For the first time it was revealed that response rate in the pigeon has a complex structure, with IRT bands that appeared largely invariant with respect to different contingencies, including stimulus control. These IRTs were in the range of 0.3 to 0.7 s, with a smaller cluster around 0.1 s. Variations in response rate as a function of different procedures were largely accounted for by changes in IRTs > 1 s. The longer IRTs appeared to be exponentially distributed (i.e., without sequential dependence). Under FR 30, a harmonic structure emerged with a fundamental band at about 0.35 s and a double at 0.7 s. This was interpreted as being a combination of a series of periodic pecks interspersed with pecks that failed to operate the key.

A number of subsequent studies have presented sequential IRT data obtained from various schedules including IRT > *t*, fixed and variable ratio, fixed and variable interval, and Sidman avoidance. The subjects have included rats, pigeons, and monkeys (Angle, 1970; Crossman, Trapp, Bonem, & Bonem, 1985; Davison, 1969; Gentry et al., 1983; Gott & Weiss, 1972; Kintsch, 1965; Mazur & Hyslop, 1983; Pear, 1985; Pear & Rector, 1979; Pear, Rector, & Legris, 1982; Weiss, 1970; Weiss & Gott, 1972; Wertheim, 1965; Williams, 1968). Although there are quantitative variants, the general qualitative findings are that (a) responding is comprised of periodic and nonperiodic IRTs; (b) there can be dependencies between successive IRTs; (c) there is a stable structure relatively insensitive to variations of schedule, schedule parameter, or the elapsing period between reinforcer delivery; and (d) variation in response rate is largely a function of the relative frequency of long IRTs.

Return plots are a more common method

for displaying data from mechanical systems. This is vividly illustrated in a monograph by Robert Shaw (1984) entitled *The Dripping Faucet as a Model Chaotic System*. This work might be considered a paradigmatic approach to the analysis of the dynamics of virtually any behavioral system, not simply in the use of return plots, but with other methods as well. Shaw begins by invoking familiar themes:

> We live in a whirl of moving structures, swept by social, economic and personal currents whose dominant theme is one of unpredictability. Yet laws, constraints of some sort seem to be operating, as evinced by our ability to function. The central issue of physics, that of *predictability,* is in fact addressed as a practical matter by each newborn infant: How do we construct a *model* from a stream of experimental data which we have not seen before? How do we use the model to make predictions? What are the limits of our predictive ability? Simple experiments, as well as the experience of daily living, still have much to teach us. Here, as a case in point, is an experimental study of a dripping faucet. (p. 1)

The dripping faucet is a system that is periodic for certain flow rates, but at others becomes aperiodic. The system is, moreover, intractable to an exact mathematical description. Shaw built a simple apparatus to measure the interdrip times (IDTs) of the falling drops as a function of variation in flow rate. The IDTs constituted the sole data for analysis of the system dynamics.

Plotting IDT_{n+1} against IDT_n yielded a two-dimensional return plot. When the drips were simply periodic, a single spot appeared. As the flow rate increased, variations in frequency occurred, smearing out the spot. As the flow rate increased further, quite suddenly a period doubling bifurcation occurred, yielding two spots. Thus two drops had to fall before the cycle repeated. At higher rates of flow, a sort of fuzzy parabola emerges. There is an apparent randomness, but with structure, a principal property of a chaotic system. The attractor becomes more and more complex, requiring a third dimensional presentation; that is, IDT_{n+2} versus IDT_{n+1} versus IDT_n. In a lower dimensional space, each drip time no longer uniquely determines the succeeding drip time. Eventually, three dimensions are not adequate to picture the events. Shaw was able to describe some of the properties of the system with a relatively simple nonlinear differential equation that possessed both periodic and chaotic solutions. However, the equation in no way encompassed all the specifiable properties of the system.

The use of a crude return plot appears early in the history of behavior analysis in a study by Mechner (1958). Using rats as subjects, Mechner established a performance in a two-lever arrangement whereby a minimum of eight consecutive presses of Lever A had to be emitted before a reinforcer would be delivered following a press of Lever B. The median length of a run was plotted as a function of the length of the preceding run. A sequential dependency was revealed by a linear function with positive slope showing that successive response runs tended to be correlated (i.e., long runs followed by long runs, etc.). The median run is a summary statistic; what is needed is a more detailed picture of the dynamics gained from plotting individual values as a function of the preceding value.

