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RELATIONSHIP BETWEEN RESPONSE RATE AND REINFORCEMENT FREQUENCY IN VARIABLE-INTERVAL SCHEDULES: II. EFFECT OF THE VOLUME OF SUCROSE REINFORCEMENT

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Three rats were exposed to variable-interval schedules specifying a range of different reinforcement frequencies, using three different volumes of .32 molar sucrose (.10, .05, and .02 milliliters) as the reinforcer. With each of the three volumes, the rates of responding of all three rats were increasing, negatively accelerated functions of reinforcement frequency, the data conforming closely to Herrnstein's equation. In each rat the value of the constant K_H, which expresses the reinforcement frequency needed to obtain the half-maximal response rate, increased with decreasing reinforcer volume, the values obtained with .02 milliliters being significantly greater than the values obtained with .10 milliliters. The values of the constant R_{max}, which expresses the theoretical maximum response rate, were not systematically related to reinforcer volume. The effect of reinforcer volume upon the relationship between response rate and reinforcement frequency is thus different from the effect of the concentration of sucrose reinforcement: In a previous experiment (Bradshaw, Szabadi, & Bevan, 1978) it was found that sucrose concentration influenced the values of both constants, R_{max} increasing and K_H decreasing with increasing sucrose concentration.

Key words: Herrnstein's equation, response rate, reinforcement frequency, reinforcer volume, variable interval, lever press, rats

It is now well established that the rate of responding in variable-interval (VI) schedules of reinforcement is an increasing, negatively accelerated function of reinforcement frequency (for review, see de Villiers, 1977). Herrnstein (1970) has proposed an equation of the following form to describe this relationship:

$$R = R_{max} \cdot r/(K_H + r), \qquad (1)$$

where R is the rate of responding and r is the frequency of reinforcement. R_{max} and K_H are constants for a given organism in a given experimental situation: R_{max} is the theoretical maximum response rate which can be generated in a VI schedule (Herrnstein, 1974), and K_H is the reinforcement frequency needed to obtain the half-maximal response rate.[1] Equation 1 defines a rectangular hyperbola.

Herrnstein (1974) predicted that parameters of reinforcement, such as reinforcer magnitude and immediacy of reinforcement, should be reflected in the value of K_H (weaker reinforcers being associated with higher values of K_H), but should not influence the value of R_{max}. On the other hand, response parameters, such as effort requirement, should be reflected in the value of R_{max}, but should not affect the value of K_H. De Villiers and Herrnstein (1976), and de Villiers (1977), in a retrospective analysis of data from the literature, found that the great majority of available information was consistent with these predictions.

In the previous experiment in this series (Bradshaw, Szabadi, & Bevan 1978), the relationship between response rate and reinforcement frequency was examined using three different reinforcers: .32 M sucrose, .05 M sucrose, and distilled water. In accordance with Herrnstein's predictions, there was an inverse relationship between sucrose concentration and the value of K_H. However, there was also an unexpected relationship between R_{max} and sucrose concentration, higher values of R_{max} being associated with the more concentrated su-

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[1] $r = K_H$ when $R = R_{max}/2$. K_H is mathematically equivalent to r_o in Herrnstein's (1970) formulation and to C in Catania's (1973) formulation. However, since K_H is defined in purely mathematical terms, it carries none of the theoretical assumptions associated with r_o (see Bradshaw, Szabadi, & Bevan, 1976).

crose solution. In the present experiment we have examined the effects of another parameter of reinforcement, the volume of liquid reinforcer, upon the relationship between response rate and reinforcement frequency.

METHOD

Subjects

Three experimentally naive female albino Wistar rats (R62, R63, and R65), bred in our laboratory and aged about 4.5 months at the start of training, were used. (A fourth rat, R64, started the experiment but died during the second phase; data from this animal have not been included in this report.) The rats were housed individually under a constant cycle of 12-hr light and 12-hr darkness, and were maintained at approximately 80% of their initial free-feeding body weights throughout the experiment. Tap water was freely available in the home cages.

Apparatus

The rats were trained in a standard operant conditioning chamber (Campden Instruments Ltd., Model 410) measuring 20 cm high by 23 cm wide by 22.5 cm long. The front wall contained a recess into which a motor-operated dipper, fitted with a cup, could deliver a small volume of liquid reinforcer (.32 *M* sucrose). (Cups of different capacities were used during the various phases of the experiment [see below]). The dipper was normally out of reach of the animal; reinforcer delivery consisted of raising the dipper into the recess for 5 sec. An aperture was situated 5.0 cm above and 2.5 cm to the right of the recess; a motor-driven retractable lever could be inserted into the chamber through this aperture. The lever could be depressed by a force of approximately .2 N. The chamber was fitted with a 2.8-W houselight in the center of the roof. It was enclosed in a sound attenuating chest, and masking noise was provided by a rotary fan. Conventional electromechanical programming and recording apparatus was situated in an adjoining room.

