CONCURRENT-SCHEDULE PERFORMANCE: EFFECTS OF RELATIVE AND OVERALL REINFORCER RATE

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Six pigeons were trained to respond on two keys, each of which provided reinforcers on an arithmetic variable-interval schedule. These concurrent schedules ran nonindependently with a 2-s changeover delay. Six sets of conditions were conducted. Within each set of conditions the ratio of reinforcers available on the two alternatives was varied, but the arranged overall reinforcer rate remained constant. Each set of conditions used a different overall reinforcer rate, ranging from 0.22 reinforcers per minute to 10 reinforcers per minute. The generalized matching law fit the data from each set of conditions, but sensitivity to reinforcer frequency (a) decreased as the overall reinforcer rate decreased for both time allocation and response allocation based analyses of the data. Overall response rates did not vary with changes in relative reinforcer rate, but decreased with decreases in overall reinforcer rate. However, as explanations based on changeover rate seem unable to deal with the changes in generalized matching sensitivity, discrimination accounts of choice may offer a more promising interpretation.

Key words: concurrent schedule, generalized matching law, sensitivity, reinforcer rate, response rate, changeover rate, key peck, pigeon

In a concurrent schedule of reinforcement, two or more simple schedules are made simultaneously available to a subject, and the major research interest is in the way in which behavior is allocated between these alternatives. Behavior on two-alternative concurrent schedules has been repeatedly shown to conform to the following equation:

$$\frac{B_1}{B_2} = c \left(\frac{R_1}{R_2}\right)^a.$$
 (1)

In Equation 1, known as the generalized matching law (e.g., Baum, 1974), B represents the amount of behavior at each alternative (in terms of either number of responses or time spent responding), R represents the number of reinforcers obtained from each alternative, and the subscripts 1 and 2 denote the two response alternatives. The multiplier c represents bias, or a preference for one or other response alternative, which is constant across

variations in the obtained reinforcer ratio. The power a is called sensitivity to reinforcement and describes the extent to which changes in the distribution of reinforcers produce changes in the allocation of behavior between the alternatives.

Many studies have investigated performance on concurrent variable-interval (VI) variable-interval schedules (see e.g., Wearden & Burgess, 1982), and much attention has been directed to whether such performance is better characterized by a sensitivity parameter of 1.0 or by a value of less than 1.0. When a = 1.0(and c = 1.0), Equation 1 reduces to the strict matching law proposed by Herrnstein (1961, 1970). This law states that the proportion of responses emitted, or time spent, at one alternative equals the proportion of reinforcers obtained from that alternative. When a is less than 1.0, the allocation of behavior is said to undermatch the distribution of reinforcers. Although some controversy still exists, several extensive reviews of the concurrent-schedule literature (Baum, 1979; Myers & Myers, 1977; Taylor & Davison, 1983; Wearden & Burgess, 1982) suggest that undermatching is the more typical result, especially when VI schedules derived from an arithmetic progression are used, and response rather than time allocation is the dependent variable. Under these conditions, the average value of the sensitivity pa-

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rameter approximates 0.80 (Lobb & Davison, 1975; Taylor & Davison, 1983), and about 80% of the values reported by Taylor and Davison lie between 0.60 and 1.0.

The generalized matching law has been applied successfully to behavior in concurrentchoice situations involving a wide variety of species, responses, reinforcers, and schedules (for an extensive review, see Davison & McCarthy, in press). One potential area of generality that has received little attention, however, is the absolute frequency with which reinforcers are delivered by the schedule. Most studies confirming the generalized matching relation (summarized in the reviews listed above) have used concurrent schedules that give overall reinforcer rates between one and two reinforcers per minute. Because the generalized matching law states that choice is controlled by the distribution of reinforcers rather than by their absolute frequency, manipulation of overall reinforcer rate constitutes an important empirical test of the generality of the law.

One experiment that addresses this question was reported by Fantino, Squires, Delbrück, and Peterson (1972). They conducted three conditions using independent concurrent VI schedules in which reinforcer ratio was held constant at 2:1 while the overall reinforcer rate permitted by the schedule was either 0.15, 1.5, or 15 reinforcers per minute. They found that the proportion of responses to the preferred VI schedule increased with increasing overall reinforcer rate, reaching almost exclusive preference in the condition arranging 15 reinforcers per minute. Because they used an independent scheduling procedure, there was a concomitant increase in the proportion of reinforcers obtained from the preferred alternative. Analyzing their data in terms of the unsigned deviation of the choice proportion from that predicted by strict matching to the obtained relative reinforcer rate, Fantino et al. reported that relative responding approximately matched relative reinforcement in all conditions, and that this relation was unaffected by overall reinforcer rate.

Reanalysis of the data of Fantino et al. (1972) suggests a different conclusion, however. Point estimates of the value of a in Equation 1 were calculated by dividing the log ratio of responses on the two keys by the log ratio of obtained reinforcers (derived from the relative rates re-

ported by Fantino et al.). The mean a values obtained were, for the 0.15, 1.5, and 15 reinforcers per minute conditions, respectively, 0.63, 0.80, and 0.99. This procedure assumes that there is no bias (c in Equation 1 = 1.0) if the absolute values of the estimated sensitivities are to be taken as veridical. However, if the principal emphasis is to be on the ordering across conditions of the estimated sensitivities, as here, the analysis requires only that bias be constant across conditions. Estimates of sensitivity to reinforcement for the performance of individual subjects showed an increasing trend with increasing overall reinforcer rate on a nonparametric trend test (Ferguson, 1965; two-tailed p < .05).

Fantino et al. (1972) also conducted a replication of the 15 reinforcers per minute condition with 4 of their original 7 subjects using a forced-choice (dependent scheduling) procedure. The results of the reanalysis of this condition were less clear. For 2 of the 4 birds, sensitivity to reinforcement was less than in the equivalent independently scheduled condition, but still much greater than in the 1.5 reinforcers per minute condition. One bird showed matching in both 15 reinforcers per minute conditions, irrespective of schedule dependency, whereas the fourth bird showed a much lower sensitivity in the dependent condition than in any other condition.

