HEAT TRANSFER FROM SPHERES AND OTHER ANIMAL FORMS

JOHN W. MITCHELL

From the Mechanical Engineering Department, University of Wisconsin, Madison, Wisconsin 53706

ABSTRACT A general predictive relation for the convection heat transfer from animal forms is developed. This relation is based on the convection equation for a sphere, and employs a simple, unique characteristic dimension to represent the animal which is the cube root of the animal volume. The accuracy of this relation is established through comparison with available convection results from animal shapes ranging in size and shape from spiders to cows. This relation allows an extrapolation to animal shapes for which data are not available. Results are also presented for the enhancement of convection heat transfer due to natural turbulence. A procedure is outlined for estimating the convecture heat loss from an animal in the natural outdoor environment.

INTRODUCTION

In recent years there has been increasing interest in modeling heat exchange between animals and their environments. These studies are generally aimed toward an assessment of the role of different mechanisms of energy flow on the energy balance, thermal stresses, and temperatures of animals. Such models incorporate basic relations for the transfer of thermal energy by solar and long wave radiation, convection, conduction, and evaporation. The net thermal energy flow is equated to the rates of thermal energy storage and generation to yield a thermal energy balance for the animal.

Convective heat transfer is one mode of thermal energy exchange between an animal and its environment. It often plays a significant, but not necessarily dominant, role in the energy balance. The equation for convection is

$$q = hA(T_s - T_a),$$

where q is heat flow, h the heat transfer coefficient, A the surface area, T_s the surface temperature, and T_a the air temperature. The heat transfer coefficient is a complicated function of geometry and wind speed. Many engineering studies have been oriented toward analytical and experimental evaluations of the heat transfer coefficient, and the mechanisms are quite well understood for many simple geometries (e.g., Krieth, 1958). For more complicated geometries, such as animals, only experimental results are available. These empirical relationships are for a limited number of animal forms, and at present, there is no general theory for predicting the coefficients for new forms.

Modeling an animal in its natural environment is further complicated by changes in both animal behavior and environmental conditions. An animal may not maintain the same orientation with respect to wind direction or the same elevation above the surface for long periods. The wind itself has unpredictable large and small scale fluctuations in speed and direction. The instantaneous value of the heat flow, computed from Eq. 1, thus varies significantly, rapidly, and unpredictably with time. It is only feasible to model an animal using the average values over some time period based on a best estimate of the average animal behavior and environmental conditions. It is also probably more meaningful to model in this manner, in that the animal itself can only respond to some average value of heat flow.

These considerations obviate the need for highly accurate relations for convection, as has been recognized by a number of investigators. Gates (1962) and Porter and Gates (1969) recommend the use of convection relations for circular cylinders or spheres using the animal diameter as the characteristic dimension. This approach has not been extensively verified, and may not be applicable to animals of quite different shapes. Further, this approach uses wind tunnel correlations, and the natural turbulence in the outdoor environment augments the heat transfer coefficient.

It is the objective of this paper to develop a generalized predictive relation for use in modeling the convective heat flow between an animal and its environment. This relation is based on convection relations for a sphere, and employs a readily obtainable characteristic dimension for the animal based on animal mass. Use of the relation is verified through comparison with available results from a wide variety of animal forms. This comparison allows extrapolation to animals for which results are not presently available. Finally the enhancement in heat transfer due to turbulence in the outdoor environment is discussed.

ANALYTICAL DEVELOPMENT

Considerable research has been done to determine geometric scaling laws for animals and plants. The relations between a linear dimension, surface area, and mass are usually formulated as power relationships (e.g. see Altman, 1958; Brody, 1945).

For dimensional consistency, and to facilitate the later development, organism mass is replaced by volume using the bulk body density ρ

$$V = m/\rho. \tag{1}$$

The general scaling relations are then of the form:

$$l = k_l V^{1/3}, \tag{2}$$

$$A = k_A V^{2/3}, \tag{3}$$

where l is some characteristic dimension (e.g., trunk diameter or length), A is surface area, m is body mass and k_l and k_A are constants. The constants are very much dependent on the particular shape of the animal, and whether a diameter or a length is taken as the characteristic dimension.