Procedure

After acclimatization to the food-deprivation regime, the rats were trained to press the lever by the method of successive approximations. After three sessions' exposure to a continuous reinforcement schedule, they were subjected to a series of variable-interval schedules as described below. Experimental sessions took place daily, at the same time each day. Each session was terminated by withdrawal of the lever after 50 reinforcements or 60 min, whichever occurred sooner.

Variable-interval schedules were used throughout the experiment. The distribution of the intervals was as described by Catania and Reynolds (1968, Appendix II). The reinforcer, a .32 *M* solution of sucrose in distilled water, was prepared daily before each experimental session.

The experiment consisted of four phases. During Phase I the rats were exposed to a series of variable-interval schedules, exposure to each schedule continuing through 30 successive daily sessions. The values of the variable-interval schedules (in seconds) were 9.2, 25.0, 43.0, 76.5, 191.5, and 384.0; a reinforcer cup of .10 ml capacity was used during Phase I. During Phase II the entire procedure was repeated, using a reinforcer cup of .05 ml capacity. During Phase III the entire procedure was again repeated, using a cup of .02 ml capacity. Finally, during Phase IV the animals were reexposed to variable-interval 43.0, first using the .01 ml cup (30 sessions), and then using the .02 ml cup (30 sessions).

RESULTS

Mean response rates (± standard error of the mean) recorded in the last five sessions' exposure to each schedule were calculated separately for each rat. (Throughout this paper "response rate" refers to the number of responses emitted per unit available time, i.e. reinforcer time was excluded.) Figure 1 shows, in the case of each rat, the relationship between response rate and delivered reinforcement frequency for the three reinforcer volumes. In general, the response rates maintained under any given schedule were lower when reinforcer cups of smaller volume were used. This effect is most readily apparent when the data obtained using the .10 ml cup and the .02 ml cup are compared. In the case of each rat and each schedule response rates were lower when the .02 ml cup was used than when the .01 ml cup was used, the magnitude of the discrepancy being greatest under intermediate frequencies of reinforcement.