The present experiment was designed as a parametric investigation of the effect suggested by the data of Fantino et al. (1972). Dependent scheduling was used throughout, and the VI schedules used were derived from an arithmetic progression. As a secondary aim, we sought to confirm, within subjects and across a wide range of reinforcer rates, Taylor and Davison's (1983) and Wearden and Burgess' (1982) finding that time allocation tends to be more sensitive than response allocation when arithmetic VI schedules are used.

METHOD

Subjects

Six adult ex-racing pigeons, numbered 131 to 136, were maintained at 85% $(\pm 15 \text{ g})$ of their free-feeding body weights. Water and grit were freely available in their home cages, and postsession mixed grain was delivered, if necessary, to maintain their set weights. Five subjects had no prior operant training, whereas Bird 132 had been trained in automaintenance and negative automaintenance procedures.

Apparatus

The standard pigeon operant chamber (width 33 cm, depth 31 cm, height 32 cm) contained three response keys 2 cm in diameter, 9.5 cm apart, and 25 cm above the grid floor. The center key was dark and inoperable throughout. The two side keys could be transilluminated red. An effective response on either key required a force of approximately 0.1 Nand produced a click. Responses to darkened keys were ineffective and not recorded. A reinforcer consisted of 3-s access to a grain hopper situated 10 cm below the center key. During reinforcement, the hopper was illuminated and the side keys were darkened. No other sources of illumination were provided. A ventilation fan provided masking noise. Solid-state control equipment situated remote from the chamber arranged experimental events and recorded the data.

Procedure

The 5 naive subjects were autoshaped using a variety of key colors. All subjects then received 30 days of single-schedule training on a variety of VI schedules, keys, and key colors. Concurrent schedules were arranged on the two side keys, and over seven sessions, equal VI schedules and a changeover delay (COD) were increased in duration until the subjects were responding on a concurrent VI 60-s VI 60-s schedule with a 2-s COD. The COD prevented responses on a given alternative from producing a reinforcer until 2 s had elapsed since the last response on the other alternative. Condition 1 then began (Table 1).

In each condition a VI schedule was available on each side key. These concurrent schedules were arranged nonindependently; that is, when one timer arranged a reinforcer and stopped, the other timer was also halted until the reinforcer was delivered. The VI schedules consisted of intervals randomized from the first 12 terms of an arithmetic progression: a, a + $d, a + 2d, \ldots$, where a is 1/12th the mean interval. The 2-s COD was present in each condition.

The sequence of experimental conditions is shown in Table 1. Six sets of conditions were conducted. Within each set, the arranged overall reinforcer rate remained constant and the

The VI schedules arranged on the left and right keys, the
arranged overall reinforcer rate, the number of training
sessions, and the number of reinforcers per session for
each condition, shown in order of presentation.

Table 1

Con- dition	VI (left)	VI (right)	Ar- ranged rein- forcers/ min	Ses- sions	Rein- forc- ers/ses- sion
1	60	60	2.00	21	40
2	34	270	2.00	20	40
3	150	38	2.00	19	40
4	38	150	2.00	19	40
5	270	34	2.00	21	40
6	270	270	0.44	22	20
7	153	1,215	0.44	36	20
8	675	171	0.44	35	20
9	171	675	0.44	25	20
10	1,215	153	0.44	21	20
11	8	30	10.00	25	40
12	54	7	10.00	31	40
13	12	12	10.00	15	40
14	7	54	10.00	24	40
15	30	8	10.00	22	40
16	120	120	1.00	53	40
17	76	300	1.00	25	40
18	540	68	1.00	25	40
19	68	540	1.00	26	40
20	300	76	1.00	19	40
21	153	1,215	0.44	34	20
22	24	24	5.00	25	40
23	15	60	5.00	31	40
24	108	14	5.00	24	40
25	14	108	5.00	38	40
26	60	15	5.00	31	40
27	380	1,500	0.22	23	10
28	2,700	340	0.22	20	10
29	340	2,700	0.22	25	10
30	1,500	380	0.22	21	10
31	600	600	0.22	30	10

arranged ratio of reinforcers available on the two alternatives (left : right) was varied across the conditions (8:1, 4:1, 1:1, 1:4, 1:8). The arranged overall reinforcer rate was varied across the six sets of conditions, and ranged from 0.22 reinforcers per min to 10 reinforcers per min (Table 1). Condition 21 was a replication of Condition 7.

The daily experimental sessions began in blackout and ended in blackout after either 45 min had elapsed or a predetermined number of reinforcers had been obtained (Table 1). At the end of the session the following data were recorded: the number of responses made on each key, the number of reinforcers obtained from each alternative, the number of switches between keys, and the time spent responding on each key, measured from the first response on a particular key until the next response on the other key and excluding time occupied by reinforcer delivery.

Each experimental condition continued until all subjects reached a defined stability criterion five times, not necessarily consecutively. This criterion was that the median relative response rate on the left key over the last set of five sessions did not differ from the median over the previous set of five sessions by more than .05.

RESULTS

The data used in the analyses were the mean number of responses emitted on each key, the mean time spent on each key, the mean number of reinforcers obtained on each key, and the mean total number of changeovers over the last five sessions of each condition for each subject. All data are shown in the Appendix.

Typically, subjects obtained an overall reinforcer rate lower than the maximum permitted by the concurrent schedule. This discrepancy was most pronounced when arranged reinforcer rates were high and decreased with decreasing arranged reinforcer rate. For the sets of conditions that arranged rates of 10, 5, 2, 1, 0.44, and 0.22 reinforcers per minute, the mean obtained overall reinforcer rates were, respectively, 6.29, 3.30, 1.89, 0.96, 0.44, and 0.20 reinforcers per minute. For every subject, the overall reinforcer rates obtained in each set of conditions were ordered in the same way as the arranged rates.

