A NEW CONSTITUTIVE FORMULATION FOR CHARACTERIZING THE MECHANICAL BEHAVIOR OF SOFT TISSUES

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ABSTRACT We present a new constitutive formulation that combines certain desirable features of two previously used approaches (phenomenological and microstructural). Specifically, we assume that certain soft tissues can be idealized as composed of various families of noninteracting fibers and a homogeneous matrix. Both the fibers and the matrix are assumed to follow the gross deformation. Within the usual framework of pseudoelasticity, incompressibility, homogeneity, and the continuum hypothesis, a pseudostrain-energy function (W) is proposed wherein W is expressed in terms of matrix and fibrous contributions. Unlike phenomenological approaches where a W is usually chosen in an ad hoc manner, the present approach can be used to postulate reasonable forms of W based on limited structural information and multiaxial stress–strain data. Illustrative applications of the theory are discussed for visceral pleura and myocardium. Concise structurally motivated constitutive relations result, wherein load-dependent anisotropy, nonlinear material behavior, finite deformations, and incompressibility are accounted for.

INTRODUCTION

Equations that characterize a material and its response to applied loads are called constitutive relations since they describe the gross behavior resulting from the internal constitution of a material (Malvern, 1969). Fung (1967) directed attention to the importance of quantifying constitutive relations for soft tissues when he wrote, "... there are many problems in physiology whose solutions require a detailed knowledge of the mechanical properties of the tissues involved. Hence, the stress–strain relationship of living tissues is of fundamental interest." Fung also emphasized that theoretical formulations of constitutive relations should be experimentally guided, and he identified many characteristics of soft tissues that should be accounted for within constitutive relations.

Experimental observations on soft tissues are obtained primarily from two sources: in vitro mechanical tests on excised specimens and histological examination. From mechanical tests, we know that many biological soft tissues are incompressible or nearly so, undergo large deformations, display nonlinear material behavior, exhibit viscoelastic character, and are anisotropic (Fung, 1981). Histological examination has revealed that many soft tissues are complex composite materials composed of, among other constituents, elastic fibers, collagen fibers, muscle, amorphous substances, and various fluids. It is, of course, the collective contribution of the individual components that yields the gross behavior.

Soft tissues are probably best described as nonlinearly viscoelastic composite materials. Fung (1973, 1981) has argued convincingly, however, that preconditioned tissues can be treated as two hyperelastic materials since unique stress–strain responses are obtained separately in particular loading and unloading protocols. Because soft tissues are not truly hyperelastic, Fung calls them pseudoelastic. That is, the tissues are described via a pseudostrain-energy function, W, with parameters determined separately for loading and unloading.

Many different constitutive formulations for various tissues have been proposed. One approach is to formulate constitutive relations, independent of structural considerations, to fit experimental data reasonably well. This "phenomenological" approach is commonly used but has led to a number of difficulties in describing the mechanical behavior of tissues as detailed below. Alternatively, one can formulate constitutive relations for tissues by accurately modeling their microstructure. This is extremely challenging, however, because of their structural complexity.

Here we propose a new constitutive formulation that is structurally motivated but avoids the necessity of detailed modeling of the complex microstructure. We consider the description of gross mechanical behavior only and use both the continuum hypothesis and the concept of pseudoelasticity. Illustrative examples of the application of the proposed approach are presented for the description of visceral pleura and passive myocardium, and this work is compared

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with previous approaches. Two previous approaches are briefly reviewed below.

PREVIOUS APPROACHES

Phenomenological Approach

The basic premise of the phenomenological approach is that one can use data from gross mechanical experiments, without requiring detailed structural correlates, to formulate reasonable constitutive relations. The major a priori restrictions imposed on the form of the relations are the usual ones: the relations must be frame-indifferent, tensorially correct, and should represent any experimentally observed material symmetries and kinematic constraints. Within these restrictions, one normally tests the ability of a number of reasonable relations to fit the data and then picks the equations that provide the best fit.

For example, Fung (1967, 1973) derived a one-dimensional exponential stress–stretch relation based on data that showed that the stiffness (i.e., derivative of stress with respect to stretch) of many tissues is linearly related to stress. Subsequently, he proposed analogous multidimensional relations in terms of exponentials:

$$W = (1/2) b_{MNKL} E_{MN} E_{KL} + c[e^{Q} - 1] \tag{1a}$$

where,

$$Q = (1/2) a_{MNKL} E_{MN} E_{KL} \qquad M, N, K, L = 1, 2, 3. \tag{1b}$$

b's, c, and a's are material parameters and E_{MN} are components of the Green strain tensor. Summation is implied over repeated indices. Eq. 1 was used, for example, by Tong and Fung (1976) and Lanir (1979*a*) for describing skin, Fung et al. (1979) and Chuong and Fung (1983) for arteries, Chew et al. (1986) for pericardium, Humphrey et al. (1987) for pleura, and Yin et al. (1987) for myocardium. Eq. 1 was recently referred to as "Fung elasticity" by Cowin (1985). There are, of course, many different phenomenological descriptions of various tissues (see Skalak and Chien, 1987).

A problem with the phenomenological approach, however, is the wide variability in the material parameters for (*a*) different experimental protocols for a given specimen, and (*b*) slight cycle-to-cycle variations within a single protocol. Consequently, it is difficult to make reliable interpretations of a tissues's behavior (Yin et al., 1986).