Curves having the form defined by Equation

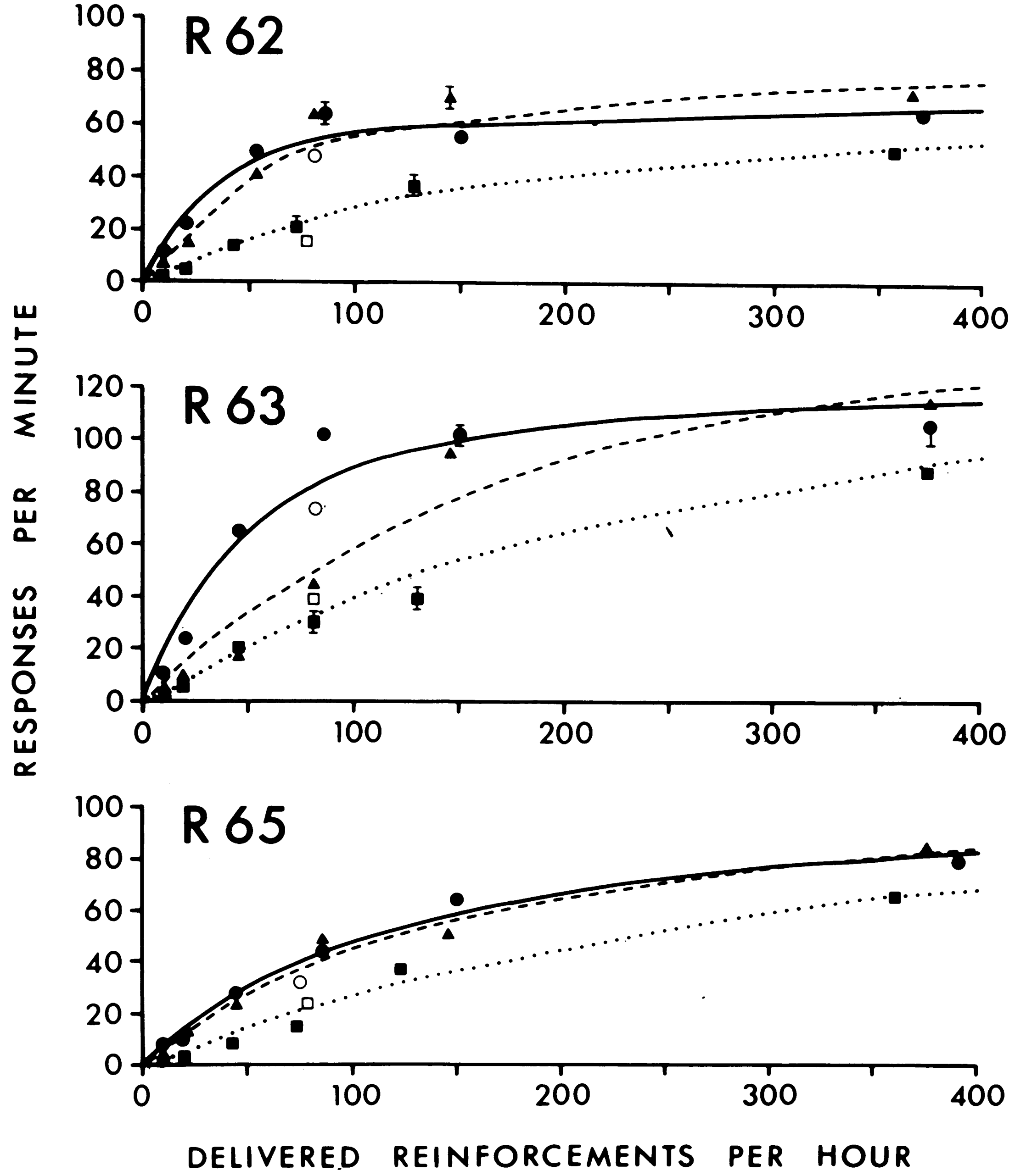

Fig. 1. Relationship between response rate and delivered reinforcement frequency for three reinforcer volumes. Each graph was obtained from one rat. Points show mean response rates for the last five sessions' exposure to a schedule; vertical bars indicate ± s.e.m., where this was greater than ± 3.0 responses per minute. Curves are rectangular hyperbolae fitted by nonlinear regression analysis. Circles and continuous curve: .10 ml; triangles and broken curve: .05 ml; squares and dotted curve: .02 ml. Unfilled symbols indicate redeterminations.

1 were fitted to the data obtained from each rat with each of the three reinforcer volumes, using nonlinear regression analysis (Wilkinson, 1961). The estimated values of R_{max} and K_H (± standard error of the estimate) derived from this analysis are shown in Table 1. Also shown in Table 1 are the indices of determination (p^2) calculated for each curve; this expresses the proportion of the variance in the response rates (R) which can be accounted for

Table 1

Values of R_{max} and K_H derived from nonlinear regression analysis.

Subject	*Volume of Reinforcer*	*Theoretical Maximum Response Rate, R_{max} (resp/min ± s.e.est.)*	*Reinforcement Frequency Needed to Obtain Half-Maximal Response Rate, K_H (reinf/hr ± s.e.est.)*	p^2
R62	.10 ml	71.8 (± 8.5)	31.8 (± 13.7)	.871
	.05 ml	91.5 (±11.0)	50.8 (± 19.4)	.942
	.02 ml	82.4 (±13.7)	209.0 (± 68.6)**	.957
R63	.10 ml	131.6 (±17.3)	51.0 (± 21.2)	.906
	.05 ml	194.4 (±45.4)	226.8 (±129.9)	.936
	.02 ml	164.0 (±27.9)	314.9 (± 97.5)***	.970
R65	.10 ml	107.6 (±12.4)	128.9 (± 33.2)	.965
	.05 ml	110.7 (±13.7)	132.1 (± 38.3)	.970
	.02 ml	136.4 (±29.8)	407.7 (±158.6)*	.966

Significance of difference from value obtained with .10 ml (normal distribution test): $*p < .05$, $**p < .01$, $***p < .005$.

Table 2

Analysis of Variance of Data Obtained from Each Rat

Subject	*Source of Variation*	*d.f.*	*MS*	*F*
R62	Volume of reinforcer	2	113.2	7.81*
	Schedule	2	252.4	17.41*
	Time epoch	5	.1	.01
	Total	53	14.5	
R63	Volume of reinforcer	2	186.8	10.49*
	Schedule	2	201.1	11.30*
	Time epoch	5	1.0	.06
	Total	53	17.8	
R65	Volume of reinforcer	2	55.5	4.27*
	Schedule	2	223.5	17.19*
	Time epoch	5	2.6	.20
	Total	53	13.0	

$*p < .05$.

in terms of reinforcement frequency (r) using Equation 1 (Lewis, 1960; see also Bradshaw et al., 1976).

