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RELATIONSHIP BETWEEN RESPONSE RATE AND REINFORCEMENT FREQUENCY IN VARIABLE-INTERVAL SCHEDULES: THE EFFECT OF THE CONCENTRATION OF SUCROSE REINFORCEMENT

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Four rats were exposed to variable-interval schedules specifying a range of different reinforcement frequencies, using sucrose of two different concentrations and distilled water as the reinforcer. With sucrose, the rates of responding of all four rats were increasing negatively accelerated functions of reinforcement frequency, the data conforming closely to Herrnstein's equation; this was also true of the data from three of the four rats when distilled water was used as the reinforcer. The values of both constants in Herrnstein's equation were related to the sucrose concentration: the asymptotic response rate decreased, and the reinforcement frequency corresponding to the half-maximal response rate increased, with decreasing sucrose concentration.

Key words: Herrnstein's equation, response rate, reinforcement frequency, sucrose concentration, variable interval, lever press, rats

Experiments with pigeons (Catania and Reynolds, 1968), rats (Bradshaw, 1977; de Villiers, *unpublished* [see de Villiers and Herrnstein, 1976]), and humans (Bradshaw, Szabadi, and Bevan, 1976, 1977) have shown that the rate of responding (R) in variable-interval (VI) schedules of reinforcement is an increasing negatively accelerated function of reinforcement frequency (r). Herrnstein (1970) proposed an equation to describe this relationship:

$$R = \frac{R_{max} \cdot r}{K_H + r}, \qquad (1)$$

where R_{max} and K_H are constants for a given organism in a given experimental situation; R_{max} is the theoretical maximum response rate that can be generated in a VI schedule (Herrnstein, 1974), and K_H is the reinforcement frequency needed to obtain the half-maximal response rate.[2] Equation (1) defines a rectangular hyperbola.

Herrnstein (1974) suggested that the value of the theoretical maximum response rate depends only on the response characteristics and on the subject's capacity to respond, and is independent of all reinforcement parameters (*e.g.*, magnitude and immediacy of reinforcement). de Villiers and Herrnstein (1976) reanalyzed the data from a number of earlier reports in the literature, and concluded that the data are generally consistent with this suggestion. However, there have been no previous attempts to test Herrnstein's prediction directly.

In the present experiment, we examined the relationship between response rate and reinforcement frequency, using different concentrations of sucrose reinforcers.

METHOD

Subjects

Four experimentally naive, female albino Sprague-Dawley rats (R20, R21, R22, and R23), aged about 3.5 months at the start of training, were housed individually under a constant cycle of 12-hr light and 12-hr darkness, and were maintained at approximately 80% of their free-feeding body weights. Tap water was freely available in the home cages.

Apparatus

The rats were trained in a standard operant conditioning chamber measuring 23 by 28 by

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[2]$r = K_H$ when $R = R_{max}/2$. K_H is mathematically equivalent to r_o in Herrnstein's (1970) formulation and to C in Catania's (1973) formulation. However, since K_H is defined in purely mathematical terms, it carries none of the theoretical assumptions associated with r_o and C (see Bradshaw *et al.*, 1976).

19 cm (Grason Stadler rat station, model E3125-100). One wall contained a recess into which a dipper mechanism could deliver 0.05-ml liquid reinforcement. Reinforcement consisted of raising the dipper into the recess for 5 sec. A lever was situated above the recess and 7.5 cm above the floor of the chamber. This lever could be operated by a force of approximately 0.22 N. Illumination was provided by a red houselight. The whole chamber was enclosed in a sound-attenuating chest, and masking noise was provided by a rotary fan. Conventional electromechanical programming and recording equipment was situated in an adjoining room. Responses and reinforcements were recorded on counters and cumulative recorders.

Procedure

After acclimatization to the food-deprivation regime, the rats were trained to press the lever by the method of successive approximations. After three sessions of continuous reinforcement they were subjected to a series of VI schedules as described below. Training sessions took place daily, at the same time each day. Each session was terminated after 50 reinforcements or 60 min, whichever occurred first.

Variable-interval schedules were used throughout the experiment. The distribution of the intervals was as described by Catania and Reynolds (1968, Appendix II). The reinforcer, a solution of sucrose in distilled water, was prepared daily before each experimental session. The dipper and reinforcer reservoir were washed with distilled water after each session.

The schedules of reinforcement used and the numbers of sessions of exposure to each schedule are shown in Table 1. In Phase I, the rats were exposed to a series of VI schedules, each specifying a different reinforcement frequency, using an 0.32 *M* sucrose solution (isotonic sucrose) as the reinforcer. In Phase II, the rats were again exposed to a series of VI schedules, this time using an 0.05 *M* sucrose solution as the reinforcer. In Phase III, the procedure was again repeated, using distilled water as the reinforcer. Finally, in Phase IV, the subjects were re-exposed to two schedules using 0.32 *M* sucrose and one schedule using 0.05 *M* sucrose.

