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MULTIPLE SCHEDULES: EFFECTS OF THE DISTRIBUTION OF REINFORCEMENTS BETWEEN COMPONENTS ON THE DISTRIBUTION OF RESPONSES BETWEEN COMPONENTS[1]

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Two pigeons were trained to peck a key under several multiple variable-interval variable-interval schedules of reinforcement; different numbers of reinforcements were scheduled in two components of equal duration which were correlated with red and green illumination of the response key respectively. The results showed: (1) that the total number of responses in a session was proportional to the one-sixth power of the total number of reinforcements delivered in that session; and (2) that the ratio of responses between the two components was equal to the one-third power of the ratio of reinforcements between them. This latter exponent may be regarded as reflecting the sensitivity of the distribution of responses between the components to the distribution of reinforcements. It was suggested that the effects of a number of complex schedules of reinforcement could be summarized by different values of this exponent.

When two variable-interval (VI) schedules of reinforcement are combined, responding maintained by each is partly determined by reinforcements delivered under the other schedule. This interaction occurs both under concurrent schedules, in which two or more schedules operate simultaneously for responses on different keys, and under multiple schedules, in which two or more schedules operate successively for responses on a single key. This similarity between concurrent and multiple schedules, however, does not extend to the way in which responses are distributed between each of the component schedules which constitute the total schedule. Under concurrent VI VI schedules, pigeons match the distribution of responses between keys to the distribution of reinforcements between keys (Herrnstein, 1961; Catania, 1963). Under multiple VI VI schedules these distributions are not matched (Reynolds, 1963).

Catania (1963) has written a series of equations which gives a precise quantitative description of the relations between response and reinforcement rates under concurrent schedules. No comparable description of these relations under multiple schedules has been advanced, possibly because of the difficulties engendered by the absence of a matching relation. But the absence of such a relation in no way excludes the possibility of a complete quantitative description of the effects of reinforcements on response outputs under multiple schedules. The present experiment was carried out to obtain some more precise quantitative data to provide the basis for such a description. This paper suggests that performance under multiple VI VI schedules can be described by equations of the same general form as, though differing quantitatively from, those derived for concurrent schedules. Further, it suggests that the different effects of these schedules can be characterized by values of a single parameter.

One complication is that component rates of responding or reinforcement are specified differently under concurrent and multiple schedules. Under concurrent schedules, component rates are calculated in terms of total session time; under multiple schedules they are calculated in terms of the duration of each component. Thus, the terms denoting rate in these schedules are not equivalent. However, it is possible to compare performances under these schedules in terms of number of responses or reinforcements in a session of given

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length. Such a comparison requires that there be equal opportunity for responding in the presence of each component of multiple schedules, as is the case under concurrent schedules. Thus, components must be of equal duration in multiple schedules which are the subject of such a comparison. It should be noted that under these conditions, equivalent relations would hold for both number and rate under multiple schedules, since rate would be calculated by dividing number by a constant time.

METHOD

Subjects

Two adult homing pigeons, maintained at 80% of their free-feeding weights, were given daily sessions. The pigeons had had 30 prior sessions of preliminary training on single variable-interval schedules of reinforcement.

Apparatus

The experimental chamber was housed in a sound-attenuating box. A single Perspex key, 1 in. in diameter, was located in the center of one wall. This could be illuminated from behind by white, red, or green light. Pecks of 15-g force or more on this key were recorded on counters and a cumulative recorder, and could be reinforced by a 2.5-sec presentation of grain at an aperture directly below the key. A light over the feeder was on, and the light behind the key was off, during reinforcement. Experimental events and contingencies were controlled automatically by electro-mechanical circuitry.

Procedure

The pigeons were trained on a series of eight multiple schedules, each consisting of two components. Table I shows the order of presentation and the number of reinforcements delivered in each component. All reinforcements were scheduled on a variable-interval basis according to the progressions of Fleschler and Hoffman (1962). The component schedules were alternated during successive 3-min intervals, total duration of the components being equal. Experimental sessions lasted 1 hr, and each component thus occupied ten 3-min intervals per session. One component was correlated with red and the other with green illumination of the key.

Table 1

Sequence of multiple schedules specified in reinforcements per session (30 min per component).