A step in that direction was taken by Wertheim (1965), who plotted modal probabilities of transition from IRT_n to IRT_{n+1} in a Sidman avoidance procedure with rats as subjects. The IRT values were actually ranges, not individual IRTs; thus, few points (generally less than 20) were plotted. Return plots were obtained from the beginning, middle, and end of sessions, showing changes in successive IRT dependencies from "stochastic" at the beginning to a very orderly positive linear relation at the end of the session.

Using a similar procedure to study IRT dependencies in mixed and tandem IRT $> t$ schedules in the rat, Angle (1970) showed that only when an IRT was reinforced was there any significant relation between consecutive IRTs; otherwise, the distribution was stochastic.

Weiss (1970) provided the first return plot with large numbers of individual IRTs from a monkey responding under an IRT > 20-s schedule. As Blough (1963) had shown earlier with the pigeon, a very short IRT shows little relation to the preceding IRT. However, a large diffuse ball of points in the upper right quadrant indicated a positive relation between successive long IRTs. Until very recently, Weiss and his colleagues (e.g., Gentry et al., 1983; Weiss et al., 1966) have stood almost alone in their extensive investigation of the fine structure of responding using sophisticated analytical routines. In this and all the previous work of this kind, there are no indications of chaotic processes.

In this issue, Palya (1992), using band and return plot procedures as well as other techniques applied to literally tens of millions of data points, displays and describes in unprecedented and definitive detail the IRT structure of pigeon responding under several schedule arrangements. As he convincingly concludes, there is no evidence for chaotic dynamics at work here. The return plots reveal no strange attractors, and, as he demonstrates, the system is perhaps best modeled as a somewhat noisy oscillator stochastically shifting back and forth between a fundamental and its harmonics, modulated to a greater or lesser degree by the prevailing contingency of reinforcement.

It is perhaps a disappointment that strange attractors have not yet emerged from behavioral data gathered from the operant laboratory, but to focus only upon this possibility is to miss the point of the whole enterprise of behavior dynamics viewed from the perspective of nonlinear dynamical systems. Study of such systems provides both conceptual models and methodologies for revealing dynamical structure, whether chaotic or not. Nevertheless, the search for chaos in behavior dynamics is compelling, not only because of the need to understand behavioral variation within a deterministic framework, but also because of the possibility of becoming part of a fundamental and historic scientific movement.

DYNAMICS OF CHOICE

I will conclude this essay by describing a dynamical system that has the potential for capturing certain molar and molecular features of concurrent-schedule responding. We might view a concurrent VI-VI schedule as a system with two attractors. The nature of these attractors will depend on the contingencies imposed. If, for example, one of the schedules is changed to extinction, then responding will "spiral in" to a fixed point on that alternative continuing to deliver reinforcement. If, however, the VI schedules deliver equal reinforcement frequencies to the two alternatives, then responding will cycle back and forth between the alternatives; in other words, a kind of limit cycle. Unequal reinforcement frequencies will result in shifts between two different limit cycles on the two halves of the phase portrait. Of course, the actual situation is considerably more complicated than the foregoing description. Consider first the variables influencing performance. They include at a minimum both relative and absolute reinforcement rates, as well as the changeover requirements. Second, consider some of the types of behavior engendered by these contingencies. Relative responding (and time allocation) is distributed on the basis of relative reinforcement frequency, and changeover rates (or dwell times) depend on relative and overall reinforcement rates as well as changeover requirements (Alsop & Elliffe, 1988; Davison, 1991; Davison & McCarthy, 1988; Hunter & Davison, 1978). Heyman (1979) found no sequential dependencies in changeover responses; they seemed to be described by a Markovian process. However, there has been a vigorous debate on this topic (e.g., Nevin, 1982; Shimp, 1982; Silberberg & Ziriax, 1982). Silberberg and Ziriax, for example, showed the conditional probability of a changeover as a function of time was clearly not reflective of a Markov process. There are other features of concurrent-schedule responding, of course, such as bias, over- and undermatching, and so on, but the model to be presented will not directly address them.