The logarithms (base 10) of the ratios of responses, time, and reinforcers on the left and right keys were calculated for each subject in each condition. Log response ratios (log B_1/B_r) were plotted against the log reinforcer ratios (log R_1/R_r) for each subject in each set of conditions with the same arranged overall reinforcer rate, and the best fitting line calculated using the method of least-squares linear regression. Identical analyses were conducted using the time-allocation data (log T_1/T_r), rather than the response-allocation data, as the dependent variable. The results of these 72 linear regressions are shown in Table 2.

The generalized matching law (Equation 1) described the data equally well for all overall reinforcer rates. In 62 of the 72 linear regressions, the proportion of the data variance ac-

counted for by the fitted line (VAC) was at least .90. The standard deviations of the slope (a) and intercept (log c) were typically not greater than 0.10 (124 of 144), never greater than 0.20, and did not change systematically with changes in overall reinforcer rate. These latter measures of goodness of fit are emphasized because VAC may increase as the slope of the fitted line (and therefore total data variance) increases. Plots of the data used in the generalized matching analyses revealed no systematic deviations from the fitted lines. Figure 1 shows plots of both log response ratios and log time ratios, as a function of log reinforcer ratios, for 2 typical subjects in each set of conditions.

Sensitivity to changes in the obtained reinforcer ratio (a in Equation 1) was within the range of values typically obtained in experiments using concurrent arithmetic VI VI schedules (Taylor & Davison, 1983). Individual subjects' time-allocation a values were higher than their corresponding response-allocation a values in 32 of the 36 possible comparisons (Table 2, Figure 2). The size of this difference did not change systematically with changes in the overall reinforcer rate.

However, both time-allocation and response-allocation a values decreased as the overall reinforcer rate was decreased; that is, the behavior ratio became less sensitive to changes in the obtained reinforcer ratio at lower overall reinforcer rates (Figure 2). Nonparametric trend tests (Ferguson, 1965) across subjects on response-allocation and time-allocation measures of sensitivity as a function of arranged reinforcer rate were significant (response-allocation a values, SS = 40, z = 2.99, p < .01; time-allocation a values, SS = 36, z =2.68, p < .01). This change is also evident in the mean a values calculated across subjects for each set of conditions (Table 1). On the other hand, as Figure 2 shows, decreasing reinforcer rate appeared to have little effect on the a values of some subjects (e.g., Birds 131 and 133).

Response-allocation estimates of bias (log c in Equation 1) showed no consistent preference between the alternatives across subjects. For example, Birds 133 and 136 typically showed a bias toward the right key, whereas Birds 132 and 134 typically showed a left-key bias (Table 2). Bias did not change significantly as the overall reinforcer rate was de-

Table 2

The sensitivity to changes in reinforcer frequency (a in Equation 1) and the bias (log c) estimated from least-squares linear regression for each subject in each set of conditions for both responseallocation and time-allocation data. The standard deviations of the estimates are shown in parentheses.

Set R/min	C 10.00	E 5.00	A 2.00	D 1.00	В 0.44	F 0.22
			og response ratio			
ו יח		L	og response ratio	a values		
Bird					a aa (a)	
131	0.67 (0.08)	0.86 (0.10)	0.87 (0.12)	0.87 (0.10)	0.82 (0.11)	0.71 (0.13
132	0.82 (0.05)	0.77 (0.02)	0.51 (0.08)	0.51 (0.06)	0.53 (0.06)	0.47 (0.10
133	0.64 (0.09)	0.45 (0.08)	0.75 (0.08)	0.62 (0.01)	0.64 (0.05)	0.60 (0.16
134	0.67 (0.13)	0.66 (0.08)	0.45 (0.08)	0.60 (0.11)	0.51 (0.09)	0.53 (0.11
135	0.67 (0.06)	0.54 (0.04)	0.58 (0.07)	0.53 (0.05)	0.56 (0.03)	0.40 (0.07
136	0.83 (0.09)	0.65 (0.16)	0.47 (0.10)	0.43 (0.06)	0.42 (0.14)	0.43 (0.06
Mean	0.72	0.66	0.61	0.59	0.58	0.52
		Log	; response ratio lo	g c values		
131	-0.25 (0.05)	0.08 (0.07)	0.08 (0.08)	-0.11 (0.07)	0.03 (0.09)	0.02 (0.08
132	0.15 (0.03)	0.22 (0.01)	-0.01 (0.05)	0.09 (0.04)	0.04 (0.04)	0.10 (0.07
133	-0.32 (0.06)	-0.12 (0.06)	-0.28 (0.06)	-0.02 (0.00)	-0.06 (0.03)	-0.25 (0.11
134	0.06 (0.09)	0.15 (0.05)	0.06 (0.05)	0.09 (0.08)	-0.01 (0.06)	0.04 (0.08
135	-0.10 (0.04)	-0.05 (0.03)	0.12 (0.05)	-0.04 (0.03)	-0.09 (0.02)	-0.02 (0.05
136	-0.34 (0.06)	-0.31 (0.10)	0.06 (0.07)	-0.38 (0.04)	-0.05 (0.09)	-0.29 (0.06
Mean	-0.13	-0.01	0.01	-0.06	-0.02	-0.07
			Log time ratio a	values		
131	0.87 (0.06)	0.88 (0.10)	1.09 (0.17)	0.97 (0.17)	0.79 (0.15)	0.76 (0.03
132	0.86 (0.03)	0.77 (0.01)	0.56 (0.11)	0.60 (0.05)	0.68 (0.07)	0.50 (0.08
133	0.69 (0.06)	0.67 (0.10)	0.83 (0.09)	0.58 (0.03)	0.64 (0.07)	0.66 (0.13
134	0.80 (0.11)	0.67 (0.05)	0.65 (0.05)	0.55 (0.06)	0.60 (0.07)	0.64 (0.09
135	0.74 (0.06)	0.67 (0.02)	0.66 (0.08)	0.70 (0.05)	0.66 (0.15)	0.59 (0.10
136	0.89 (0.09)	0.72 (0.08)	0.66 (0.09)	0.47 (0.03)	0.73 (0.08)	0.60 (0.05
Mean	0.81	0.73	0.74	0.65	0.68	0.63
		L	og time ratio log	c values		
131	0.01 (0.04)	0.17 (0.07)	0.08 (0.11)	-0.22 (0.12)	-0.30 (0.12)	-0.30 (0.02
132	0.11 (0.02)	0.20 (0.00)	0.03 (0.07)	-0.06 (0.04)	-0.12 (0.04)	-0.08 (0.06
133	-0.15(0.04)	0.07 (0.07)	-0.11(0.07)	0.01 (0.02)	-0.03 (0.05)	-0.39 (0.09
134	-0.10(0.07)	-0.01(0.03)	-0.03(0.04)	-0.03 (0.04)	-0.10 (0.05)	-0.14 (0.07
135	-0.11(0.04)	-0.02(0.01)	0.15 (0.05)	-0.03(0.04)	-0.30(0.10)	-0.12 (0.06
136	-0.11(0.06)	-0.05(0.05)	-0.01(0.07)	-0.22(0.02)	-0.06 (0.06)	-0.14 (0.05
Mean	-0.06	0.06	0.02	-0.09	-0.15	-0.20