Microstructural Approach

A number of investigators attempt to describe gross behavior of tissues on the basis of microstructural geometry and properties (e.g., Comminou and Yannas, 1976; Decraemer et al., 1980*a*, *b*; Lanir, 1979*b*, 1983*a*; and Aspen, 1986) For example, Lanir advocates modeling certain soft tissues (e.g., skin) by assuming that tissue response is due to the additive contributions of a fluid matrix, elastin fibers, and collagen fibers. The fluid matrix is assumed to contribute only a hydrostatic pressure to the stress field, whereas the elastin and collagen fibers are assumed to behave linearly. Knowledge of the volume fractions, fiber waviness distributions, fiber orientations, and interactions is required to complete the model. This approach was employed by Fung and Sobin (1982) and Lanir (1983*b*) for describing lung tissue, and Horowitz et al. (1986) for describing passive myocardium. Thus far, however, these microstructural relations have not described the data better than phenomenological relations.

The microstructural approach is attractive, in principle, since it seeks to yield constitutive relations in terms of physically meaningful material parameters that represent the actual composition of the material. Furthermore, much of the requisite information may be determined from histological data. The microstructural relations are mathematically complex, however, requiring precise quantification of the tissue architecture including constituent interactions. Until the requisite data becomes available, this approach appears to be difficult to implement.

PRESENT APPROACH

Preliminaries

The goal of the present work is to formulate a constitutive theory capable of describing the gross behavior of certain biological soft tissues. Due to the complexities of both the behavior of the tissue and its microstructure, we seek a compromise between structural accuracy and mathematical simplicity. Consequently, the approach adopted is a synthesis of certain desirable features from the phenomenological and the microstructural approaches. Like most constitutive theories (Malvern, 1969), our approach describes only an idealized (or representative) response of the material under specific circumstances and not the actual material. Although we use the terms "fiber" and "matrix" in the formulation of a particular W, we emphasize that, except in special cases, the constituent parameters should not be construed to be the specific moduli of the individual constituents.

Formulation

We assume that the mechanical behavior of certain biological tissues can be represented by an idealized constitutive relation that is based on the assumptions of homogeneity of the material, pseudoelasticity (Fung, 1981), and the continuum hypothesis. As in other approaches, we assume that the actual behavior of the material can be represented by a pseudostrain-energy function $W(\mathbf{C})$, where $\mathbf{C} = \mathbf{F}^{\mathrm{T}} \cdot \mathbf{F}$ is the right Cauchy–Green deformation tensor, $\mathbf{F}$ is the deformation gradient tensor, and the superscript T denotes transpose. An applicable constitutive statement is, therefore (Truesdell and Noll, 1965),

$$\mathbf{t} = -p\mathbf{I} + 2\mathbf{F} \cdot (\partial W / \partial \mathbf{C}) \cdot \mathbf{F}^{\mathrm{T}}, \tag{2}$$

where $\mathbf{t}$ is the Cauchy (true) stress tensor, p is a Lagrange multiplier to enforce the incompressibility constraint, and $\mathbf{I}$

is the identity tensor. We require that $W(\mathbf{C})$ represent the actual strain energy stored in a unit initial volume of the tissue due to deformation, and that it describes the gross material symmetry of the tissue. Consistent with empirical data on tissues, Eq. 2 accounts for large deformations, nonlinear behavior, incompressibility, and anisotropy of the material.

Moreover, we use limited information on the microstructure of the tissue to aid us in the choice of an appropriate W. Although the ground substance matrix and extracellular fluids may contribute to the viscoelastic properties of the tissue (Minns et al., 1973; Decraemer et al., 1980*b*), it appears that their influence on the pseudoelastic properties is negligible (Wu and Yao, 1976; Lanir 1979*b*, 1983*b*; Fung and Sobin, 1982). Since some of the interconstituent coupling would be mediated through these substances/fluids, we assume that certain tissues can be idealized as composed of noninteracting families of densely distributed thin extensible fibers and a homogeneous "matrix." The "matrix" need not be an embedding material per se, but may be simply a collection of particular structural constituents. We further assume that both the fibers and the matrix follow the gross deformation field. The continuum assumption is reasonable since physiologic fiber diameters are typically orders of magnitude smaller than the gross dimensions of interest. Thus, for convenience, we assume that $W(\mathbf{C})$, which represents the overall tissue behavior, can be written as

$$W = W_m + W_f, \tag{3}$$

where W_m and W_f are "matrix" and "fiber" pseudostrain-energy functions, respectively. Furthermore, W_f may be written as

$$W_f = \Sigma W_f^{(i)} \qquad i = 1, 2, \ldots, k, \tag{4}$$

where Σ denotes summation over the appropriate fiber families, and $W_f^{(i)}$ represents the stored energy in the ith fiber family. A fiber family is defined as a collection of locally parallel fibers with identical material properties. In this way, we can account for k fiber families possessing different material properties or different original orientations. W_m and W_f are pseudostrain energies per unit initial volume of the composite. Associated energies per unit initial volume of the constituents (denoted by w) could also be used with the inclusion of the appropriate volume fractions. The volume fractions can be absorbed into the gross material constants without a loss of generality, however. Finally, notice that detailed descripitons of interfiber interactions and complex waviness and density distributions inherent to the microstructural approach have been avoided.