TABLE I

CONVECTIVE HEAT TRANSFER RELATIONS FOR ANIMAL SHAPES, $(hl/k) = C_l (vl/\nu)^n$

Animal	C_l	n	l = dimension	Notes	References
Sphere	0.37	0.6	Diameter		Kreith (1958)
Cylinder	0.615	0.466	Diameter	40 < Re < 4,000	Kreith (1958)
	0.174	0.618		4,000 < Re < 40,000	
Cow	0.65	0.53	Trunk diameter		Wiersma and Nelson (1967)
Man	1.30	0.53	Average diameter	Walking adult males, l = 0.2 m	Nishi and Gagge (1970) Rapp (1973)
Sheep	0.50	0.55	Trunk diameter		Bennett and Hutchinson (1964) Joyce et al. (1966)
Frog	0.258	0.667	Snout-vent length		Tracy (1972)
Lizard	0.35	0.6	Snout-vent length	Transverse to air flow	Porter et al. (1973)
Lizard	0.1	0.74	Snout-vent length	Parallel to air flow	Porter et al. (1973)
Lizard	1.36	0.39	Snout-vent length	Prostrate on surface, average for parallel and perpendicular to air flow	Muth (1975)*
Lizard	1.91	0.45	Snout-vent length	Elevated from surface, average for parallel and perpendicular to air flow	Muth (1975)*
Flying insects	0.0749	0.78	Average diameter		Church (1960)
Spiders	0.47	0.5	Diameter		Reichart and Tracy (1975)

*Personal communication.

Forced convection heat transfer for animals, plants, and geometric shapes is usually represented by an equation of the form

$$h = C_h v^n, \tag{4}$$

where h is the convective heat transfer coefficient and v is the velocity. The coefficients C_h and n are usually experimentally determined constants. Convection results are normally represented in nondimensional form as

$$\mathrm{Nu}_l = hl/k, \tag{5}$$

$$\mathrm{Re}_l = vl/\nu, \tag{6}$$

where Nu_l is Nusselt number, Re_l is Reynolds number, l is the characteristic dimension, k is fluid thermal conductivity, and ν is fluid kinematic viscosity. The convection relation, Eq. 4, can be rewritten using these parameters as

$$\mathrm{Nu}_l = C_l \mathrm{Re}_l^n, \tag{7}$$

where C_l is a dimensionless constant combining C_h and fluid properties.

Convection results have been obtained for a wide variety of animal sizes and shapes, ranging from spiders to cows. These data are summarized in Table I and in Fig. 1

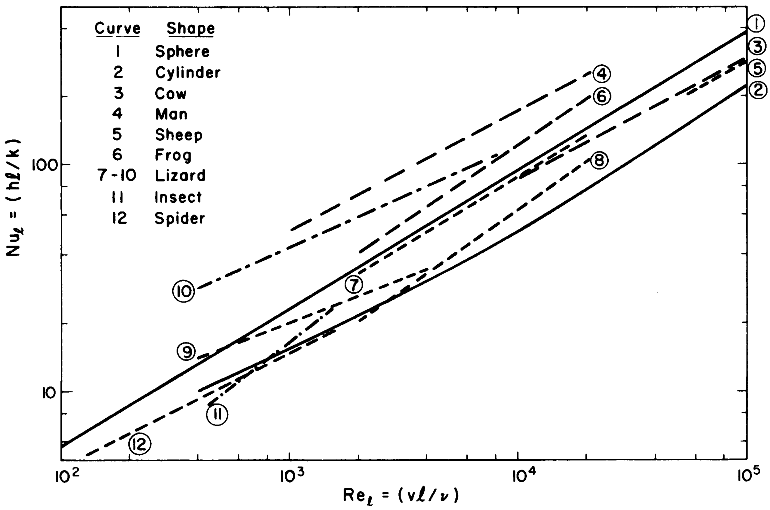

FIGURE 1 Heat transfer for various forms based on the characteristic dimension l.

where the commonly employed characteristic dimension of the shape is used. The data for frogs (Tracy, 1972), lizards (Porter et al., 1973, footnote 1) and insects (Church, 1960) were taken using castings of actual animals which preserved the surface texture of the animal. The data for man were taken from experiments on humans (Nishi and Gagge, 1970; Rapp, 1973). Also shown for reference are results for the smooth sphere and cylinder (Kreith, 1958). These engineering models are commonly employed to represent convection from animals in the absence of data. In particular, the cylinder relation has been used extensively by Gates (1962), Porter and Gates (1969), Birkebak (1966), Spotila et al. (1972).