	Component				*Component*		
Pigeon	*1*	*2*	*Sessions*	*Pigeon*	*1*	*2*	*Sessions*
12	10	10	34	14	10	10	12
	10	3	28		10	1	16
	10	10	13				
	10	0	17		10	10	26
	10	10	16		10	3	14
	10	50	15		10	10	29
	1	24	30		10	20	28
	10	1	19		10	10	40
	10	20	14		10	50	29
	10	6	19		10	10	17
					10	20	17
					10	1	30
					1	24	18
					10	0	20
					0	10	13

Changes between stimuli occurred only when reinforcements set up in their presence had been collected by a peck on the key. Thus, all scheduled reinforcements were delivered. With this procedure, a delay in collecting a reinforcement prolonged the duration of a stimulus presentation beyond 3 min by an amount equal to the delay. In practice, such delays were very infrequent and when they occurred amounted only to a few seconds per session. In the absence of comparative data, this procedure was preferred to the more common one of canceling reinforcements not collected before a stimulus change; the smallest variation in reinforcements delivered, under the latter, might be expected to have a greater effect than the small variations in stimulus durations obtained.

Seven multiple schedules arranged 10 reinforcements per session (VI 3-min) in Component 1 (red); reinforcements in Component 2 (green) varied from 0 (extinction) to 50 per session (VI 36-sec). One additional schedule provided one reinforcement per session in Component 1 (red) so that the relatively large number of reinforcements required in the other component did not increase total reinforcements to a level that would interfere with the pigeons' body weights. In one of the schedules, 10 reinforcements were arranged in each component. Performance under this schedule was employed as a baseline. Both pigeons were returned to this baseline schedule a number of times between training under other schedules in order to measure any changes in absolute level of responding during the course of the experiment. Training under all schedules was continued for at least 12 sessions and until the rate of responding in both components was judged to be stable. An additional criterion for the baseline schedule required that the difference between the number of responses under the two components be not greater than 5% of the number under either component.

RESULTS

Total Number of Responses

In Fig. 1, Panel A, the total number of responses emitted in a session is plotted as a function of the total number of reinforcements in a session for each of the schedules. Points represent the average of the last three sessions under a schedule. It can be seen that output varied considerably, as measured by the baseline schedule, particularly in the case of Pigeon 14. This variation obscures the effects of total number of reinforcements on total number of responses. However, some systematic effects can be seen relative to performance under the baseline schedule. In Fig. 1(A), points representing performance under schedules preceded by baseline training have been joined to points representing performance during that baseline training. The slope of these lines indicates the effects of changes in total number of reinforcements for a given level of output under the baseline schedule. These lines are similar in that they all have a small positive slope. This suggests that the effects of total number of reinforcements may be most appropriately analyzed in terms of changes in responding from a baseline level.

To facilitate comparison of the experimental points with one another and with a theoretical function, the data were transformed so that all baseline points coincided. This was done by multiplying the number of responses under an experimental schedule by whatever factor raised the number of responses emitted under the preceding baseline schedule to a value of 3000; this approximated the average of all measures of baseline performance. Figure 1(B) shows the corrected total numbers of responses emitted as a function of the total numbers of reinforcements delivered, for each of the schedules preceded by baseline training. The broken curve in Fig. 1(B) is a power function generated by the equation

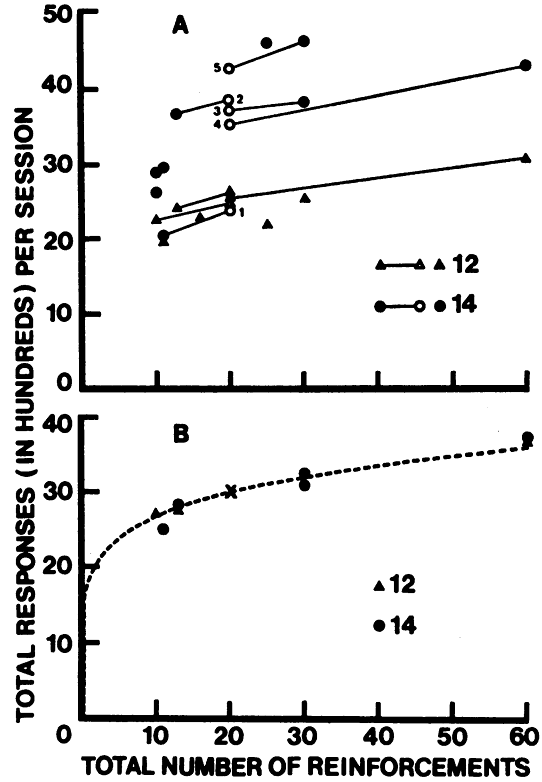

Fig. 1. A. Total number of responses per session as a function of total number of reinforcements per session. Open points represent performance under the baseline schedule. The small numerals beside points for Pigeon 14 indicate the order in which the baseline schedule was presented for this bird. The lines represent the effect of number of reinforcements for a given baseline performance. Panel B shows total number of responses corrected for preceding baseline level as a function of total number of reinforcements. The curve is a plot of equation (1). The cross at 20 reinforcements represents the level of baseline responding preceding the determination of each point: see text.