The concurrent schedule may be viewed in dynamical terms as a "double-well potential" problem (see Killeen, 1992, for further discussion of the concept of the potential in behavior analysis). The concept of the potential function was developed by Laplace. The function describing the force on a body at a point is equal to the negative derivative of the function describing the potential at that point. Imagine a marble in a hemispheric bowl. If no forces are acting on the marble, it will sit in the bottom of the bowl. This is a stable fixed-point attractor. If we place the marble up the side of the bowl near the edge and release it, it will roll down the side, up the opposite side a ways, and back and forth until it comes to rest at the bottom (assuming frictional forces are at play). The side of the bowl represents the potential function, and its slope at any point represents the net force acting on the marble. The bowl may be described as a potential well. The marble will be confined to the well unless enough energy is imparted to it to escape the potential. Now imagine that the bottom of the bowl is dented upward to make a smooth symmetrical mound inside. If the marble is rolled down the side it must climb over the mound to reach the opposite side. As

A

B

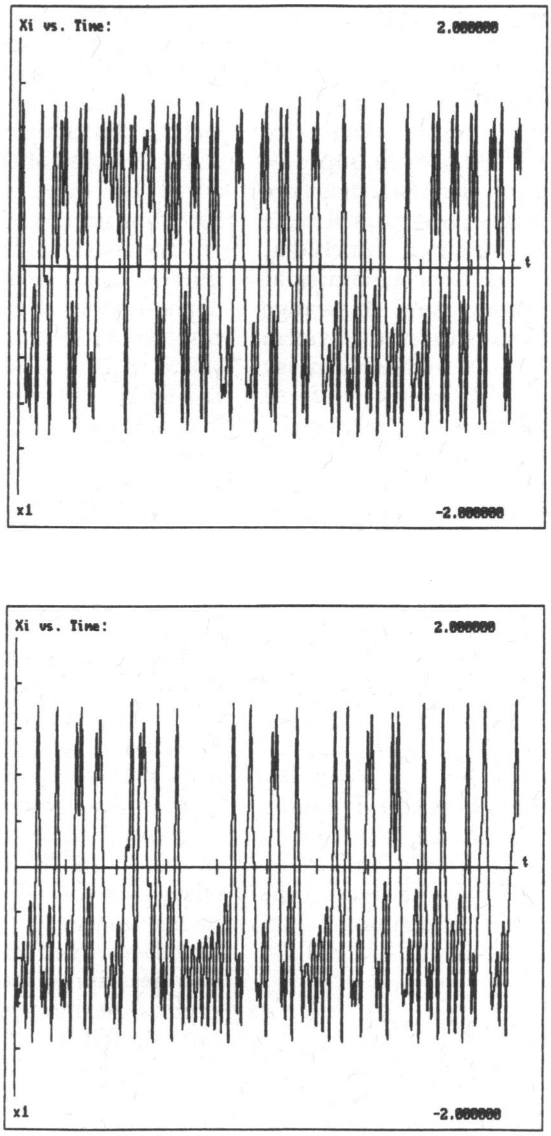

Fig. 3. A solution of Equation 5 showing the displacement of a marble in a two-well potential with a sinusoidal forcing function. The horizontal line marks the transition from one well to the other. In Panel A, the well is symmetrical with the result that, although there is much shaking about within a well, the time spent on the two sides is about equal. In Panel B, the wells are asymmetrical, with the result that more time is spent in one well than the other. Notice that occasionally in both the symmetrical and asymmetrical wells, but particularly in the latter, the marble swings back and forth with increasing amplitude before jumping to the other side. This result implies a certain degree of "sequential dependency" in dwell times, although this has not yet been investigated with this system. For more detail on correspondences with concurrent schedules, see the text.

the marble climbs the barrier it slows down; if it goes just pass the top, it will accelerate to the bottom of the opposite well. If we were to divide the bowl exactly in two and look straight on at the inside boundary of the cross-section, we would see the double-well potential function in a single plane. With no other forces acting on it, this system has two fixed-point attractors; the marble will either sit on one side or the other. The top of the mound is an unstable point.