creased. However, for the time-allocation data, subjects became more biased toward the right key as the overall reinforcer rate decreased. This change was significant on a nonparametric trend test (SS = 40, z = 2.99, p < .01).

The mean overall response rate and the mean response rates on each key were calculated across subjects for each condition. The logarithms (base 10) of these response rates have been plotted as a function of obtained relative reinforcer rate (left/left + right) for each set of conditions in Figure 3. The results from Condition 7 and its replication, Condition 21, were very similar (0.44 reinforcers per minute panel). Within each set of conditions, overall response rate did not change systematically with changes in obtained relative reinforcer rate. Across the three sets of conditions with higher obtained reinforcer rates (6.29, 3.30, and 1.89 reinforcers per minute) there was little change in overall response rate. However, with further decreases in overall reinforcer rate, the overall response rate decreased. Because the overall response rate changed with changes in overall reinforcer rate, individual subjects' *a* values for each set of conditions were plotted against their respective individual subjects' mean overall response rate for that set of conditions. However, this treatment of the results seemed less orderly than that shown in Figure 2 and is not presented.

The plots of the mean response rates on each

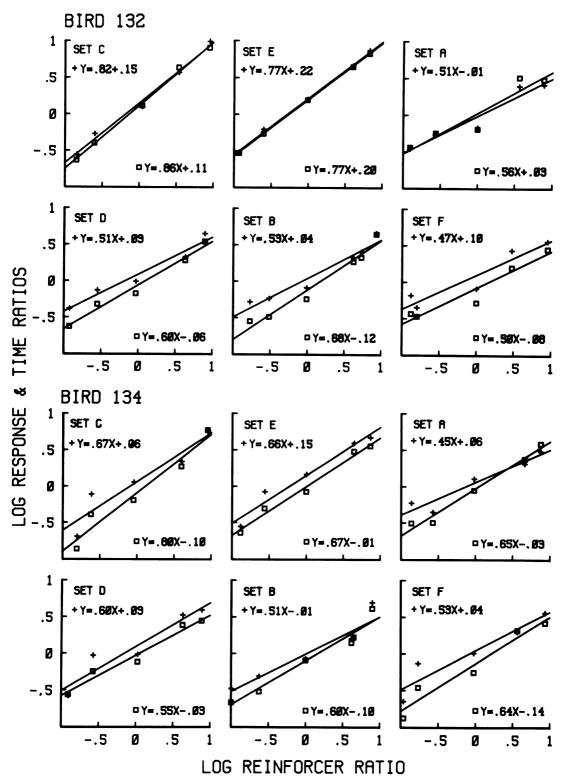


Fig. 1. The logarithms (base 10) of the ratio of responses (plus signs) and of time (squares) on the left and right keys as functions of the logarithm (base 10) of the ratio of reinforcers obtained from the left and right keys, for 2 typical subjects. Each panel shows data from a set of conditions arranging the same overall reinforcer rate (labeled, in order of decreasing overall reinforcer rate, Sets C, E, A, D, B, and F; see Table 2), and the best-fitting straight lines for response and time allocation, together with their equations.

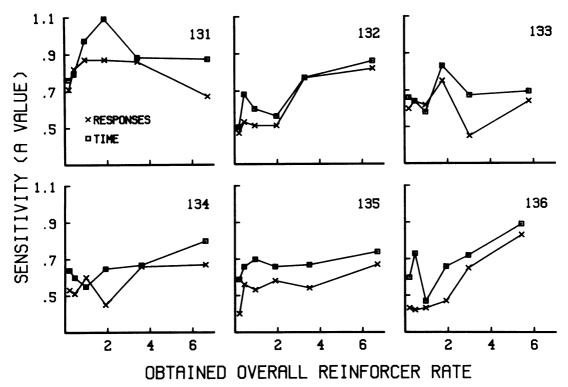


Fig. 2. Best-fitting sensitivity values for response (crosses) and for time (squares) allocation as functions of the obtained overall reinforcer rate (reinforcers per minute), for each subject.

key reflect the major finding of the generalized matching analysis. For example, when the results from the 0.20 reinforcers per minute set of conditions are compared with those from the 6.29 reinforcers per minute set of conditions, not only did the overall response rate decrease but the response rate on the alternative from which the majority of reinforcers were obtained (e.g., right key when the leftkey relative reinforcer rate = .11) decreased proportionally more than the response rate on the other alternative (e.g., left key when the left-key relative reinforcer rate = .11). This trend toward convergence of the response rates at extreme reinforcer ratios leads to the decrease in the a values obtained from the generalized matching analysis.