For thin hyperelastic fibers, W_f is assumed to depend on $\mathbf{C}$ only through the stretch ratio (α) in the direction of a fiber family. Alpha is determined as follows. Consider a differential line element, $d\mathbf{X}$, specified at a material point in the undeformed configuration and coincident with a fiber direction. Let $d\mathbf{X}$ (with magnitude dS and direction given by unit vector $\mathbf{N}$) be mapped into $d\mathbf{x}$ in the deformed configuration (with magnitude ds and direction given by unit vector $\mathbf{n}$) by the gross deformation. That is, let

$$d\mathbf{x} = \mathbf{F} \cdot d\mathbf{X}. \tag{5}$$

Defining a stretch ratio in the direction of a fiber to be $\alpha = (ds/dS)$ and noting that

$$ds^2 = d\mathbf{x}^T \cdot d\mathbf{x} = (\mathbf{F} \cdot \mathbf{N})^T \cdot (\mathbf{F} \cdot \mathbf{N})\, dS^2 \tag{6}$$

we obtain

$$\alpha^2 = \mathbf{N}^T \cdot \mathbf{C} \cdot \mathbf{N}, \tag{7}$$

indicating that α is determined from the initial orientation ($\mathbf{N} \equiv \mathbf{N}^T$) and the gross deformation ($\mathbf{C}$). No ambiguity should arise due to the α^2 term on the left hand side of Eq. 7 since α must be nonnegative physically. The deformed fiber orientation is known from the deformation and the original orientation from Eq. 5, viz.,

$$\mathbf{n} = \mathbf{F} \cdot \mathbf{N}/\alpha. \tag{8}$$

For the special case in which the behavior of the matrix is reasonably isotropic, W_m can be written as a function of the gross strain invariants:

$$W(\mathbf{C}) = W(I_1, I_2), \tag{9}$$

where

$$I_1 = \text{tr}\mathbf{C} = \text{tr}\mathbf{B} \tag{10a}$$

$$2I_2 = (\text{tr}\mathbf{C})^2 - \text{tr}\mathbf{C}^2 = (\text{tr}\mathbf{B})^2 - \text{tr}\mathbf{B}^2. \tag{10b}$$

The trace of a tensor is denoted by tr, $I_3 = 1$ due to incompressibility, and $\mathbf{B} = \mathbf{F} \cdot \mathbf{F}^T$ is the left Cauchy–Green deformation tensor. Thus, for this special case, all of the tissue anisotropy is accounted for through the various fiber families. This concept is also used by Spencer (1972) for describing materials containing inextensible fibers.

The $\partial W/\partial \mathbf{C}$ in Eq. 2, with W given by Eq. 3, can be determined by invoking the chain rule for differentiation. That is, for example, for a single fiber family and an isotropic "matrix," Eq. 2 becomes

$$\mathbf{t} = -p\mathbf{I} + 2W_1\mathbf{B} - 2W_2\mathbf{B}^{-1} + (W_\alpha/\alpha)\mathbf{F} \cdot \mathbf{N}^T \otimes \mathbf{N} \cdot \mathbf{F}^T, \tag{11}$$

where $\partial W_f/\partial \mathbf{C} = (\partial W_f/\partial \alpha)(\partial \alpha/\partial \mathbf{C})$ and $\partial \alpha/\partial \mathbf{C} = \mathbf{N}^T \otimes \mathbf{N}/2\alpha$. The other terms are obtained similarly. Additionally, $W_1 = \partial W_m/\partial I_1$, $W_2 = \partial W_m/\partial I_2$, $W_\alpha = \partial W_f/\partial \alpha$, -1 denotes inverse and $\otimes$ denotes tensor product. Finally, the relations for multiple families of fibers are straightforward extensions of Eq. 11.

Determination of W

The specific form of W, including the parameters, completely specifies the behavior of a particular material.

Although restrictions on the form of W are based on theoretical arguments, the specific functional form and the values of the parameters must come from experimental observations. Hereafter, we discuss two methods for determining the functional form of W (or, the form of the derivatives of W with respect to invariant quantities). Illustrative special cases are presented, and extensions to certain additional cases are straightforward.

Method 1

Consider the biaxial deformation of a thin planar sheet composed of two families of fibers and a homogeneous isotropic matrix. Material particles originally at X_A go to x_i after deformation, namely,

$$x_1 = \lambda_1 X_1 + \kappa_1 X_2, \quad x_2 = \lambda_2 X_2 + \kappa_2 X_1, \quad x_3 = \lambda_3 X_3, \quad (12)$$

where λ_i are stretch ratios and κ_i are measures of shear. The only nonzero physical components (Spencer, 1980) of $\mathbf{F}$ are: $F_{11} = \lambda_1$, $F_{12} = \kappa_1$, $F_{21} = \kappa_2$, $F_{22} = \lambda_2$, and $F_{33} = \lambda_3$. Incompressibility ($\det\mathbf{F} = 1$) requires that $\lambda_3 = 1/(\lambda_1\lambda_2 - \kappa_1\kappa_2)$ so only four of the five deformation measures are independent. The physical components of $\mathbf{B}$ and $\mathbf{C}$ are determined thus: $B_{ij} = F_{iM}F_{jM}$ and $C_{AB} = F_{mA}F_{mB}$ where summation is implied over repeated indices, and lowercase and uppercase indices denote quantities associated with the deformed and undeformed configurations, respectively. From Eq. 3, we let

$$W = W_m(I_1, I_2) + W_f(\alpha) + W_f(\beta), \quad (13)$$

where I_1 and I_2 are given in Eq. 10, and stretch ratios in the directions of the two families of fibers are