Now shake the bowl back and forth in one plane (assume that the marble will only move in one plane). The marble will shift from one side to the other in a manner that depends on the shape of the bowl (i.e., characteristics of the potential function) and the driving force on the bowl. A dynamical system with these kinds of properties is described by a generalized Duffing equation:

$$d^2x/dt^2 + adx/dt - bx + cx^2 + dx^3 = F(t). \quad (5)$$

(See Gluckenheimer & Holmes, 1983, and Moon, 1987, for detailed discussions of the Duffing equation.) This is a nonlinear second-order differential equation solvable only by numerical methods. The first term is the acceleration of the marble at any point, the second represents frictional forces, and the remaining left-hand terms relate to the potential function described above. The right-hand term is the forcing function defining the shaking of the bowl. There are a large number of parameters [a, b, c, d, and at least two hidden in $F(t)$] all of whose values critically determine the nature of the solutions, many of which lie in the chaotic domain.

How does all this relate to concurrent schedules? As mentioned above, each alternative is a well of the potential function. The "hump" in the middle represents a changeover contingency. The overall reinforcement contingency establishes and maintains switching, and is thus embodied in the forcing function $F(t)$. Without the reinforcing consequences, behavior, like the marble, will come to rest. The simplest form of $F(t)$ is sinusoidal; for example, $F(t) = A \cos(2\pi ft)$, where A is the amplitude and f is the frequency. The overall frequency of switching is related to f, but also depends on the potential function.

Under concurrent VI-VI schedules, the higher the overall frequency of reinforcement, the greater the switching rate, and the more similar the alternative reinforcer rates, the greater the switching rate (Alsop & Elliffe, 1988). If the two alternative reinforcer rates are equal, the potential function is symmetrical (the parameter c above would be zero). With a symmetrical potential, it would be expected that the relative dwell times would be equal. With unequal rates, the potential becomes asymmetrical ($c > 0$), and the dwell times should favor the deeper potential well.

These two conditions are illustrated in Figure 3, which shows the displacement of our marble as a function of time. The marble moves in an irregular fashion back and forth within a well; then, after some unpredictable time, it crosses to the other side, and so on. For present purposes the *time spent* on each side of the double well is the variable of interest. (Perhaps the within-well variation in displacement before switching is reflective of ambivalence!) For a symmetrical well (Panel A), as predicted, the time allocation to each side appears about the same. With an asymmetrical potential (Panel B), however, the tendency is to spend more time in the deeper well. The words "about" and "tendency" are important here, because, as in concurrent-schedule performance, there is moment-to-moment variation in the state of the system. This is a chaotic system, given the assigned parameter values. The system is not chaotic for all such values, however. For example, under some conditions, the marble will bounce back and fourth periodically, as did Herrnstein's pigeons in his original experiment (1961) when no changeover delay was in effect. Or, the marble may remain on one side, as in the case of, say, concurrent VI extinction schedules.

This model is only just being explored in its details. As mentioned above, there are many parameters and initial conditions to examine. An essential domain of information is lacking, namely the stochastic properties of the chaotic solutions. This is a largely unexplored area of chaotic systems in general, but in the present case we need to know, for example, whether the dwell times have distributional and sequential properties characteristic of actual concurrent-schedule performance.

The properties of the potential function are such that they may encompass interaction effects between the changeover contingency and relative and overall reinforcement rates. These also remain matters for further investigation in the operant laboratory. Vital to the present approach is the analysis and display of data in ways to reveal dynamical processes. For example, data reside now in many laboratories that could be used to determine the sequential properties of dwell times by simply plotting T_n against T_{n+1} at various relative and overall reinforcement frequencies. Until we understand better the appropriate details of performance, we cannot hope to develop adequate dynamical models.

A FINAL WORD

As a cursory glance at Skinner's *The Behavior of Organisms* (1938) affirms, we began with behavior dynamics. Now after many years of an odyssey through the stable baseline, the steady state, and the static condition, we are at last returning to the Ithaca of change, of transition, of variation—in a word, dynamics. I have attempted to show that in developing a dynamics of behavior, we can look to the edifice of physics for models and methods. This is not the only approach, nor is it free of serious criticism (see, e.g., Zeiler, 1992). But the principal concern should not be how we approach the topic of behavior change, but that we deal vigorously and effectively with it, whatever our methods. The contributions to this issue affirm this enterprise.