RESULTS

Mean response rates (± s.e.m.) for the last five sessions' exposure to each schedule were calculated separately for each rat, and graphs were plotted of response rate *versus* delivered reinforcement frequency. Curves having the

Table 1

Order of presentation of the variable-interval schedules used in the experiment

Phase of Experiment	*Sucrose Concentration*	*VI Schedule (in seconds)*	*Number of Sessions*
I	0.32 *M*	VI 13.6	30
		VI 38.0	30
		VI 73.5	30
		VI 180.0	30
		VI 360.0	30
		VI 654.5	30
II	0.05 *M*	VI 13.6	30
		VI 38.0	30
		VI 73.5	30
		VI 180.0	30
		VI 360.0	30
III	0.00 *M* (distilled water)	VI 13.6	30
		VI 38.0	30
		VI 73.5	30
		VI 112.5	30
		VI 360.0	30
IV	0.32 *M* (redetermination)	VI 13.6	12
	0.32 *M* (redetermination)	VI 360.0	12
	0.05 *M* (redetermination)	VI 73.5	12 ‡

‡R21 died during the first few sessions' exposure to this schedule.

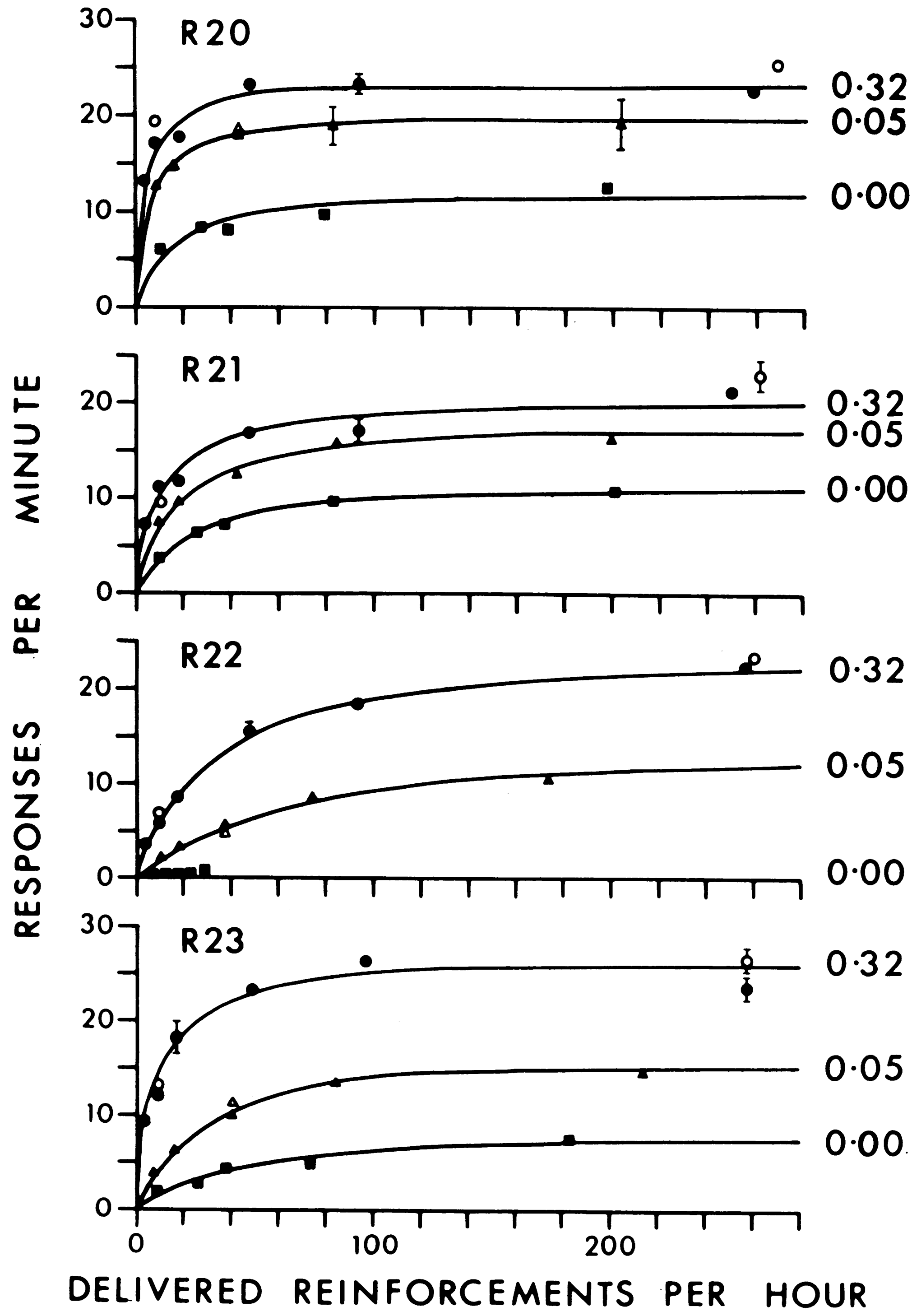

Fig. 1. Relationship between response rate and delivered reinforcement frequency for three sucrose concentrations. Each graph was obtained from one rat. Points show mean response rates for the last five sessions' exposure to a schedule; vertical bars indicate ± s.e.m., where this was greater than ± 1.0 responses per minute; circles: 0.32 *M* sucrose; triangles: 0.05 *M* sucrose; squares: 0.00 *M* sucrose (distilled water). Curves are best-fit rectangular hyperbolae, fitted by nonlinear regression analysis. Open symbols indicate redetermination, obtained during Phase IV; these values were not used in the curve-fitting procedure.

form defined by Equation (1) were fitted to the data derived from each rat, using nonlinear regression analysis (Wilkinson, 1961). This method gives estimates (± s.e.est) of the values of the thoretical maximum response rate, and the reinforcement frequency needed to obtain the half-maximal response rate (K_H). Figure 1 shows the data obtained from all four subjects; the values of the constants are shown in Table 2. Also shown in Table 2 are the indices of determination (p^2) calculated for each curve. The statistic p^2 expresses the proportion of the data variance that can be accounted for by the predictions of a fitted curvilinear function (Lewis, 1960).