$$\alpha^2 = \mathbf{N}^T \cdot \mathbf{C} \cdot \mathbf{N} \quad (14a)$$

$$\beta^2 = \mathbf{M}^T \cdot \mathbf{C} \cdot \mathbf{M} \quad (14b)$$

wherein $\mathbf{N} = (\cos\Phi, \sin\Phi, 0)$ and $\mathbf{M} = (\cos\Psi, \sin\Psi, 0)$. Our constitutive relation becomes, therefore,

$$\mathbf{t} = -p\mathbf{I} + 2W_1\mathbf{B} - 2W_2\mathbf{B}^{-1} + (W_\alpha/\alpha)\,\mathbf{F}\cdot\mathbf{N}^T \otimes \mathbf{N}\cdot\mathbf{F}^T + (W_\beta/\beta)\,\mathbf{F}\cdot\mathbf{M}^T \otimes \mathbf{M}\cdot\mathbf{F}^T. \quad (15)$$

The Lagrange multiplier (p) is determined from the generalized plane-stress boundary condition ($t_{33} = 0$), and the nonzero physical components of $\mathbf{t}$ are

$$t_{11} = 2W_1(B_{11} - B_{33}) - 2W_2(B_{11}^{-1} - B_{33}^{-1}) + W_\alpha(\xi_1/\alpha) + W_\beta(\xi_2/\beta) \quad (16a)$$

$$t_{22} = 2W_1(B_{22} - B_{33}) - 2W_2(B_{22}^{-1} - B_{33}^{-1}) + W_\alpha(\xi_3/\alpha) + W_\beta(\xi_4/\beta) \quad (16b)$$

$$t_{12} = 2W_1(B_{12}) - 2W_2(B_{12}^{-1}) + W_\alpha(\xi_5/\alpha) + W_\beta(\xi_6/\beta), \quad (16c)$$

where ξ_i ($i = 1, 2, \ldots, 6$) are functions of $\mathbf{F}$ and $\mathbf{N}$ or $\mathbf{M}$ as, for example,

$$\xi_5 = \lambda_1\kappa_2\cos^2\Phi + (\lambda_1\lambda_2 + \kappa_1\kappa_2)\cos\Phi\sin\Phi + \kappa_1\lambda_2\sin^2\Phi. \quad (17)$$

Eqs. 16a–c represent three equations in terms of four unknowns (W_1, W_2, W_α, W_β) and the experimentally measurable quantities: α, β, and the components of $\mathbf{t}$, $\mathbf{C}$, and $\mathbf{B}$. Hence, a biaxial experiment of the type prescribed by Eq. 12, (*a*) is not sufficient to completely determine the functional form of W in Eq. 13, (*b*) can be used to determine W_1, W_2, and W_α or W_β if either W_α or W_β is known from additional information, (*c*) can be used to determine W_α, W_β, and W_1 or W_2 if W_m is a function of either I_1 or I_2 alone, (*d*) can be used to determine W_f for three separate fiber families if W_m contributes only a hydrostatic pressure to the stress field, and, (*e*) can be used to determine W if additional information specifies the functional form of all partial derivatives of W in excess of three.

We note, however, that t_{12} can be difficult to measure. If t_{12} is not measured, then the biaxial experiment described by Eq. 12 would yield only two of the partial derivatives of $W(\mathbf{C})$. In this instance, a W would be determinable only for (*a*) a material composed of two fiber families with a matrix that yields a hydrostatic pressure contribution to the stress field, (*b*) a material composed of one fiber family with W_m a function of I_1 or I_2 only, or (*c*) an isotropic material with no fiber families.

One possible way to obtain the functional form of each w_f is to perform uniaxial tests on excised fibers. Although typical biological fibers can be isolated and their properties determined individually (e.g., elastin from ligamentum nuchae or collagen from tendon; see Fung, 1981), their geometric complexities within the composite tissue such as waviness of, interactions between and networks of, also need to be quantified. Hence, although a w_f can be determined, the requisite data that are needed to complete the description of the contribution of the fibers to the overall behavior of the tissue are often unknown. Thus, uniaxial fiber experiments may be useful only for certain fiber types in certain tissues.

For those cases in which uniaxial tests on fibers may be useful, note that the first Piola–Kirchhoff stress, $\mathbf{P}$, is given by (Malvern, 1969):

$$\mathbf{P} = \partial W/\partial\mathbf{F}^T \quad (18a)$$

from which the first Piola–Kirchhoff stress in the direction of the fiber is

$$P_{11} = \partial w_f/\partial\alpha \quad (18b)$$

for uniaxial extension. Moreover, $P_{11} = f(\alpha)/A_o$ where A_o is the original cross-sectional area of the fiber, and $f(\alpha)$ is the experimentally measured force required to extend the fiber to a stretch ratio α. Thus, w_f could be determined from a uniaxial fiber experiment, namely,

$$w_f = \int_1^\alpha [f(\zeta)/A_o]\,d\zeta, \quad (19)$$

where ζ is a dummy variable. The functional form of W_f in Eq. 13 is, therefore, easily prescribed from Eq. 19. For example, if $f(\alpha)$ vs. α is found experimentally to be linear (e.g., $f(\alpha) = c(\alpha - 1)$), then

$$w_f = (c/2A_o)\,[\alpha^2 - 2\alpha + 1]. \tag{20a}$$

Similarly, for a linear Cauchy stress–stretch fiber relationship,

$$w_f = b[\alpha - \ln\alpha - 1], \tag{20b}$$

whereas for an exponential $f(\alpha)$ vs. α relation reported by Wilson (1981), the expression is

$$w_f = (c/2A_o)\,[e^{a(\alpha-1)^2} - 1]. \tag{20c}$$

Eqs. 20a–c are provided as examples to illustrate how w_f can be determined from Eq. 19 given experimental $f(\alpha)$ vs. α data. A series of uniaxial tests could determine $w_f^{(i)}$ (with $i = 1, 2, \ldots k$) for k families of fibers with different material properties.