$$(N_1 + N_2) = K\,(n_1 + n_2)^{1/6} \qquad (1)$$

where N and n represent the number of responses and reinforcements respectively, and the subscripts represent the components of the multiple schedule. K is a constant of proportionality, the value of which (1820.5) was selected so that the curve passed through the same point as that representing performances under the baseline schedule.

These results indicate that, although the total number of responses emitted under a multiple schedule is rather insensitive to the total number of reinforcements delivered, there is a systematic relation between them. Equation (1) appears to describe this relation quite adequately. This equation has the same form and exponent as that found by Catania (1963) to describe the relation between total rates of responding and reinforcement under concurrent VI VI schedules.

Relative Number of Responses

The relative number of responses emitted under Component 1 (red) of each schedule is plotted in Fig. 2 as a function of the relative number of reinforcements delivered under that component. Each point represents the average of the last three sessions under a schedule. In the case of the baseline schedule, training was continued until the difference in relative numbers of responses between components was minimal. For this reason, points for this schedule (0.5, 0.5) have not been plotted. The diagonal line in Fig. 2 represents matching of the relative number of responses and reinforcements. Clearly, matching did not occur.

In general, responses tend to be more evenly distributed between components than matching would require. Nevertheless, relative number of reinforcements does seem systematically to affect relative number of responses. The relation between these relative numbers can be described reasonably adequately by the broken curve in Fig. 2, which is generated by the equation

$$\frac{N_1}{N_1 + N_2} = \frac{n_1^a}{n_1^a + n_2^a} \qquad (2)$$

in which *a* is a constant. For the present data, *a* is approximately one third. In Fig. 3, data from Reynolds' (1963) experiment with multiple VI VI schedules are reproduced to show that they are also reasonably well fitted by equation (2). This equation is also of the same form as that found to hold between relative rates under concurrent schedules, though in the latter case the exponent *a* approximates unity, indicating matching.

Number of Responses Under Components

The average number of responses emitted in the last three sessions under the components of each schedule are plotted as a function of the number of reinforcements delivered under Component 2 (green) in Fig. 4. Again, the systematic effects of reinforcement frequency tend to be obscured by variations in overall output of responses. However, systematic effects for a given baseline level are suggested by the lines drawn between points representing performance under a component and under the preceding baseline schedule. For this reason, these data were also corrected to provide comparisons in terms of a given baseline value. This was done by multiplying the number of

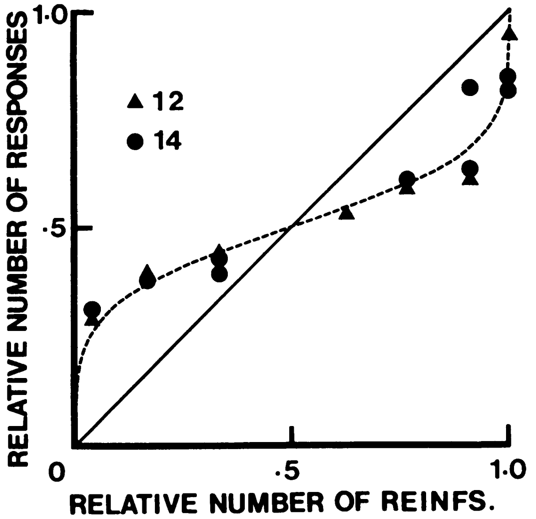

Fig. 2. Relative number of responses as a function of relative number of reinforcements. The diagonal line and broken curve are plots of equation (2) for two values of the exponent *a;* unity and one third respectively.