The mean overall changeover rate was calculated across subjects for each condition. This has been plotted as a function of obtained relative reinforcer rate for each set of conditions in Figure 4. The results from Condition 7 and its replication, Condition 21, were similar. Within each set of conditions, as the relative reinforcer rates became more extreme, the changeover rate usually decreased, giving an inverted U-shaped function. Again, there was little difference in the results from the three sets of conditions with the higher overall reinforcer rates. However, further decreases in overall reinforcer rate affected this function in two ways: Changeover rate decreased, and the

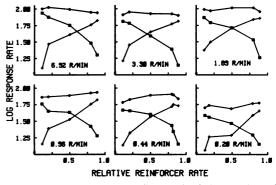


Fig. 3. The logarithms (base 10) of the number of responses per minute on the left (plus signs), on the right (squares), and on both (triangles) keys as functions of the obtained relative reinforcer rate on the left key. Data are means across all subjects, and the mean overall reinforcer rate (reinforcers per minute) for each set of conditions is shown in each panel.

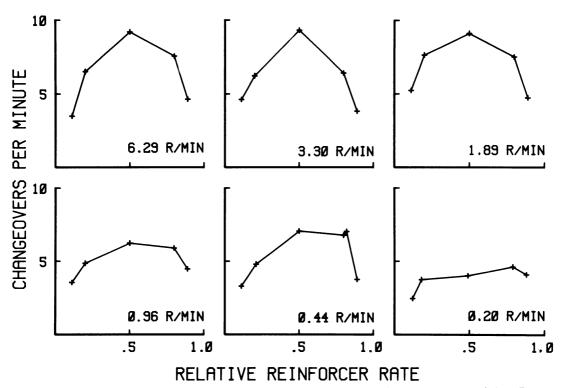
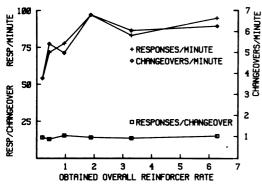


Fig. 4. Changeovers per minute as a function of the obtained relative reinforcer rate on the left key. Data are means across all subjects, and the mean overall reinforcer rate (reinforcers per minute) for each set of conditions is shown in each panel.

inverted U-shaped functions were less peaked (even when the general decrease in changeover rate is taken into account).

As Figures 3 and 4 show, both overall response rate and overall changeover rate decreased with decreases in overall reinforcer rate.



rate and the mean overall response rate across subjects and conditions for each set of conditions plotted as a function of the overall obtained reinforcer rate. When the axis for the changeover-rate data is scaled linearly to extend over the same range as the axis for the response-rate data, the two functions are very similar. Hence, plotting the mean overall number of responses per changeover in each set of conditions as a function of mean overall obtained reinforcer rate (Figure 5) gives a flat line; that is, mean responses per changeover in each set of conditions remained constant (mean across subjects and conditions = 14.2 responses per changeover).

Figure 5 shows the mean overall changeover

DISCUSSION

Fig. 5. Overall responses per minute (plus signs, left axis), responses per changeover (squares, left axis), and changeovers per minute (diamonds, right axis) as functions of obtained overall reinforcer rate (reinforcers per minute). Data are means across subjects and conditions arranging the same overall reinforcer rate.

The data reported here provide additional support for the generalized matching law as a useful description of concurrent-schedule performance. The logarithmic transformation of Equation 1 described the allocation of both responses and time well in all sets of conditions. Undermatching characterized choice at all overall reinforcer rates, and was more marked for response allocation than for time allocation (cf. Taylor & Davison, 1983; Wearden & Burgess, 1982). The mean sensitivity values for time (0.71) and response (0.61) allocation fell close to the range of typical values reported by Wearden and Burgess (1982) and by Taylor and Davison (1983) for concurrent arithmetic VI VI performance. These general findings may be taken as further evidence against the view that strict matching (Herrnstein, 1961, 1970) constitutes the best description of concurrent-schedule performance and that sensitivity parameters other than 1.0 are best seen as statistical deviations from strict matching (e.g., Baum, 1979).

As the reanalysis of the data of Fantino et al. (1972) suggested, the value of the sensitivity parameter in Equation 1 increased as a function of increasing overall reinforcer rate, for both time-based and response-based analyses. Figure 6 shows mean sensitivity values, as a function of overall arranged reinforcer rate, for the results of Fantino et al. and for the present results in both response and time analyses. (The abscissa shows arranged reinforcer rates because Fantino et al. did not report obtained rates.) Although our data showed lower mean sensitivities than did those of Fantino et al., in both experiments sensitivity was an increasing, concave downward, function of overall reinforcer rate. Again, this systematic deviation from strict matching with changes in overall reinforcer rate argues against the view that choice performance is best characterized by statistical fluctuations from a norm of strict matching (e.g., Baum, 1979, 1983).

Figure 3 illustrates log absolute response rates as a function of relative reinforcer rate within each set of conditions. With increasing overall reinforcer rate, the functions for log response rate on each key progressively diverge (at least when the extremes are compared), reflecting the change in sensitivity shown in the generalized matching analysis. Overall response rates were not systematically affected by relative reinforcer rate, but did increase with increasing overall reinforcer rate. As shown more clearly in Figure 5, this increase was again negatively accelerated, and there was little change in overall response rate across the three highest overall reinforcer rates. This function resembles that for response rate in a

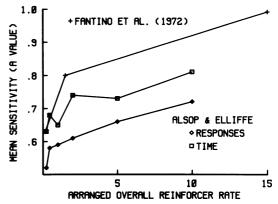


Fig. 6. Sensitivity as a function of the arranged overall reinforcer rate (reinforcers per minute), for the present data in both response (diamonds) and time (squares) analyses, and for the data of Fantino et al. (1972) (plus signs). All values are means across subjects.

single VI schedule (Catania & Reynolds, 1968) and would similarly be well described by Herrnstein's (1970) hyperbola.

Changeover rate was an inverted U-shaped function of relative reinforcer rate within each set of conditions, as was reported by Brownstein and Pliskoff (1968). However, the sharpness of the peak in this function increased with increasing overall reinforcer rate, again most sharply over the low end of the range (Figure 4). Finally, Figure 5 shows that mean changeover rate within a set of conditions also increased with increasing overall reinforcer rate, and that this increase corresponded closely to that for overall response rate. As a consequence, the mean number of responses per changeover remained constant across changes in overall reinforcer rate (Figure 5).