Method 2

The first method for determining the functional form of W was illustrated for a material composed of a small number of families of fibers and an isotropic matrix. Complex experimental protocols are required to determine W even for these simple tissues. Obviously, to determine a suitable W for a material with multiple families of fibers and an anisotropic matrix would require even more complicated experiments that could be difficult to perform. Hence, a second method for determining the form of W is desirable. In this method, specific functional forms of W_f and W_m can be postulated based on qualitative multiaxial data and limited structural information. In this way, functional forms of W can be "built up" in a systematic fashion, independent of particular experiments. We will illustrate this method for two tissues for which limited biaxial stress–strain data are available.

Pleura

Visceral pleura is a thin serous membrane that completely invests the lungs and is intimately adherent to the underlying parenchyma. It is composed of, among other constituents, a dense plexus of collagen and elastin (Nagaishi, 1972), and has recently been thought to contribute substantially to the mechanics of the lung (Hajji et al., 1979). Stamenovic (1984) and Humphrey et al. (1987) recently presented phenomenological constitutive relations for describing pleural behavior. Humphrey et al. used Eq. 1, whereas Stamenovic considered the behavior to result from a planar network of fibers randomly distributed in the plane of the tissue.

We idealize the pleura as a fluid matrix and randomly oriented elastin and collagen fibers in the plane of the tissue (i.e., initially transversely isotropic with respect to a normal to the surface). Consequently, from Eqs. 3 and 4 we obtain

$$W = W_m + \Sigma\,(W^{(e)} + W^{(c)}) \tag{21}$$

where $W^{(e)}$ and $W^{(c)}$ are the elastin and collagen pseudo-strain-energy functions, per unit initial volume of the composite. Since elastin tends to exist as relatively straight fibers with linear behavior, we let (Eq. 20b)

$$W^{(e)} = b[\alpha - \ln\alpha - 1], \tag{22}$$

where α is a stretch ratio in the direction of an elastin fiber. Collagen normally exists as an initially wavy fiber that gradually straightens upon loading. Since the exact undulated geometry and degree of straightening of collagen is difficult to specify, the net effect of many such fibers is approximated with an exponential W_f, viz. Eq. 20c:

$$W^{(c)} = A[e^{a(\beta-1)^2} - 1], \tag{23}$$

where β is the stretch ratio in the direction of a collagen fiber.

Furthermore, we let $\mathbf{N} = (\cos\Phi, \sin\Phi, 0)$ be a unit vector defining an elastin fiber direction in the undeformed configuration and $\mathbf{M} = (\cos\Psi, \sin\Psi, 0)$ be a similar unit vector associated with the collagen. Without a loss of generality, we let $\Psi = \Phi$ which yields $\mathbf{M} = \mathbf{N}$ and $\beta = \alpha$. To account for the observed initial transversely isotropic material symmetry (Humphrey et al., 1986), we sum all the fiber families from $\Phi = 0$ to π.

Finally, if we assume that W_m contributes only a hydrostatic fluid pressure ($f_p\mathbf{I}$) to the stress field, then Eq. 2 becomes

$$\mathbf{t} = (f_p - p)\mathbf{I} + 2\mathbf{F}\cdot(\partial/\partial\mathbf{C})\left[\int_0^{\pi}(W^{(e)} + W^{(c)})\,d\Phi\right]\cdot\mathbf{F}^T, \tag{24}$$

which can be written as (assuming the fields are sufficiently smooth)

$$\mathbf{t} = (f_p - p)\mathbf{I} + \mathbf{F}\cdot\left\{\int_0^{\pi}\{b(1 - 1/\alpha) + 2a(\alpha - 1)Ae^{a(\alpha-1)^2}\}(1/\alpha)\mathbf{N}^T\otimes\mathbf{N}\,d\Phi\right\}\cdot\mathbf{F}^T, \tag{25}$$

where α is a function of Φ from Eq. 7. The operations of $\mathbf{F}$ on $\mathbf{N}$ can be carried out within the integral. For biaxial stretching of visceral pleura, t_{11}, t_{22}, and t_{12} can be calculated easily. Moreover, $p = f_p$ from the traction-free boundary condition that $t_{33} = 0$. These relations were easily integrated numerically using a standard Romberg technique.