The data obtained from all four rats with 0.32 *M* sucrose and with 0.05 *M* sucrose conformed closely to Equation (1), the values of p^2 being in every case greater than 0.9. When distilled water was used as the reinforcer, the data from R21 and R23 again yielded p^2 values in excess of 0.9, and the data from R20 yielded a value of about 0.8. However, in the case of R22, distilled water failed to maintain responding, even under the highest scheduled reinforcement frequency.

For all four subjects, the values of both constants in Equation (1) varied as a function of sucrose concentration. The value of the theoretical maximum response rate (R_{max}) declined, and the reinforcement frequency needed to obtain the half-maximal response rate (K_H) increased, with decreasing concentrations of sucrose.

When 0.32 *M* sucrose was used as the reinforcer, the delivered reinforcement frequencies were in every case within 5% of the reinforcement frequencies specified by the schedules. However, the lower rates of responding maintained by 0.05 *M* sucrose and distilled water were reflected in greater departures of delivered reinforcement frequency from scheduled

Table 2

Values of constants

Subject	*Sucrose Concentration (M)*	*Reinforcement Frequency Needed to Obtain Half-Maximal Response Rate, K_H (reinf/hr ± s.e. est)*	*Theoretical Maximum Response Rate, R_{max} (resp/min ± s.e. est)*	p^2
A. *Delivered reinforcement frequency*				
R20	0.32	4.2 (± 0.9)	23.6 (± 0.9)	0.91
	0.05	6.3 (± 0.9)	20.2 (± 0.5)	0.97
	0.00	14.3 (± 6.4)	12.1 (± 1.4)	0.81
R21	0.32	10.8 (± 2.9)	20.6 (± 1.5)	0.91
	0.05	17.0 (± 3.5)	18.6 (± 1.1)	0.96
	0.00	25.7 (± 4.1)	12.2 (± 0.6)	0.98
R22	0.32	35.1 (± 3.4)	25.6 (± 0.8)	0.99
	0.05	67.3 (± 14.3)	15.3 (± 1.4)	0.98
	0.00	(equation not fitted)*		
R23	0.32	9.2 (± 2.1)	26.7 (± 1.5)	0.95
	0.05	30.4 (± 5.1)	16.9 (± 0.9)	0.98
	0.00	54.7 (± 22.4)	9.4 (± 1.5)	0.92
B. *Scheduled reinforcement frequency*				
R20	0.32	4.1 (± 0.9)	23.6 (± 0.9)	0.91
	0.05	6.2 (± 0.8)	20.0 (± 0.4)	0.97
	0.00	13.6 (± 6.9)	11.6 (± 1.4)	0.77
R21	0.32	11.2 (± 3.2)	20.6 (± 1.5)	0.91
	0.05	16.9 (± 4.1)	18.0 (± 1.1)	0.95
	0.00	28.6 (± 6.4)	12.0 (± 0.8)	0.97
R22	0.32	36.5 (± 3.1)	25.8 (± 0.7)	0.99
	0.05	69.3 (± 13.7)	13.9 (± 1.1)	0.98
	0.00	(equation not fitted)*		
R23	0.32	10.3 (± 2.4)	27.0 (± 1.6)	0.94
	0.05	32.9 (± 6.3)	16.6 (± 1.0)	0.98
	0.00	60.1 (± 26.1)	8.8 (± 1.3)	0.91