REFERENCES

Alsop, B., & Elliffe, D. (1988). Concurrent-schedule performance: Effects of relative and overall reinforcer rate. *Journal of the Experimental Analysis of Behavior,* **49,** 21-36.

Anger, D. (1956). The dependence of interresponse times upon the relative reinforcement of different interresponse times. *Journal of Experimental Psychology,* **52,** 145-161.

Angle, H. V. (1970). The role of reinforcement in controlling sequential IRT dependencies. *Journal of the Experimental Analysis of Behavior,* **14,** 145-151.

Baker, G. L., & Gollub, J. P. (1990). *Chaotic dynamics.* Cambridge: Cambridge University Press.

Barrett, J. E. (1985). Modification of the behavioral effects of drugs by environmental variables. In L. S. Seiden & R. L. Balster (Eds.), *Behavioral pharmacology: The current status* (pp. 7-22). New York: Alan R. Liss.

Baum, W. M. (1973). The correlation-based law of effect. *Journal of the Experimental Analysis of Behavior,* **20,** 137-153.

Baum, W. M. (1989). Quantitative prediction and molar description of the environment. *The Behavior Analyst,* **12,** 167-176.

Bickel, W. K., & Etzel, B. C. (1985). The quantal nature of controlling stimulus-response relations as measured in tests of stimulus generalization. *Journal of the Experimental Analysis of Behavior,* **44,** 245-270.

Blough, D. S. (1963). Interresponse time as a function of continuous variables: A new method and some data. *Journal of the Experimental Analysis of Behavior,* **6,** 237-246.

Blough, D. S. (1966). The reinforcement of least-frequent interresponse times. *Journal of the Experimental Analysis of Behavior,* **9,** 581-591.

Catania, A. C. (1962). Independence of concurrent responding maintained by interval schedules of reinforcement. *Journal of the Experimental Analysis of Behavior,* **5,** 175-184.

Commons, M. L., Mazur, J. E., Nevin, J. A., & Rachlin, H. (Eds.). (1987). *Quantitative analyses of behavior: Vol. 5. The effect of delay and of intervening events on reinforcement value.* Hillsdale, NJ: Erlbaum.

Crossman, E. K., Trapp, N. L., Bonem, E. J., & Bonem, M. K. (1985). Temporal patterns of responding in small fixed-ratio schedules. *Journal of the Experimental Analysis of Behavior,* **43,** 115-130.

Davison, M. C. (1969). Successive interresponse times in fixed-ratio and second-order fixed-ratio performance. *Journal of the Experimental Analysis of Behavior,* **12,** 385-389.

Davison, M. (1991). Choice, changeover, and travel: A quantitative model. *Journal of the Experimental Analysis of Behavior,* **55,** 47-61.

Davison, M., & McCarthy, D. (1988). *The matching law: A research review.* Hillsdale, NJ: Erlbaum.

Dugas, R. (1988). *A history of mechanics.* New York: Dover.

Ferster, C. B., & Skinner, B. F. (1957). *Schedules of reinforcement.* New York: Appleton-Century-Crofts.

Galbicka, G., & Branch, M. N. (1981). Selective punishment of interresponse times. *Journal of the Experimental Analysis of Behavior,* **35,** 311-322.

Galbicka, G., & Platt, J. R. (1984). Interresponse time punishment: A basis for shock-maintained behavior. *Journal of the Experimental Analysis of Behavior,* **41,** 291-308.

Galbicka, G., & Platt, J. R. (1986). Parametric manipulation of interresponse-time contingency independent of reinforcement frequency. *Journal of Experimental Psychology: Animal Behavior Processes,* **12,** 371-380.

Gentry, G. D., Weiss, B., & Laties, V. G. (1983). The microanalysis of fixed-interval responding. *Journal of the Experimental Analysis of Behavior,* **39,** 327-343.