The data obtained from all three rats with each reinforcer volume conformed closely to Equation 1, the values of p^2 being in every case greater than .85, and in 8 out of the 9 cases greater than .90. In the case of each rat, the value of K_H increased with decreasing volume of the reinforcer. The differences between the values of K_H obtained with .05 ml and .10 ml did not achieve statistical significance; however, for each rat the value of K_H obtained with .02 ml was significantly greater than the value obtained with .10 ml (normal distribution test: see Table 1 for individual significance levels). In no case did the values of R_{max} obtained with .05 ml and .02 ml differ significantly from the values obtained with .10 ml.

As the criterion for terminating a session was either the delivery of 50 reinforcers or the completion of 60 min (see Method), it follows that the duration of sessions varied between conditions. Therefore a statistical analysis was carried out in order to determine whether response rate varied systematically within sessions. Performance maintained under the three schedules associated with the longest session time (VI 76.5, VI 191.5, and VI 384.0) was analyzed by dividing the total session time into 10-min epochs, and determining the response rate separately for each epoch by measurement of the cumulative records. Three-factor analyses of variance were carried out in which the variables were reinforcer volume (.10 ml, .05 ml, and .02 ml), schedule, and time epoch within a session. The data obtained from each rat were analyzed separately. The results of the analyses are shown in Table 2. In the case of each rat, significant effects of volume and schedule were found; however, time epoch had no significant effect.

DISCUSSION

The results obtained from all three rats using all three reinforcer volumes conformed closely to Equation 1. Thus the present results are in agreement with earlier findings (Bradshaw, 1977; Bradshaw et al., 1978; see discussion of de Villiers' unpublished data, de Villiers & Herrnstein, 1976) that the behavior of rats in VI schedules is accurately described by Herrnstein's equation. The proportion of the data variance accounted for in the present experiment (87.0% to 97.0%) is comparable to that reported in the previous studies.

In the case of all three rats, the value of K_H was inversely related to the volume of sucrose reinforcer. This is in accordance with the predicition derived by Herrnstein (1974). According to Herrnstein's interpretation of K_H ("r_O" in Herrnstein's terminology), this term reflects the frequency of extraneous, or unscheduled reinforcement. Thus it is to be expected that when this term is expressed in the units of a more powerful reinforcer, such as .10 ml of a .32 *M* sucrose solution, its value will be lower than when it is expressed in units of a weaker reinforcer, such as .05 ml or .02 ml of the same solution (see also de Villiers & Herrnstein, 1976).

The effect of the volume of sucrose reinforcer upon the value of K_H seen in this experiment is similar to the effect of the *concentration* of sucrose reinforcer upon the value of this constant (Bradshaw et al., 1978). However, reinforcer volume and reinforcer concentration exerted different effects upon the value of R_{max}. In the previous study it was found that the value of R_{max} increased with increasing sucrose concentrations. In the present experiment no such direct relationship was found between R_{max} and reinforcer volume. It is apparent, therefore, that these two parameters of reinforcement are not equivalent with respect to their effects upon the relationship defined by Equation 1.

Although no significant differences were found, in any of the subjects, between the values of R_{max} obtained with the different reinforcer volumes, it is noteworthy that in all three subjects the value of R_{max} was somewhat lower when the .10 ml cup was used than when either of the other two cups was used. It is possible that this reflects an interruption of operant responding by postprandial activities, such as licking in the vicinity of the dipper aperture (cf. Iversen, 1978; Staddon, 1977), which may be more prevalent following the consumption of the larger volume of the sucrose solution. Such an effect would have a more marked effect on response rate under high reinforcement frequencies, and might therefore result in a reduction in the value of R_{max}.

The term 'magnitude of reinforcement' has been used to refer to a variety of different dimensions of reinforcing stimuli, including the size, weight, and number of food particles, the duration of exposure to food, and the concentration and volume of liquid reinforcers (see Guttman, 1953). The literature relating to the effects of different magnitudes of reinforcement contains many apparently contradictory findings: some authors have reported little or no effect of manipulations of reinforcement magnitude, while others have reported large and durable effects (for review, see Neuringer, 1967; Schrier, 1958). At least two procedural factors are known to be partly responsible for these discrepancies. Firstly, the differential effects of different magnitudes of reinforcement upon performance are greater when subjects are exposed to several magnitudes within a single session ("shift condition") than when they receive extended training under each magnitude ("nonshift condition") (Schrier, 1958). Secondly, the differential effects of different magnitudes of reinforcement are greater when subjects are able to choose between different magnitudes ("response-contingent magnitude") than when magnitude is manipulated independently of the subjects' behaviour ("noncontingent magnitude") (Neuringer, 1967). The present results, taken together with the results of the previous experiment (Bradshaw et al., 1978) suggest that in addition to these procedural factors, the particular dimension of the reinforcing event which is manipulated in order to vary the magnitude of reinforcement may also influence the nature of the effects observed. In both experiments, differential effects of different reinforcement magnitudes upon response rate were observed; however, when the *concentration* of the sucrose reinforcer was varied these effects were apparent under all frequencies of reinforcement (see Figure 1 in Bradshaw et al., 1978), whereas when the *volume* of the sucrose reinforcer was varied the effects were much more marked under intermediate reinforcement frequencies than under high reinforcement frequencies (see Figure 1).