*Response rates < 1.0 responses per minute under all five schedules.

reinforcement frequency; this was especially marked in schedules specifying high reinforcement frequencies (Figure 1). Because of these discrepancies, the curve-fitting procedure was carried out, using both delivered and scheduled reinforcement frequency as the independent variable. The results of the two analyses are shown in Tables 2A and 2B respectively. It is apparent that very similar values of both constants were obtained from the two analyses (in no case was there a statistically significant difference between the two values obtained: normal distribution, $p > 0.1$ in every case), and the two analyses yielded almost identical values of p^2.

During Phase IV, the subjects were re-exposed to two schedules using 0.32 *M* sucrose and (except for R21, which died at this stage in the experiment) to one schedule using 0.05 *M* sucrose. In general, the response rates during Phase IV closely approximated the response rates in the corresponding schedules during Phases I and II. (An exception, however, was noted for R20 with both schedules using 0.32 *M* sucrose, where the response rates observed in Phase IV were somewhat higher than those observed in Phase I.)

DISCUSSION

In agreement with earlier findings (Bradshaw, 1977; de Villiers, *unpublished* [see de Villiers and Herrnstein, 1976]) the present results indicate that the behavior of rats under VI schedules conforms to Herrnstein's equation. The goodness of fit of Equation (1) and the values of the constants obtained were almost identical whether delivered or scheduled reinforcement frequency was used as the independent variable. This presumably reflects the fact that significant deviations of delivered reinforcement frequency from scheduled reinforcement frequency occurred only in high-density schedules, where the response rates were close to their asymptotic values.

The main finding from the present study was that the values of both constants in Herrnstein's equation, K_H and R_{max}, were systematically related to the concentration of the sucrose reinforcement, K_H increasing, and R_{max} decreasing, with decreasing sucrose concentrations. The relationship between the value of K_H and sucrose concentration is compatible with Herrnstein's formulation. According to Herrnstein's own interpretation of K_H ("r_o" in Herrnstein's terminology), this term reflects the frequency of extraneous reinforcement *measured in the same units as the reference reinforcement*. Thus, it is to be expected that when K_H is expressed in units of a more powerful reinforcer, such as 0.32 *M* sucrose, its value will be lower than when it is expressed in units of a weaker reinforcer, such as 0.05 *M* sucrose (see de Villiers and Herrnstein, 1976).

The relationship between the value of R_{max} and sucrose concentration is more surprising. According to Herrnstein (1974), this term ("k" in Herrnstein's terminology), depends only on the response characteristics and on the subject's capacity to respond, and is independent of all parameters of reinforcement. The present results are in conflict with this suggestion. The dependence of R_{max} on the characteristics of the reinforcer may, however, be accommodated by a simple modification to Equation (1):

$$R = \frac{(e \cdot V) \cdot r}{K_H + r}, \text{ where } eV = R_{max}. \qquad (2)$$

The constant V expresses the theoretical maximum response rate for a particular subject; the value of V depends only on the response characteristics, and is independent of all parameters of reinforcement (it is thus conceptually similar to Herrnstein's k). The term e (the "efficacy" of the reinforcer) may have a value between 0 and 1.0 depending on the nature of the reinforcer. Only in the case of a highly efficacious reinforcer, where $e \simeq 1$, will the value of R_{max} accurately reflect the maximum response capabilities of the organism (V). Further experiments are needed to delineate the features of the reinforcing stimulus which determine the value of e. For instance, it would be of interest to determine whether e is related monotonically or bitonically to sucrose concentration.

In three of the four rats (R20, R21, R23), substantial rates of lever pressing were maintained under schedules of distilled water delivery, and the relationship between response rate and reinforcement frequency conformed closely to the hyperbolic function. The most immediate explanation for this unexpected effectiveness of distilled water as a reinforcer would be that the operation of the dipper and the presentation of 0.05 ml of fluid acquired reinforcing properties by virtue of their long-standing association with sucrose. It is noteworthy, however, that the three rats in question were still responding for distilled water five months (150 sessions) after the sucrose had been withdrawn.

Staddon (1977) discussed various theoretical models from which Herrnstein's equation may be derived. One of these models is based on the principle of "momentary maximizing" (Shimp, 1966). Staddon has argued that this model implies a positive correlation between the values of R_{max} and K_H. The data presented in this paper would seem to be in conflict with this suggestion, since in all the animals studied there was an inverse relationship between the values of these two constants.

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