Glass, L., & Mackey, M. C. (1988). *From clocks to chaos.* Princeton, NJ: Princeton University Press.

Gleick, J. (1987). *Chaos: Making a new science.* New York: Viking.

Gluckenheimer, J., & Holmes, P. (1983). *Nonlinear oscillations, dynamical systems, and bifurcations of vector fields.* New York: Springer-Verlag.

Gott, C. T., & Weiss, B. (1972). The development of fixed-ratio performance under the influence of ribonucleic acid. *Journal of the Experimental Analysis of Behavior,* **18,** 481-497.

Herrnstein, R. J. (1961). Relative and absolute strength of a response as a function of frequency of reinforcement. *Journal of the Experimental Analysis of Behavior,* **4,** 267-272.

Herrnstein, R. J. (1982). Melioration as behavioral dynamism. In M. L. Commons, R. J. Herrnstein, & H. Rachlin (Eds.), *Quantitative analyses of behavior: Vol. 2. Matching and maximizing accounts* (pp. 433-458). Cambridge, MA: Ballinger.

Herrnstein, R. J., & Morse, W. H. (1958). A conjunctive schedule of reinforcement. *Journal of the Experimental Analysis of Behavior,* **1,** 15-24.

Heyman, G. M. (1979). A Markov model description of changeover probabilities on concurrent variable-interval schedules. *Journal of the Experimental Analysis of Behavior,* **31,** 41-51.

Hunter, I. W., & Davison, M. C. (1978). Response rate and changeover performance on concurrent variable-interval schedules. *Journal of the Experimental Analysis of Behavior,* **29,** 535-556.

Hunter, I., & Davison, M. (1985). Determination of a behavioral transfer function: White-noise analysis of session-to-session response-ratio dynamics on concurrent VI VI schedules. *Journal of the Experimental Analysis of Behavior,* **43,** 43-59.

Jackson, E. A. (1989). *Perspectives of nonlinear dynamics.* Cambridge: Cambridge University Press.

Johnson, D. F., & Wheeler, H. P., III. (1982). Comparisons between one-key and two-key versions of the sine wave schedule for pigeons. *Journal of the Experimental Analysis of Behavior,* **38,** 101–108.

Killeen, P. R. (1988). The reflex reserve. *Journal of the Experimental Analysis of Behavior,* **50,** 319–331.

Killeen, P. R. (1992). Mechanics of the animate. *Journal of the Experimental Analysis of Behavior,* **57,** 429–463.

Kintsch, W. (1965). Frequency distribution of interresponse times during VI and VR reinforcement. *Journal of the Experimental Analysis of Behavior,* **8,** 347–352.

Marr, M. J. (1984). Conceptual approaches and issues. *Journal of the Experimental Analysis of Behavior,* **42,** 353–362.

Marr, M. J. (1985). 'Tis the gift to be simple: A retrospective appreciation of Mach's *The Science of Mechanics. Journal of the Experimental Analysis of Behavior,* **44,** 129–138.

Marr, M. J. (1986). Mathematics and verbal behavior. In T. Thompson & M. D. Zeiler (Eds.), *Analysis and integration of behavioral units* (pp. 161–183). Hillsdale, NJ: Erlbaum.

Marr, M. J. (1989). Some remarks on the quantitative analysis of behavior. *The Behavior Analyst,* **12,** 143–151.

Marr, M. J. (1990). El lenguaje y la mecánica cuántica. In E. Ribes & P. Harzem (Eds.), *Lenguaje y conducta* (pp. 267–297). Mexico City: Trillas.

Mazur, J. E., & Hyslop, M. E. (1983). Fixed-ratio performance with and without a postreinforcement timeout. *Journal of the Experimental Analysis of Behavior,* **38,** 143–155.

McDowell, J. J, & Sulzen, H. M. (1981). Dynamic equilibrium on a cyclic interval schedule with a ramp. *Journal of the Experimental Analysis of Behavior,* **36,** 9–19.

McDowell, J. J, & Wixted, J. T. (1986). Variable-ratio schedules as variable-interval schedules with linear feedback loops. *Journal of the Experimental Analysis of Behavior,* **46,** 315–329.