The parameters, b, A, and a were determined by minimizing the error between predicted (Eq. 25) and experimental (Humphrey et al., 1986) Cauchy stresses in a nonlinear least-squares sense (Patitucci, 1983). They are listed in Table I for two experiments for each of seven specimens. Fig. 1 illustrates the goodness of the fit to the data by Eq. 25 for one specimen with the parameters determined from the data in the figure. Although the form of W was postulated partly on the basis of structural

TABLE I

Spec	b	A	a	R_1	R_2
1	9.474	0.385	11.92	0.997	0.998
1	8.951	0.544	11.52	0.992	0.994
2	5.045	3.049	9.428	0.996	0.996
2	7.727	0.813	12.49	0.999	0.996
3	7.867	2.069	10.19	0.999	0.999
3	7.302	2.654	9.187	0.999	0.999
4	9.041	0.629	8.595	0.998	0.998
4	4.651	6.942	4.842	0.994	0.996
5	6.570	2.671	8.252	0.992	0.998
5	6.854	3.120	8.390	0.992	0.994
6	2.020	44.50	6.442	0.994	0.997
6	0.203	59.90	5.296	0.990	0.992
7	7.736	2.367	9.544	0.983	0.982
7	4.686	16.51	5.302	0.961	0.973

Pseudoelastic parameters for Eq. 25 for two equibiaxial stretching experiments on each of seven different specimens. R_1 and R_2 are the correlation coefficients between experimental and predicted in-plane normal Cauchy stress. The units of b and A are 10^5 dyn/cm^2 and 10^3 dyn/cm^2, respectively, whereas a is dimensionless. The data are from Humphrey et al. (1986). Spec, specimen number.

considerations, the specific W (and therefore the stress–strain relations) is in terms of the best fit gross material constants.

Eq. 25 fit the data nearly as well as the purely phenomenological four parameter Humphrey et al. (1987) model, but has the advantage that it may be able to account for some load-dependent behavior through the W_f terms. Note, too, that the ad hoc phenomenological W of Humphrey et al., which was found to provide the best fit to data, included an exponential and a leading linear term in the stress–strain relations. The need for such terms was based solely on the fit-to-data, but now appears consistent with the separate collagen and elastin contributions. The Stamenovic model, which used an ad hoc description of the fibers based on alveolar data, neglected the linear term that is

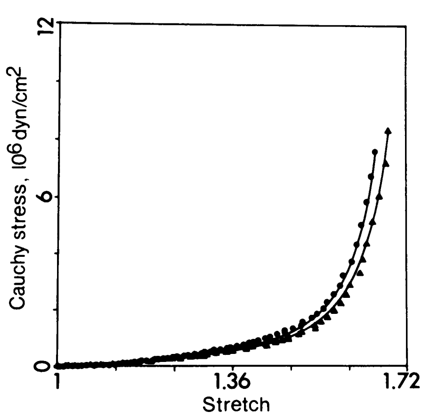

FIGURE 1 Fit of Eq. 25 to biaxial pleural data of Humphrey et al. (1986). Results are for one specimen with the parameters determined from the following data: *circles*, axis one data; *triangles*, axis two data; *solid lines*, theory.

seen to be structurally motivated. Finally, both the Humphrey and Stamenovic relations were posed in terms of principal strains. In the present case, the need for using principal strains to keep the expressions concise was not necessary.

Myocardium

A number of phenomenological myocardial stress–strain relations have been proposed based on both uniaxial and biaxial data (e.g., Pinto and Fung, 1973 or Yin et al., 1987). These relations (as well as others) have not been uniformly successful, however, in describing the available experimental data, in particular the load-dependent behavior and the anisotropy.

Although the composition of cardiac tissue is known qualitatively, the exact orientations, interactions, and volume fractions of the different constituents are not known. From micrographs, however, we can consider to a first approximation that the nonmuscular constituents are nearly randomly distributed in certain configurations (Borg and Caulfield, 1981; Robinson, 1983). Streeter (1979) has reported extensive data showing that myocardial fibers are locally parallel in layers that are distributed throughout the heart wall in a very well-defined manner.

Thus, we consider passive cardiac tissue to be transversely isotropic with respect to current muscle fiber directions, and assume that the behavior of the nonmuscular constituents can be represented by an isotropic matrix (i.e., W_m (I_1, I_2)). The anisotropic behavior (Demer and Yin, 1983) is accounted for, therefore, by a single family of muscle fibers that is assumed to be present in each layer. Specifically, based on prior experimental data (e.g., Pinto and Fung, 1973; Yin et al., 1987) indicating exponential stress–strain myocardial behavior, we assume the following W:

$$W = c\,[e^{b(I_1-3)} - 1] + A\,[e^{a(\alpha-1)^2} - 1], \qquad (26)$$

where c, b, A, and a are parameters, and α is the stretch ratio in the direction of a muscle fiber. W_m is the form used by Demiray (1976), whereas W_f is similar to the form suggested by Wilson (1981). Biaxial data from Yin et al. (1987) were used to determine the parameters using a nonlinear least-squared regression as was done with the pleura. Yin et al. performed equibiaxial stretching tests on thin sheets of passive myocardium (i.e., a layer) wherein the predominant fiber direction coincided with a stretching axis (i.e., $\mathbf{N} = (1, 0, 0)$). The corresponding nonzero Cauchy stresses are easily calculated with p obtained from the boundary condition that $t_{33} = 0$ on the top and bottom surfaces, and $\alpha = \lambda_1$ from Eq. 7. Fig. 2 illustrates the fit of Eq. 26 to data from one specimen. The mean ± SD of the parameters for three specimens were: $c = 2.078 \pm 0.977$ g/cm^2, $b = 9.448 \pm 0.468$, $A = 3.462 \pm 2.951$ g/cm^2, and $a = 65.86 \pm 12.32$ with mean correlation coefficients of 0.994 and 0.995 in the fiber and cross-fiber directions, respectively.