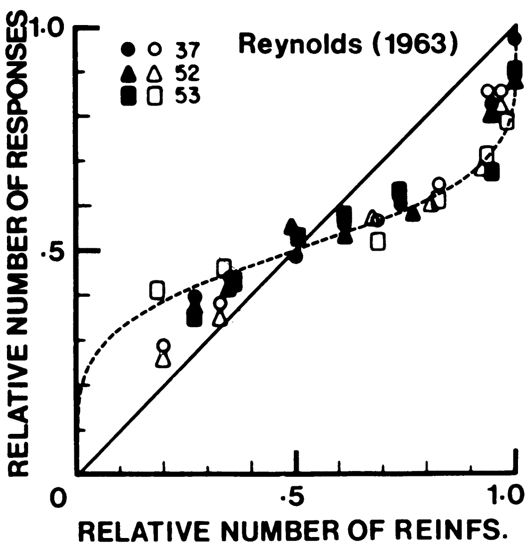

Fig. 3. Points are reproduced from Reynolds' (1963) relative response data for comparison with the functions generated by equation (2).

responses under each component by the factor required to raise the average number of responses under a component of the preceding baseline schedule to a value of 1500, which approximated the average of all baseline performances.

These transformed data are plotted in Fig. 5 for each schedule preceded by training under the baseline schedule. These results indicate systematic effects of the number of reinforcements delivered in Component 2 on the number of responses emitted under each component. These effects may be compared with the solid curves in Fig. 5, generated by the equation obtained by solving for N in equation (1) and (2). The solution for N_1 is:

$$N_1 = \left(\frac{n_1^a}{n_1^a + n_2^a}\right) K\,(n_1 + n_2)^{1/6} \qquad (3)$$

Although this expression is rather ungainly, the correspondence of the experimental points to the curves generated by it is close enough to suggest that it describes the data adequately.

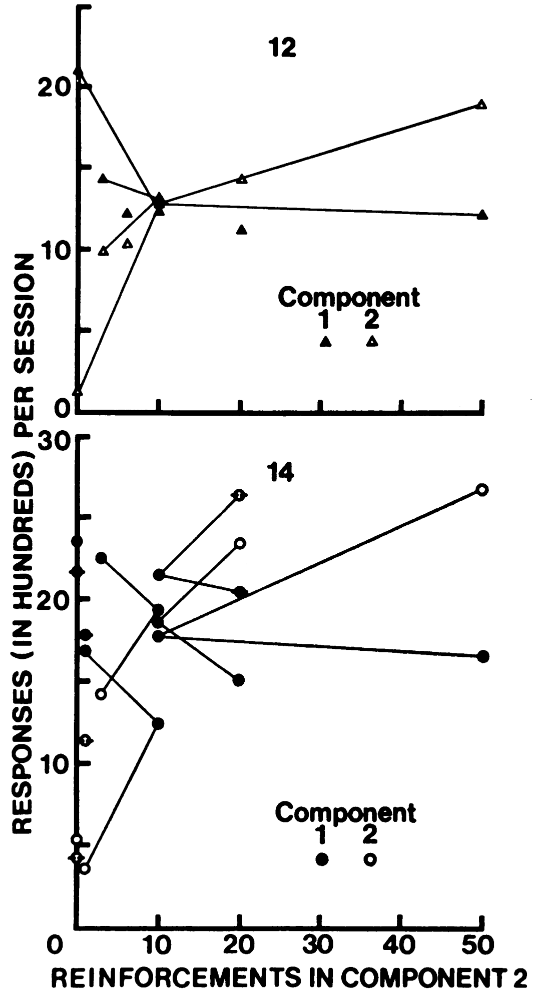

Fig. 4. Number of responses per session under a component as a function of number of reinforcements in Component 2. The number of reinforcements per session in Component 1 was constant. Points are filled for Component 1 and open for Component 2. A horizontal line through points for Pigeon 14 indicates a second determination of performance under a schedule. The lines represent the effect of number of reinforcements for a given baseline performance (points at 10 reinforcements on the abscissa).

In Fig. 5, a dashed line and a dotted curve have been plotted to represent the number of responses that would be predicted under Components 1 and 2 respectively if no interactions occurred; *i.e.*, if responding under a component depended solely on the number of reinforcements delivered under that component. These predictions are based on the assumption that the number of responses emitted under an independent schedule would simply be proportional to the one-sixth power of the number of reinforcements delivered under that schedule; *i.e.*, $N = Kn^{1/6}$ (*cf.* Catania, 1963). Insofar as this assumption is correct, the effects of interaction are represented under each component by deviations of the empirical points from the broken lines in Fig. 5. These data indicated that such effects occurred under both components and that they were all in the direction predicted by equation (3), which generated the solid curves. It can also be seen from the broken lines in Fig. 5 that equation (3) implies a difference in the size of the effects of interaction under the different components of a given schedule. More specifically, the effect of interaction is greater in that component in which the greater number of reinforcements is scheduled. The empirical points appear to be consistent with this rule.