The analyses of variance shown in Table 2 indicate that response rates maintained under a particular schedule did not change systematically as a function of time within a session. However, significant effects of both the reinforcer volume and the schedule were obtained. Thus the analyses provide statistical confirmation of the dependence of response rate upon both reinforcement frequency and reinforcer volume.

It is of interest to relate the present findings with single variable-interval schedules to previous observations with concurrent variable-interval variable-interval schedules in which dissimilar reinforcers were associated with the different component schedules (Fantino, Squires, Delbruck, & Peterson, 1972; Hamblin & Miller, 1977; Hollard & Davison, 1971; Keller & Gollub, 1977; Miller, 1976; Schneider, 1973; Todorov, 1973). These studies generally showed that preference for one reinforcer (u) over the other reinforcer (v) could be characterized by a bias factor, c, in the Matching Law (Baum, 1974; Herrnstein, 1970):

$$R_u/R_v = c\,(r_u/r_v). \qquad (2)$$

A consistent bias in favor of one type of reinforcer in concurrent schedules is consistent with the present observations of performance in single schedules. For example, in the case where u and v stand for different volumes of a liquid reinforcer, response rates in single variable-interval schedules can be expressed as

$$R_u = R_{max(u)} \cdot r_u/(K_{H(u)} + r_u) \qquad (3)$$

and

$$R_v = R_{max(v)} \cdot r_v/(K_{H(v)} + r_v). \qquad (4)$$

The present finding that different reinforcer volumes are associated with different values of K_H can be expressed as

$$K_{H(v)} = n \cdot K_{H(u)}. \qquad (5)$$

(Note that the present results suggest that in the case of reinforcer volume $R_{max(u)} = R_{max(v)}$.) Equation 4 may now be rewritten

$$R_v = R_{max(v)} \cdot r_v/(n \cdot K_{H(u)} + r_v)$$

or

$$R_v = (R_{max(v)} \cdot r_v/n)/(K_{H(u)} + [r_v/n]). \qquad (4a)$$

It follows from Equations 3 and 4a that, provided that $R_{max(u)} = R_{max(v)}$, equal values of r_u and r_v/n are associated with equal rates of responding. Thus n may be regarded as a scaling constant which enables r_u and r_v to be expressed in "behaviorally equivalent" units (cf Herrnstein, 1974). This scaling constant may now be used to derive the ratio of the response rates in the two components of a concurrent schedule in which the two reinforcer volumes are associated respectively with the two components.

The absolute rates of responding in the two components, A and B, of a concurrent schedule are given by

$$R_A = R_{max} \cdot r_A/(K_H + r_A + r_B) \qquad (6)$$

and

$$R_B = R_{max} \cdot r_B/(K_H + r_A + r_B) \qquad (7)$$

(Herrnstein, 1970). If the values of R_{max} and K_H are assumed to be invariant between the two components, Equations 6 and 7 may be combined to yield the Matching Law:

$$R_A/R_B = r_A/r_B. \qquad (8)$$

In the case considered here, where different reinforcer volumes, u and v, are associated with the two components, the term n can be used to rescale r_v so as to express r_u and r_v in "behaviorally equivalent" units. Equations 6 and 7 then become

$$R_u = R_{max(u)} \cdot r_u/(K_{H(u)} + r_u + [r_v/n]) \qquad (6a)$$

and

$$R_v = (R_{max(v)} \cdot r_v/n)/(K_{H(u)} + r_u + [r_v/n]). \qquad (7a)$$

Assuming equality of $R_{max(u)}$ and $R_{max(v)}$ (see above), Equations 6a and 7a may be combined to yield

$$R_u/R_v = n\,(r_u/r_v). \qquad (8a)$$

Equation 8a is formally identical to Equation 2, n having the status of a bias factor; thus the ratio of the values of K_H obtained for two reinforcer volumes in single variable-interval schedules should be equal to the bias in favor of the preferred volume when the two volumes are associated respectively with the two components of a concurrent schedule.

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