The relations in Fig. 1 show that at a given Reynolds number, the nondimensional heat transfer coefficient (Nusselt number) varies between animal shapes by as much as a factor of four. The sphere and the cylinder, which use the same characteristic dimension, differ by almost a factor of two. Thus, there appears to be no "universal" heat transfer relationship for all shapes that uses the traditional characteristic dimensions.

Fig. 1 also suggests that new experiments need to be conducted to determine convection coefficients for an animal form that has not already been tested. Fig. 1 might be used as a guide, but, as is readily apparent from the spread of the results, considerably uncertainty would exist. For example, the use of the cylinder relation to predict the heat transfer coefficient for humans would yield a result low by a factor of four. As will be shown, a characteristic dimension can be chosen to allow accurate estimations of the convection coefficient.

The geometric relations, Eqs. 2 and 3, show that volume (or mass) is the primary determinant for the linear dimensions of a given animal form. The coefficients k_l and

[1] Muth, A. 1975. Personal communication.

TABLE II

GEOMETRIC RELATIONSHIPS FOR ANIMAL SHAPES

	$l = K_l V^{1/3}$		$A = K_A V^{2/3}$		Notes
Animal	K_l	Reference	K_A	Reference	
Sphere	1.24		4.83		Geometry $d = (6/\pi)^{1/3} V^{1/3}$ $A = (36\pi)^{1/3} V^{2/3}$
Cow	1.18	Brody (1945)	9.4	Brody (1945)	
			9.3–11	Altman (1958)	
Man	—		10.8	Brody (1945)	Area relation is fit to relation of Brody $A = 8.8\ V^{0.685}$
Sheep	—		8.3–10.7	Altman (1958)	
Frog	2.27	Tracy (1972)	11.0	Tracy (1972)	
Lizard	3.3	Norris (1965)	11.0	Norris (1965)	
Flying insects	—		—		Data on convection given in terms of mean diameter assumed spherical shape
Spider	2.3	Reichart and Tracy (1975)	—	—	

k_A do not vary by orders of magnitude among different forms (Altman, 1958; Brody, 1945). This suggests that a characteristic dimension for convection should be related directly to volume. The use of a new characteristic dimension defined as the cube root of the volume will be tested. This dimension is given by

$$L = V^{1/3}. \tag{8}$$

The relation between the physical dimension l used in the previous correlations and the volume-related dimension L can be readily determined. Eq. 2 relates l and V through the empirical constant k_l. The values of k_l and the proportional coefficient for area, k_A, together with the corresponding references, are given in Table II. It should be noted that the cylinder is omitted from this tabulation. A cylinder of infinite extent has an undefined relation between its diameter and volume. Thus, L is undefined, and cylinder results are excluded from further consideration.

The heat transfer parameters are also defined in terms of L as

$$\mathrm{Nu}_L = hL/k, \tag{9}$$

$$\mathrm{Re}_L = \mathrm{v}L/\nu, \tag{10}$$

and the convection heat transfer relation, Eq. 7, becomes

$$\mathrm{Nu}_L = C_L \mathrm{Re}_L^n. \tag{11}$$

The value of the proportional coefficient C_L, changes from that of C_l in Eq. 7, but the exponent n remains the same. The convection relations using the dimension L are summarized in Table III and Fig. 2.