The surprising feature of the present data is the observed dependence of generalized matching sensitivity (a in Equation 1) on the overall reinforcer rate arranged by the concurrent schedule. This effect was apparent in the data from several, but not all, birds (Figure 2). The implications of this finding for models of choice behavior will now be examined. Two major types of explanation for the present results will be considered: those based on the observed changes in the rate of changing over between the two keys, and those based on possible changes in the discriminability of the reinforcer rates obtained from the two alternatives at high and low overall reinforcer rates.

Herrnstein's (1961, 1970) approach to complex schedule performance was to propose that the proportion of responses allocated to an alternative matched the proportion of reinforcers obtained from that alternative, relative to all other reinforcers obtained in the experimental situation. This gives the equation

$$\frac{B_1}{B_2} = \frac{R_1(R_2 + mR_1 + R_e)}{R_2(R_1 + mR_2 + R_e)},$$
 (2)

where R_{e} represents reinforcers obtained from sources other than those arranged by the explicit response alternatives, and m is a parameter describing the interaction between the two programmed sources of reinforcement. For concurrent schedules, m = 1.0 (interaction between the two schedules is maximal) and Equation 2 predicts strict matching at all overall reinforcer rates. If m is allowed to take values less than 1.0, variation in absolute reinforcer rates, relative to the extraneous reinforcer rate R_e , does influence behavior allocation. Herrnstein and Loveland (1974) used this feature of Equation 2 to predict changes in behavior allocation in multiple schedules when level of food deprivation was varied. Applied to the present experiment, however, their reasoning predicts that sensitivity, in a generalized matching analysis, should decrease, not increase, with increases in overall reinforcer rate (see Herrnstein & Loveland, 1974, Figure 1).

The present results are consistent with Herrnstein's system of equations only if the parameter m in Equation 2 is assumed to decrease with decreasing overall reinforcer rate and if this decrease is sufficiently large to counteract the direct effect of absolute reinforcer rate described above. Two explanations of the present results in these terms are possible, but neither is convincing to us. Figure 5 shows that the rate of changing over between keys decreased with decreasing overall reinforcer rate, and this may perhaps be taken as an index of a change in the degree of interaction between the two alternatives. The decrease in changeover rate observed was small, however, and does not seem sufficient to offset the large changes in overall reinforcer rate. Moreover, the identification of changeover rate with amount of interaction is debatable. Shimp and Wheatley (1971) reported that generalized matching sensitivity in a multiple schedule increased to 1.0 with decreasing component duration (increasing rate of changing over between components), an effect that is consistent with increasing interactions between components in Herrnstein's analysis. However, Charman and Davison (1982) found, in a parametric study, that multiple-schedule sensitivity did not consistently depend on component duration.

The present changeover-rate data therefore do not suggest an explanation in terms of Herrnstein's interaction parameter. It may, however, be more in keeping with the spirit of Herrnstein's analysis to suppose that interaction depends on the temporal distance between a response on one alternative and the last reinforcer, rather than last response, on the other alternative. This position predicts that concurrent interaction, and therefore generalized matching sensitivity, will decrease with decreasing overall reinforcer rate. An explanation in these terms, although not driven by the present data, cannot be dismissed by them either. However, we find such an approach unsatisfying, because it requires that m in Equation 2 be a direct function of the reinforcer rates on each alternative, when these reinforcer rates themselves appear in the equation. That is, a parameter of Equation 2 must itself be a function of the variables in the equation. Nevin (1984) argues cogently for parameter invariance as an important goal of the experimental analysis of behavior, and the present case seems to be a particularly serious violation of that goal. The role of a parameter should be to describe the effects on choice of variables other than those represented in the equation, and the parameter should itself be independent of those variables.

Wearden (1983) proposed a model for concurrent-schedule performance in which subjects strictly match response ratios to reinforcer ratios during periods in which their responding is under the control of the schedules but periodically emit bursts of random responding. These bursts of random responding have the effect, in a generalized matching analysis, of decreasing the observed sensitivity to reinforcer rate, thereby producing undermatching. This interpretation is consistent with the present results if it is assumed that bursts of random responding occurred more frequently at lower overall reinforcer rates. Such an assumption seems plausible: If the concurrent schedule delivers a low overall reinforcer rate, control of responding by the schedule may well be attenuated. However, increases in random responding should presumably be accompanied by increases in changing over between the keys. Figure 5 shows that changeovers were in fact less frequent at low overall reinforcer rates and that decreases in changeover rate mirrored decreases in overall response rate, suggesting that the probability of bursts of random responding was independent of overall reinforcer rate.

Baum (1974) suggested that undermatching in concurrent schedules could be due in part to poor discrimination of the alternatives. This suggestion may be relevant to the present results: It may be that subjects discriminated the reinforcer rates produced by the two keys less well when overall reinforcer rate was low, because their behavior came into contact with the contingencies less often. The discriminability interpretation of undermatching has been developed in detail by Davison and Jenkins (1985). Their model proposes that responses strictly match the ratio of reinforcers obtained from the two alternatives, but that some reinforcers obtained from each alternative are misallocated to the other alternative. A contingency-discriminability parameter describes the extent to which subjects can discriminate which alternative produced a reinforcer, and takes values from 1.0 (random allocation of reinforcers to alternatives, generalized matching sensitivity is zero) to infinity (perfect allocation of reinforcers to alternatives, producing overall strict matching). The model does not directly predict how changes in a molar variable, overall reinforcer rate, might produce changes in a relatively molecular behavioral process, discrimination of contingencies. Elaborate explanations in terms of attentional processes are certainly possible, but go beyond the scope of the present paper.