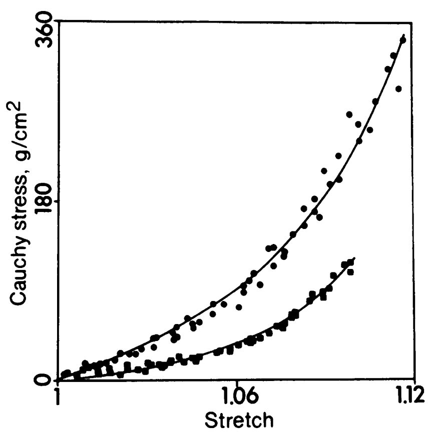

FIGURE 2 Fit of Eq. 26 to biaxial potassium-arrested myocardial data of Yin et al. (1987). Results are for one specimen with the parameters determined from the following data: *circles*, fiber data; *squares*, cross-fiber data; *solid lines*, theory.

Observe that for $W = W_m(I_1) + W_f(\alpha)$, biaxial data are sufficient, in principle, to determine the functional forms of W in a way similar to that presented in Eq. 15. For the purposes of this paper, this was not done since our intent was to illustrate the employment of the second method, and not rigorously to determine the best W.

Previously, the myocardium has been assumed to be isotropic (e.g., Demiray, 1976 or Mirsky, 1973) or to be a fluid-filled fiber-reinforced continuum (e.g., Feit, 1979; Chadwick, 1982; Tozeren 1983 and others). However, neither of these two approaches can describe the available biaxial data.

CONCLUSIONS

Two commonly used constitutive approaches for describing soft tissue behavior were briefly reviewed. Whereas the phenomenological approach offers mathematical simplicity, it has limitations in describing the load-dependent behavior exhibited by many tissues. A possible mechanism for the load-dependency is material reorganization under different loading conditions. Since many tissues are fibrous materials, it is quite possible that fiber reorientation may play a role in the load-dependency. The microstructural approach attempts to model both the material properties and the architecture of the individual constituents within the material, and may therefore be able to account for some of the load-dependent behavior. But because of the complex microstructure, the lack of complete structural data, and the difficulty in quantifying the interactions between the constituents, this approach becomes mathematically involved and difficult to implement.

Hence, in this paper, a new experimentally guided and structurally motivated constitutive formulation was presented. It is a compromise between mathematical simplicity (i.e., phenomenological approach) and physical accuracy (i.e., microstructural approach). The actual behavior of a tissue is idealized by using a pseudostrain-energy function that is written in terms of matrix and fiber contributions. Additionally, it is assumed that the material is incompressible, homogeneous, and continuous. It was shown that pseudostrain-energy functions can be explicitly determined from empirical data or can be "built-up" on the basis of limited structural information and qualitative experimental data. Because only limited information is used, the material parameters are, except in special cases, simply best fit parameters as in phenomenological approaches, and will accurately reflect the moduli of the individual constituents only in special cases. Possible advantages of our approach are that (*a*) concise constitutive relations, which may be able to account for some of the load-dependent behavior, are easily formulated, and (*b*) nonlinear material behavior, finite deformations, incompressibility, and anisotropy are easily accounted for.

Constitutive relatives were constructed to illustrate the application of the theory to visceral pleura and myocardium. Previous biaxial data were used to show how specific material constants are obtained. A surprisingly good-fit to these data was obtained even though no attempt was made to refine our original forms for W. Extensive testing to identify the most appropriate W's for any tissue awaits further study and is beyond the scope of the present work.

The present approach is not without certain disadvantages, however, and will undoubtedly be refined in the future as additional experimental data guides us. In particular, the modeling of constituent interactions will likely be needed for certain tissues.

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REFERENCES

Aspen, R. M. 1986. Relation between structure and mechanical behavior of fibre-reinforced composite materials at large strain. *Proc. R. Soc. Lond. B. Biol. Sci.* A406:287–298.

Borg, T. K., and J. B. Caulfield. 1981. The collagen matrix of the heart. *Fed. Proc.* 40:2037–2041.

Chadwick, R. S. 1982. Mechanics of the left ventricle. *Biophys. J.* 39:279–288.

Chew, P. H., F. C. P. Yin, and S. L. Zeger. 1986. Biaxial stress-strain properties of canine pericardium. *J. Mol. Cell. Cardiol.* 18:567–578.

Chuong, C. J., and Y. C. Fung. 1983. Three dimensional stress distribution in arteries. *ASME J. Biomech. Eng.* 105:268–274.

Comminou, M., and I. V. Yannas. 1976. Dependence of stress-strain nonlinearity of connective tissues on the geometry of collagen fibers. *J. Biomech.* 9:427–433.

Cowin, S. C. 1985. A model of stress adaptive anisotropy for soft tissue. *ASME Adv. Bioeng.* 26–27.

Decraemer, W. F., M. A. Maes, and V. J. Vanhuyse. 1980*a*. An elastic stress-strain relation for soft biological tissues based on a structural model. *J. Biomech.* 13:463–468.

Decraemer, W. F., M. A. Maes, V. J. Vanhuyse, and P. Vanpeperstraete. 1980*b*. A nonlinear viscoelastic constitutive equation for soft biological tissues based upon a structural model. *J. Biomech.* 13:559–564.

Demer, L. L., and F. C. P. Yin. 1983. Passive biaxial mechanical properties of isolated canine myocardium. *J. Physiol. (Lond.)*. 339:615–630.