DISCUSSION

The present results confirm Reynolds' (1963) finding that pigeons under multiple VI VI schedules do not match relative number of responses in a component to relative number of reinforcements in that component. At the same time, they suggest that number of reinforcements systematically affects number of responses emitted under these schedules which may be described adequately by the set of three equations presented. From inspection

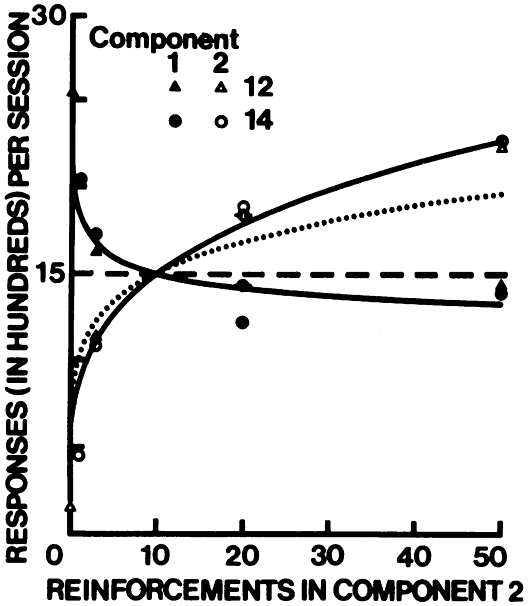

Fig. 5. Number of responses per session under each component corrected for preceding baseline level as a function of number of reinforcements in Component 2. Points are filled for Component 1 and open for Component 2. The solid curves are plots of equation (3) for Components 1 and 2 respectively. The dashed line and dotted curve represent the equation $N = Kn^{1/6}$ for Components 1 and 2 respectively. The intersection of the curves at 10 reinforcements on the abscissa represents the level of baseline responding preceding each schedule.

it appears that equation (3) would not fit Reynolds' (1963) component response rate data as well, perhaps because irregularities were produced in Reynolds' data by shifts in the overall level of responding during the experiment. In the present experiment, in which this form of variability was corrected for in terms of baseline performance, the data appear more orderly.

The results illustrated in Fig. 5 indicate the effects of interactions between the components. Since these effects are in opposite directions under the two components, they are termed behavioral contrast. Further, the results suggest that the amount of contrast is greater under that component in which the greater number of reinforcements is scheduled. Contrast under this component is termed positive, because it involves an increase in the absolute number of responses emitted; contrast under the other component is termed negative, because it involves a decrease in the absolute number of responses emitted. Another way of stating this result is to say that the positive contrast effect under a given schedule is always greater than the negative contrast effect under that schedule. This appears consistent with the results of Reynolds' (1961) investigation of behavioral contrast under multiple VI VI schedules. These showed considerably larger positive than negative contrast effects relative to the effects of control schedules. Thus, there is evidence that the positive effects of interactions are greater than the negative effects under multiple schedules.

The results show that the total number of responses emitted under a multiple VI VI schedule increases with the sixth root of the absolute number of reinforcements (Fig. 1(B) and equation 1). However, because the exponent is so small, the expression $(n_1 + n_2)^{1/6}$ does not differ very markedly from constancy over a large part of the range of values employed. Other investigators (*e.g.*, Smith and Hoy, 1954; Herrnstein, 1964) have opted for the simplification that the total level of responding remains independent of the total number of reinforcements delivered under a schedule. In practice, with any appreciable number of reinforcements, it would prove difficult to discriminate between the sixth-root function and independence. However, the simplification of a constant total response output must fail in the limiting case of no reinforcements, and therefore the sixth-root function is preferable on logical grounds. In addition, the data of Fig. 1, and those illustrated in Catania's (1963) Fig. 1, all have slopes discriminable from zero. It may be concluded that while the effect of adding reinforcements to a multiple VI VI schedule is one of rapidly diminishing returns, it is not correct to say that it has no effect at all. In this respect both multiple and concurrent schedules follow the same function, which provides a welcome invariance between these schedules.

The results obtained indicate that performance under multiple and concurrent VI VI schedules is comparable in other respects also. These schedules can be conveniently compared in terms of the relation between the ratios of component responses and reinforcements. Thus:

$$\frac{N_1}{N_2} = \left(\frac{n_1}{n_2}\right)^a \qquad (4)$$

This is an alternative form of equation (2) above. It applies to the distribution of responses and reinforcements between components both in multiple and concurrent schedules. Again, the exponent *a* would approximate one third in the case of multiple schedules and unity in the case of concurrent schedules. Since the total response output follows the same function under multiple schedules as it does under concurrent schedules, the quantitative differences between the effects of these schedules can be specified in terms of this single parameter, *a*. The smaller value of *a* required in the case of multiple schedules indicates a more even distribution of responses between components than in the case of concurrent schedules under which matching of response and reinforcement distributions occurs.