As shown in Fig. 2, the results are very well correlated over a threefold range of Re_L using L as the characteristic dimension. All the results, except for the two lizard re-

TABLE III

CONVECTIVE HEAT TRANSFER RELATIONS FOR ANIMAL SHAPES

$(hL/k) = C_L(vL/\nu)^n;\ L = V^{1/3} = (m/\rho)^{1/3}$

Animal	C_L	n	Re range	K_A
Sphere	0.34	0.6	All	4.83
Cow	0.6	0.53	10^4–1.5×10^5	9.4
Man	0.63	0.53	5×10^3– 5×10^4	10.8
Sheep	0.49	0.55	5×10^4–1.3×10^5	9.2
Frog	0.196	0.667	9×10^2– 9×10^3	11
Lizard	0.56	0.6	8×10^2– 5×10^3	11
Lizard	0.096	0.74	8×10^2– 5×10^3	11
Lizard	1.09	0.39	3×10^2– 3×10^3	11
Lizard	1.00	0.45	3×10^2– 3×10^3	11
Flying insects	0.0714	0.78	4×10^2–1.5×10^5	—
Spider	0.52	0.5	90–1.4×10^3	—

lations numbered 7 and 8, fall within about ±20% of the sphere relation. The use of the sphere relation gives a satisfactory estimate of the convection coefficient for all of the diverse geometries shown here. The convection from a sphere is given in terms of L by

$$\mathrm{Nu}_L = 0.34\ \mathrm{Re}_L^{0.6}. \qquad (12)$$

Curve number 7 is for a lizard model transverse to the air flow. This shape in this orientation appears very much like a cylinder with respect to air flow. Thus, the volume related dimension L is apparently not the significant one. Curve number 8 is for a

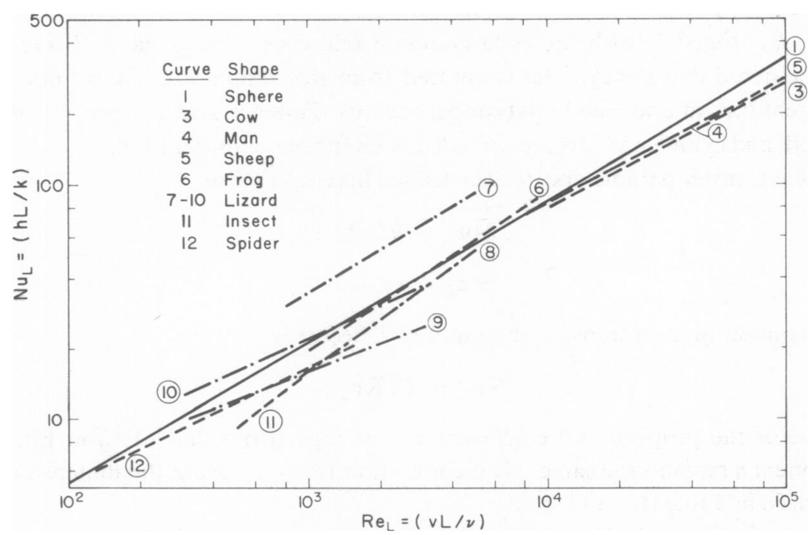

FIGURE 2 Heat transfer for various form based on the characteristic dimension L.

lizard prostrate on the substrate. The air flow over the model is restricted by the close proximity to the substrate such that a large proportion of the surface is probably exposed to low speed air. This would explain the relatively low heat transfer results for this model. Even with these differences in orientation, the sphere relation would still predict a coefficient within about 50% of the actual value.

The results shown in Fig. 2 demonstrate that wind tunnel tests of diverse animal shapes and surfaces can be well correlated using L as the characteristic dimension. In the field, the turbulence level and scale in the wind is higher than that usually found in wind tunnels. This increased turbulence increases the heat transfer coefficient for objects in the outdoor environment (Pearman, et al., 1972; Jackson, 1975).

A recent study by Kowalski and Mitchell (1975) was directed toward evaluating the influence of turbulence on heat transfer from spheres under field conditions. Heat transfer from spheres of three different diameters (3.81, 5.08, and 7.62 cm) was measured at heights of 10, 40, and 200 cm above the ground surface. Wind speeds over the short grass prairie varied from 1 to 8.5 m/s.

The enhancement of heat transfer defined as the ratio of Nusselt number in the field (Nu_T) to that at the same velocity in the wind tunnel, was found to vary from 1.0 to 2.2, with a mean value of 1.23. Thus, on the average, the heat transfer coefficient in the field is 23% higher than that predicted from the wind tunnel relation, Eq. 12.