A similar, discriminability-based, argument can be applied within the melioration framework proposed by Herrnstein and Vaughan (e.g., 1980). According to the melioration hypothesis, matching is the equilibrium point of a dynamic process in which time is allocated to the alternatives in such a way as to equalize the local reinforcer rates obtained on each. When the overall reinforcer rate is low, discriminating differences in local reinforcer rates may be more difficult. In a related line of reasoning, feedback functions describing the local reinforcer rate on each alternative as a function of the allocation of behavior between the alternatives will be shallower at lower overall rates of reinforcement. Here, the subject's task can be described as one of discriminating marginal changes in local reinforcer rate. Both these effects imply that a melioration process may stop short of the equilibrium (matching) point, and that this deviation from matching will increase with decreasing overall reinforcer rate.

In summary, the validity or usefulness of the generalized matching law, seen as a purely descriptive tool, is not attacked by the present results. Choice was described equally well by Equation 1 at all overall reinforcer rates (see Table 2), and changes in the sensitivity parameter a clearly described changes in the control of behavior allocation by the distribution of reinforcers. This is consistent with Prelec's (1984) derivation from primitive assumptions of a power-function relationship between response and reinforcer ratios in concurrent schedules. However, seen as a predictive tool, the generalized matching law does not handle the present results well, because changes in sensitivity, although permitted by the model, are not in any sense predicted by it. In fact, when used predictively, the generalized matching law states that behavior allocation is controlled by the relationship between concurrent reinforcer rates and not by their absolute level. Changes in the value of the sensitivity parameter, a, must therefore be ascribed to changes in some other variable which is itself affected by absolute reinforcer rate. Despite the attractiveness of the observed correlation between changeover rate and sensitivity in the present results, changeover rate does not seem to be a likely candidate for this explanatory variable, as shown above. It is more reasonable to conclude that changeover rate simply reflects the overall level of schedule-related behavior, as seen in the correlation between overall response and changeover rates shown in Figure 5. Explanations in terms of changes in the discriminability of reinforcement alternatives, whether within a melioration or a contingency-discriminability framework, seem more promising.

Across changes in overall reinforcer rate, the only measure of concurrent performance to remain constant was the number of responses per changeover. These effects of overall reinforcer rate, a variable which had been implicitly presumed irrelevant, argue for caution in attempting to generalize from experimental results obtained at a particular overall reinforcer rate. Moreover, it is apparent that results from experiments in which one alternative reinforcer rate was held constant, while the other was varied, should be interpreted very conservatively.

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APPENDIX

The mean number of responses, time spent responding, number of reinforcers obtained on the left and right keys, and the total number of changeovers between keys, calculated from the last five sessions in each condition for each subject.

Condi	Resp	onses	Ti	me	Reini	forcers	Change-
	Left	Right	Left	Right	Left	Right	overs
Bird 131							
1	1,296	1,283	664	607	19.4	20.6	170
2	1,962	209	1,232	92	35.8	4.6	36
3	823	1,366	478	778	8.4	31.6	147
4	1,871	555	1,093	221	33.2	7.0	94
5	243	1,542	93	1,202	4.8	35.2	43
6	2,111	1,860	992	1,713	9.8	9.8	255
7	3,226	401	1,990	425	15.4	3.2	108
8	605	1,744	342	2,176	3.4	15.0	99
9	3,161	889	1,725	1,019	15.8	3.2	195
10	219	1,556	172	2,540	2.0	17.0	47
11	400	234	290	66	32.0	8.0	35
12	77	748	38	282	4.0	36.0	23
13	334	503	199	210	19.0	21.0	68
14	331	200	289	64	35.2	4.8	31
15	167	695	85	274	7.6	32.4	42
16	1,165	1,612	775	1,765 1,037	20.4 32.2	18.2 7.4	144
17	2,284 457	1,183	1,545 284	2,187	4.6	7.4 34.4	234 72
18 19	4,294	2,864 576	2,260	2,187	35.4	4.2	92
20	4,294 564	2,297	315	2,245	8.2	4.2 30.4	103
20	3,150	473	1,831	802	17.0	1.0	105
22	703	627	303	325	19.8	20.2	100
22	916	340	534	112	32.6	7.4	54
23	252	1,226	177	645	4.8	35.2	62
25	1,280	110	616	45	36.0	4.0	19
26	522	1,140	296	500	8.4	31.6	84
27	2,615	583	1,611	1,012	6.4	1.4	130
28	718	1,569	468	2,311	2.2	6.6	88
29	2,460	447	2,084	625	7.2	0.6	125
30	482	806	477	2,102	2.0	6.6	79
31	487	774	862	1,834	4.4	4.2	118
Bird 132							
1	978	1,436	471	724	20.0	20.0	201
2	1,661	638	906	295	35.6	4.6	186
3	877	1,602	456	802	8.6	31.6	209
4	1,841	737	960	292	32.0	8.8	140
5	662	1,899	326	895	4.4	35.8	145
6	1,570	1,925	973	1,693	9.4	9.6	257
7	2,457	968	1,785	831	16.2	3.0	312
8	1,131	1,951	645	1,984	4.6	15.0	233
9	2,289	1,088	1,719	917	15.2	3.6	321
10	845	1,630	561	1,981	2.8	16.6	169
11	543	148	341	79	31.0	9.0	36
12	111	401	60	257	5.0	35.0	23
13	341	271	249	191	21.0	19.0	76
14	445	46	294	37	36.0	4.0	16
15	178	332	107	265	8.0	32.0	37
16	1,312	1,342	978	1,460	18.8	20.2	314
17	2,148	1,009	1,731	900	32.0	7.4 35.4	342
18	672	1,622	481	2,033	4.2	35.4	165
19 20	1,972	444	2,005	577	34.8	4.4	183
20	1,087	1,477	803	1,671	8.8 17.2	31.2	233
21	1,951	425 352	2,158 456	484 289	17.2 19.8	2.0 20.2	188 115
22 23	570 737	159	436 586	131	32.0	8.0	59
23	151	137	200	131	52.0	0.0	J7

APPENDIX

(Continued)