Mechner, F. (1958). Sequential dependencies of the lengths of consecutive response runs. *Journal of the Experimental Analysis of Behavior,* **1,** 229–233.

Moon, F. C. (1987). *Chaotic vibrations.* New York: Wiley.

Morse, W. H. (1966). Intermittent reinforcement. In W. K. Honig (Ed.), *Operant behavior: Areas of research and application* (pp. 52–108). New York: Appleton-Century-Crofts.

Morse, W. H., & Kelleher, R. T. (1977). Determinants of reinforcement and punishment. In W. K. Honig & J. E. R. Staddon (Eds.), *Handbook of operant behavior* (pp. 174–200). Englewood Cliffs, NJ: Prentice-Hall.

Mueller, C. G. (1950). Theoretical relationships among some measures of conditioning. *Proceedings of the National Academy of Sciences,* **36,** 123–130.

Myerson, J., & Hale, S. (1988). Choice in transition: A comparison of melioration and the kinetic model. *Journal of the Experimental Analysis of Behavior,* **49,** 291–302.

Myerson, J., & Miezin, F. M. (1980). The kinetics of choice: An operant systems analysis. *Psychological Review,* **87,** 160–174.

Nevin, J. A. (1973). The maintenance of behavior. In J. A. Nevin (Ed.), *The study of behavior* (pp. 200–236). Glenview, IL: Scott, Foresman.

Nevin, J. A. (1982). Some persistent issues in the study of matching and maximizing. In M. L. Commons, R. J. Herrnstein, & H. Rachlin (Eds.), *Quantitative analyses of behavior: Vol. 2. Matching and maximizing accounts* (pp. 153–165). Cambridge, MA: Ballinger.

Nevin, J. A. (1992). An integrative model for the study of behavioral momentum. *Journal of the Experimental Analysis of Behavior,* **57,** 301–316.

Palya, W. L. (1992). Dynamics in the fine structure of schedule-controlled behavior. *Journal of the Experimental Analysis of Behavior,* **57,** 267–287.

Pear, J. J. (1985). Spatiotemporal patterns of behavior produced by variable-interval schedules of reinforcement. *Journal of the Experimental Analysis of Behavior,* **44,** 217–231.

Pear, J. J., & Rector, B. L. (1979). Constituents of response rate. *Journal of the Experimental Analysis of Behavior,* **32,** 341–362.

Pear, J. J., Rector, B. L., & Legris, J. A. (1982). Toward analyzing the continuity of behavior. In M. L. Commons, R. J. Herrnstein, & H. Rachlin (Eds.), *Quantitative analyses of behavior: Vol. 2. Matching and maximizing accounts* (pp. 3–24). Cambridge, MA: Ballinger.

Prelec, D. (1982). Matching, maximizing, and the hyperbolic reinforcement feedback function. *Psychological Review,* **89,** 189–230.

Rachlin, H. (1978). A molar theory of reinforcement schedules. *Journal of the Experimental Analysis of Behavior,* **30,** 345–360.

Rasband, S. N. (1990). *Chaotic dynamics of nonlinear systems.* New York: Wiley.

Ruelle, D. (1989). *Chaotic evolution and strange attractors.* Cambridge: Cambridge University Press.

Schoenfeld, W. N. (Ed.). (1970). *The theory of reinforcement schedules.* New York: Appleton-Century-Crofts.

Shaw, R. (1984). *The dripping faucet as a model chaotic system.* Santa Cruz, CA: Aerial Press.

Shimp, C. P. (1968). Magnitude and frequency of reinforcement and frequency of interresponse times. *Journal of the Experimental Analysis of Behavior,* **11,** 525–535.

Shimp, C. P. (1979). The local organization of behavior: Method and theory. In M. Zeiler & P. Harzem (Eds.), *Advances in the analysis of behavior. Vol. 1: Reinforcement and the organization of behaviour* (pp. 261–298). Chichester, England: Wiley.

Shimp, C. P. (1982). Reinforcement and the local organization of behavior. In M. L. Commons, R. J. Herrnstein, & H. Rachlin (Eds.), *Quantitative analyses of behavior: Vol. 2. Matching and maximizing accounts* (pp. 111–130). Cambridge, MA: Ballinger.