The exponent *a* in equation (4) may be regarded as indicative of the sensitivity of the distribution of responses between components to the distribution of reinforcements between those components. The larger the value of *a*, the larger the change in N_1/N_2 for a given change in n_1/n_2. Some significant values of *a* are illustrated in Fig. 6. When $a = 0$, the distribution of responses is constant over components and thus independent of the distribution of reinforcements. Thus $a = 0$ would hold when *no discrimination* occurred. When $a = 1$, the distribution of responses changes equally with the distribution of reinforcements. Thus $a = 1$ represents *matching*. When $a = \infty$, all the responses occur under whichever component has the greater proportion of reinforcements. In certain instances $a = \infty$ can be interpreted as *maximizing*.

An interesting case is that in which the effects of components are independent of one another. We have assumed that here the sixth-root function ($N = Kn^{1/6}$) would hold for each component. Given this assumption, it can be seen that a value of $a = 1/6$ would represent *no interaction* between components. This relation allows a further analysis of interaction effects under schedules where the total output obeys the same function (equation 1). In the case of these schedules, a value of *a* greater than one sixth would represent behavioral *contrast,* in that the response outputs in each component would be less like each other than would be predicted from the effects of each component in isolation. On the other hand, a value of *a* less than one sixth would represent behavioral *induction,* in that the component response outputs would be more like each other than would be predicted.

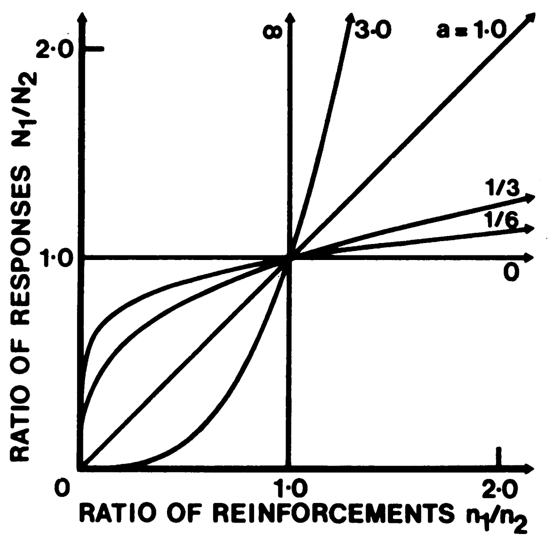

Fig. 6. Ratio of component responses as a function of ratio of component reinforcements. The lines represent plots of equation (4) for different values of the exponent *a*. The numbers denote the value of *a* for the respective curves.

Thus, by allowing *a* to take on various values, equation (4) may provide an economical summary of the characteristic effects of a number of schedules. The data from the present experiment, together with those of Reynolds (1963), suggest that $a = 1/3$ is characteristic of multiple VI VI schedules. It has also been pointed out that $a = 1$ is characteristic of concurrent VI VI schedules. A schedule that seems to be characterized by $a = \infty$ is a concurrent schedule comprised of two fixed-ratio components. If a certain number of responses is required per reinforcement on each key, Herrnstein (1958) has shown that all responses tend to be made on whichever key has the greater probability of reinforcement. Even when the probability of reinforcement is the same under each component, all responses tend to be made on one or other key.

It should be possible to characterize the effects of other complex schedules in similar fashion. For instance, some data from the experiment of Chung and Herrnstein (1967) could be accounted for by a value of *a* greater than one. In their experiment, two variable-interval schedules were presented concurrently and the immediacy of reinforcement was manipulated This manipulation also affected the number of reinforcements delivered under each component schedule. Since a value of $a = 3$ would provide a close approximation to their data, it appears that the joint manipulation of these two variables may have markedly increased the degree to which distribution of reinforcements affected distribution of responses between components.

Thus, the distribution of responses between components of a number of complex schedules can be described by equation (4), with different values of a characterizing each schedule. In other words, one way in which these different schedules can be distinguished is in terms of the sensitivity of the distribution of responses to the distribution of reinforcements. This suggests that it is a change in this sensitivity, rather than a change in the form of reinforcement-response relations, which is produced by different schedules. The origins of this differential sensitivity are presumably to be sought in the different contingencies operating in each schedule.

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