Demiray, H. 1976. Stresses in ventricular wall. *ASME J. Appl. Mech.* 43:194–197.

Feit, T. S. 1979. Diastolic pressure-volume relations and distribution of pressure and fiber extension across the wall of a model left ventricle, *Biophys. J.* 28:143–166.

Fung, Y. C. 1967. Elasticity of soft tissues in simple elongation. *Am. J. Physiol.* 213:1532–1544.

Fung, Y. C. B. 1973. Biorheology of soft tissues. *Biorheology.* 10:139–155.

Fung, Y. C., K. Fronek, and P. Patitucci. 1979. Pseudoelasticity of arteries and the choice of its mathematical expression. *Am. J. Physiol.* 237:H620–631.

Fung, Y. C. 1981. Biomechanics: Mechanical Properties of Living Tissues. Springer-Verlag, New York. 242–245.

Fung, Y. C., and S. S. Sobin. 1982. On the constitutive equation of the lung tissue. *ASME Adv. Bioeng.* 84–87.

Hajji, M. A., T. A. Wilson, and S. J. Lai-Fook. 1979. Improved measurements of shear modulus and pleural membrane tension of the lung. *J. Appl. Physiol.* 47:175–181.

Horowitz, A., Y. Lanir, M. Perl, F. C. P. Yin, P. Chew, R. Strumpf, and S. Zeger. 1986. A three-dimensional constitutive law for passive myocardium. *ASME Adv. Bioeng.* 144–145.

Humphrey, J. D., D. L. Vawter, and R. P. Vito. 1986. Mechanical behavior of excised canine visceral pleura. *Ann. Biomed. Eng.* 14:451–466.

Humphrey, J. D., D. L. Vawter, and R. P. Vito. 1987. Pseudoelasticity of excised visceral pleura. *ASME J. Biomech. Eng.* 109:115–120.

Lanir, Y. 1979*a*. Biaxial stress-strain relationship in the skin. *Isr. J. Technol.* 17:78–85.

Lanir, Y. 1979*b*. A structural theory for the homogeneous biaxial stress-strain relationship in flat collagenous tissues. *J. Biomech.* 12:423–436.

Lanir, Y. 1983*a*. Constitutive equations for fibrous connective tissues. *J. Biomech.* 16:1–12.

Lanir, Y. 1983*b*. Constitutive equations for the lung tissue. *ASME J. Biomech. Eng.* 105:374–380.

Malvern, L. E. 1969. Introduction to the Mechanics of a Continuous Media. Prentice-Hall, Inc., Englewood Cliffs, NJ. 273–278, 397.

Minns, R. J., P. D. Soden, and D. S. Jackson. 1973. The role of the fibrous components and ground substance in the mechanical properties of biological tissues: preliminary investigation. *J. Biomech.* 6:153–165.

Mirsky, I. 1973. Ventricular and arterial wall stresses based on large deformation analyses. *Biophys. J.* 13:1141–1157.

Nagaishi, C. 1972. Functional Anatomy and Histology of the Lung. University Park Press, Baltimore.

Patitucci, P. 1983. Computer programs for fitting pseudostrain energy functions to soft tissues experimental stress and strain data. Available through DECUS, One Iron Way, MR2-3/E55, Marlboro, MA.

Pinto, J. G., and Y. C. Fung. 1973. Mechanical properties of the heart muscle in the passive state. *J. Biomech.* 6:597–616.

Robinson, T. F. 1983. The physiological relationship between connective tissue and contractile filaments in heart muscle. *Einstein Q. J. Biol. Med.* 1:121–127.

Skalak, R., and S. Chien. 1987. Handbook of Bioengineering. McGraw Hill Book Co., New York.

Spencer, A. J. M. 1972. Deformations of Fibre-reinforced Materials. Clarendon Press, Oxford.

Spencer, A. J. M. 1980. Continuum Mechanics. Longman, Oxford.

Stamenovic, D. 1984. Mechanical properties of pleural membrane. *J. Appl. Physiol.* 57:1189–1194.

Streeter, D. D. 1979. Gross morphology and fiber geometry of the heart. Handbook of Physiology, Section 2: The Cardiovascular System. Vol. 1. The Heart. R. M. Berne, et al., editors. American Physiological Society, Bethesda. 61–112.

Tong, P., and Y. C. Fung. 1976. The stress-strain relationship for skin. *J. Biomech.* 9:649–657.

Tozeren, A. 1983. Static analysis of the left ventricle. *ASME J. Biomech. Eng.* 105:39–46.

Truesdell, C., and W. Noll. 1965. The nonlinear field theories. *In* Handbuch der Physik. Vol. III/3. S. Flugge, editor. Springer-Verlag, Berlin. 119–126.

Wilson, T. A. 1981. Mechanics of the pressure volume curve of the lung. *Ann. Biomed. Eng.* 9:439–449.

Wu, H-C, and R-F Yao. 1976. Mechanical behavior of the human annulus fibrosus. *J. Biomech.* 9:1–7.

Yin, F. C. P., P. H. Chew, and S. L. Zeger. 1986. An approach to quantification of biaxial tissue stress-strain data. *J. Biomech.* 19:27–37.

Yin, F. C. P., R. K. Strumpf, P. H. Chew, and S. L. Zeger. 1987. Quantification of the mechanical properties of noncontracting canine myocardium under simultaneous biaxial loading. *J. Biomech.* 20:577–589.