Further, it was found that the enhancement was a function of both sphere diameter and height above the surface. The results were correlated by the ratio of height to sphere diameter, z/D, and are shown in Fig. 3. As discussed by Kowalski and Mitchell (1975), the results of other investigators were used to estimate the results at the equivalent of low heights above the ground surface. A z/D of 0.5 represents that for a sphere on the substrate, and is the minimum physically significant value. The physical explanation for the change with height lies in the fact that the turbulent mixing length, a measure of the turbulent eddy size, increases linearly with distance above the surface.

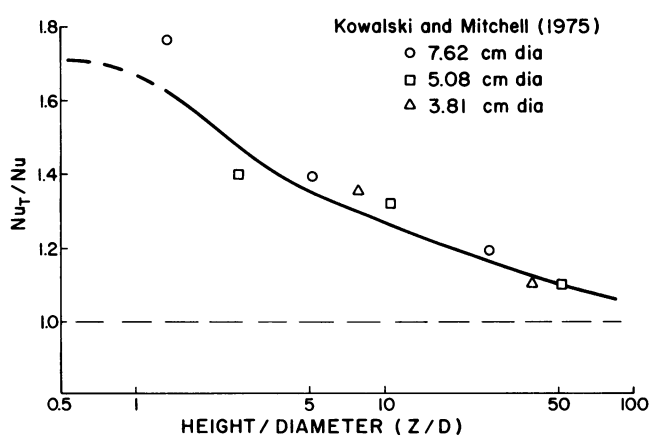

FIGURE 3 Enhancement in heat transfer as a function of height.

The mixing length x is given by the relation

$$x = Kz, \tag{13}$$

where K is Kármán's constant (0.4, Sutton, 1953).

At values of z/D of less than 1, the disturbance caused by the turbulence is amplified in the sphere boundary layer (Maisel and Sherwood, 1950), which causes a maximum increase in heat transfer. At increased heights, the eddy size (mixing length) becomes large relative to the sphere diameter. From the point of view of the sphere, the flow is unsteady with rapid fluctuations which also increase the heat transfer. As z/D increases, the frequency of the fluctuations reduces and the sphere is exposed to longer periods of relatively steady flow, with a consequent reduction in heat transfer augmentation. This explains the reduction in enhancement as z/D increases.

The enhancement for animal shapes outdoors is expected to be the same as that for spheres. No direct measurements have been made, but the results of Tanner and Goltz (1972) and Porter et al. (1973) support the correlation presented in Fig. 3.

These results indicate that the heat transfer enhancement is quite high for animals near the ground surface. The value of z/D and, also, z/L, is between 0.5 for an animal resting on the surface and 1.0 for one standing. The convection enhancement is then about 1.5 times the wind tunnel prediction. Flying and climbing animals probably have large values of z/D and z/L with little enhancement.

SUMMARY AND CONCLUSIONS

Convective heat transfer from animal shapes can be estimated using the equation for convection from a sphere. The procedure involves the following: (*a*) The volume of the animal is determined from its mass and density. The characteristic dimension L is evaluated as the one-third power of the volume. (*b*) Reynolds number is computed using the characteristic dimension L, wind speed, and kinematic viscosity. Nusselt number is computed using either Eq. 12 or Fig. 2. The heat transfer coefficient h is then computed using L and air thermal conductivity. (*c*) Enhancement of the convection coefficient due to outdoor turbulence is estimated using Fig. 3 with z/D estimated as equal to z/L. Enhancement is not very sensitive to z/D, and thus the approximation that z/D equals z/L is satisfactory. This procedure gives an estimate of the convection coefficient under field conditions. (*d*) The overall convection conductance (hA) can also be estimated. The surface area A is estimated from the volume using Eq. 3. For an animal whose surface area to volume relation is unknown, it appears that a value of 9–11 for k_A is satisfactory.

The above procedure provides an estimate of the convection conductance for an animal under field conditions, and allows an evaluation of convective heat loss. Approximations are made in the determination of both convection coefficient and surface area. However, for purposes of modeling the energy balance, energy flows, and thermal stresses on an animal, the procedure should be sufficiently accurate. Variations in animal orientation and elevation, in wind speed and direction, and in exposure to different radiating surfaces all serve to override the approximations introduced in this procedure.

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