Silberberg, A., Warren-Boulton, F. R., & Asano, T. (1988). Maximizing present value: A model to explain why moderate response rates obtain on variable-interval schedules. *Journal of the Experimental Analysis of Behavior,* **49,** 331–338.

Silberberg, A., & Ziriax, J. M. (1982). The interchangeover time as a molecular dependent variable in concurrent schedules. In M. L. Commons, R. J. Herrnstein, & H. Rachlin (Eds.), *Quantitative analyses of behavior: Vol. 2. Matching and maximizing accounts* (pp. 131–151). Cambridge, MA: Ballinger.

Skinner, B. F. (1938). *The behavior of organisms.* New York: Appleton-Century-Crofts.

Staddon, J. E. R. (1964). Reinforcement as input: Cyclic variable-interval schedule. *Science,* **145,** 410–412.

Staddon, J. E. R. (1968). Spaced responding and choice: A preliminary analysis. *Journal of the Experimental Analysis of Behavior,* **35,** 397–412.

Staddon, J. E. R. (1988). Quasi-dynamic choice models: Melioration and ratio invariance. *Journal of the Experimental Analysis of Behavior,* **49,** 303–320.

Staddon, J. E. R., & Ettinger, R. H. (1989). *Learning: An introduction to the principles of adaptive behavior.* San Diego: Harcourt Brace Jovanovich.

Stewart, I. (1989). *Does God play dice?: A mathematics of chaos.* New York: Basil Blackwell.

Thompson, J. M. T., & Stewart, H. B. (1986). *Nonlinear dynamics and chaos.* Chichester, England: Wiley.

Thompson, T., & Lubinski, D. (1986). Units of analysis and kinetic structures of behavioral repertoires. *Journal of the Experimental Analysis of Behavior,* **46,** 219–242.

Thompson, T., & Zeiler, M. D. (Eds.). (1986). *Analysis and integration of behavioral units.* Hillsdale, NJ: Erlbaum.

Vaughan, W., Jr. (1987). Dissociation of value and response strength. *Journal of the Experimental Analysis of Behavior,* **48,** 367–381.

Vaughan, W., Jr., & Miller, H. L., Jr. (1984). Optimization versus response-strength accounts of behavior. *Journal of the Experimental Analysis of Behavior,* **42,** 337–348.

Weiss, B. (1970). The fine structure of operant behavior during transition states. In W. Schoenfeld (Ed.), *The theory of reinforcement schedules* (pp. 277–311). New York: Appleton-Century-Crofts.

Weiss, B., & Gott, T. C. (1972). A microanalysis of drug effects of fixed-ratio performance in pigeons. *Journal of Pharmacology and Experimental Therapeutics,* **180,** 189–202.

Weiss, B., Laties, V. G., Siegel, L., & Goldstein, D. (1966). A computer analysis of serial interactions in spaced responding. *Journal of the Experimental Analysis of Behavior,* **9,** 619–625.

Weiss, B., Ziriax, J. M., & Newland, M. C. (1989). Serial properties of behavior and their chemical modification. *Animal Learning & Behavior,* **17,** 83–93.

Wertheim, G. A. (1965). Some sequential aspects of IRTs emitted during Sidman-avoidance behavior in the white rat. *Journal of the Experimental Analysis of Behavior,* **8,** 9–15.

Williams, D. R. (1968). The structure of response rate. *Journal of the Experimental Analysis of Behavior,* **11,** 251–258.

Zeiler, M. D. (1977). Schedules of reinforcement: The controlling variables. In W. K. Honig & J. E. R. Staddon (Eds.), *Handbook of operant behavior* (pp. 201–232). Englewood Cliffs, NJ: Prentice-Hall.

Zeiler, M. D. (1992). On immediate function. *Journal of the Experimental Analysis of Behavior,* **57,** 417–427.

Zeiler, M. D., & Harzem, P. (Eds.). (1979). *Advances in the analysis of behavior. Vol. 1: Reinforcement and the organization of behaviour.* Chichester, England: Wiley.

Received November 27, 1991
Final acceptance January 4, 1992