Condi	Resp	onses	Ti	me	Rein	forcers	- Change- overs
	Left	Right	Left	Right	Left	Right	
24	190	628	178	601	4.2	35.8	66
25	767	102	558	82	34.8	5.2	35
26	381	611	279	514	8.0	32.0	92
27	1,434	526	1,588	995	6.6	2.2	188
28	624	954	718	1,952	1.0	7.6	168
29	1,778	495	1,928	684	7.2	0.8	171
30	662	1,480	625	1,852	1.2	7.6	220
31	841	1,031	929	1,777	4.2	4.2	212
Bird 133							
1	611	972	599	811	20.8	19.4	139
2	846	290	1,151	218	35.6	4.4	68
3	271	1,187	354	996	7.6	32.8	126
4	665	631	852	435	31.8	8.2	158
5	178	2,194	131	1,310	4.2	35.8	60
6	1,222	1,053	1,321	1,192	9.8	9.2	262
7	1,534	722	1,505	876	16.4	3.6	275
8	552	1,803	705	1,920	4.0	15.4	208
9	1,678	951	1,580	871	15.2	4.8	262
10	618	2,250	569	1,958	2.6	17.2	174
11	234	219	301	160	32.6	7.4	65
12	40	446	41	318	3.8	36.2	20
13	204	502	187	315	20.0	20.0	85
14	265	144	287	100	35.0	5.0	40
15	131	416	109	286	8.6	31.4	46
16	1,546	1,668	1,257	1,157	19.6	19.8	228
17	2,186	1,062	1,721	762	31.0	9.0	198
18	677	2,475	598	1,805	4.6	35.4	164
19	2,259	636	1,866	585	35.6	4.4	204
20	913	2,447	683	1,874	7.0	32.8	193
21	1,668	502	2,279	468	16.4	2.0	155
22	410	379	383	363	20.4	19.6	110
23	388	335	513	261	31.2	8.8	112
24	223	666	252	604	4.6	35.4	87
25	264	129	671	89	36.2	3.8	39
26	226	796	217	637	7.4	32.6	80
27	944	993	1,186	1,471	6.4	1.6	167
28	389	1,881	302	2,337	1.0	8.0	84
29	1,546	473	1,815	771	7.2	1.0	128
30	458	1,774	375	2,265	1.4	6.6	90
31	379	1,474	439	2,086	4.0	4.6	96
Bird 134				/=-	a c c		
1	1,423	1,108	599	679	20.0	20.8	256
2	1,503	493	994	258	36.0	4.8	148
3	633	1,425	313	992	8.4	31.6	173
4	1,479	713	928	389	33.0	7.2	210
5	729	1,229	284	916	4.8	35.2	134
6	2,096	2,484	1,150	1,422	9.4	9.4	454
7	2,946	1,872	1,409	1,010	15.8	3.8	465
8	1,451	2,979	594	1,973	3.6	15.2	269
9	2,889	1,857	1,691	1,013	16.0	3.6	397
10	1,035	3,137	459	2,138	1.6	18.0	240
11	439	201	235	127	32.0	8.0	62
12	95	468	43	313	5.4	34.6	20
13	450	398	159	250	19.0	21.0	67
14	535	95	278	48	36.0	4.0	24
15	324	421	107	266	7.8	32.2	47
16	2,384	2,472	1,061	1,385	20.6	19.4	384

AP	PE.	ND	IX
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(Continued)

	Resp	onses	Ti	me	Rein	forcers	_ Change-
Condi- tion	Left	Right	Left	Right	Left	Right	overs
17	4,908	1,470	1,756	721	32.4	7.6	243
18	907	3,455	528	1,934	4.4	35.6	196
19	5,010	1,284	1,741	623	35.4	4.6	195
20	2,194	2,362	885	1,565	8.4	31.6	318
21	5,217	1,055	2,064	503	17.4	2.2	205
22	671	462	285	342	20.0	20.0	123
23	1,052	269	478	160	32.6	7.4	70
24	302	1,086	140	616	4.6	35.4	58
25	1,045	226	499	141	35.2	4.8	56
26	582	693	228	464	8.6	31.4	109
20	2,805	1,409	1,733	839	7.4	2.0	292
27	682		338				
		3,049		2,528	0.8	7.4	123
29	4,236	1,172	1,848	706	7.0	0.8	326
30	1,929	2,607	646	1,867	1.4	8.2	254
31	2,219	2,193	908	1,639	4.2	4.4	326
Bird 135			/ 		a c :		
1	1,045	792	687	556	20.4	20.0	164
2	1,658	317	1,162	170	35.8	4.6	77
3	618	1,225	433	867	9.0	31.0	150
4	1,472	629	920	324	31.4	8.8	132
5	591	1,179	395	876	4.4	35.6	149
6	1,032	1,374	817	1,956	10.0	9.8	216
7	1,490	694	1,154	1,444	16.0	3.4	298
8	666	1,824	406	2,235	4.0	15.4	125
9	1,738	1,039	1,384	1,291	14.8	3.2	272
10	454	1,877	336	2,328	1.8	17.0	106
11	460	259	264	125	31.0	9.0	67
12	70	446	36	291	3.6	36.4	15
13	314	470	168	251	21.0	19.0	60
14	514	151	261	78	35.2	4.8	38
15	199	501	90	243	8.0	32.0	38
16	1,182	1,667	1,099	1,411	18.8	20.2	268
17	1,825	828	1,949	607	32.2	7.8	203
18	526	1,665	501	1,991	5.0	35.0	136
19		703		559	35.0		207
20	1,867 830		1,938 656		8.6	4.4	207 199
		1,549		1,805		31.0	
21	1,405	608	2,097	518	16.2	2.8	145
22	456	547	331	324	19.6	20.4	108
23	595	370	438	188	32.0	8.0	82
24	248	932	144	649	4.2	35.8	65
25	712	209	500	118	36.2	3.8	59
26	372	712	220	562	8.4	31.6	72
27	1,276	644	1,964	869	6.6	2.2	185
28	522	1,318	456	2,207	1.0	8.4	142
29	834	470	1,741	971	7.8	1.2	122
30	659	1,106	718	2,018	1.8	7.4	162
31	722	951	1,028	1,718	4.4	4.4	150
Bird 136							
1	1,227	1,041	617	784	20.4	19.8	249
2	1,525	346	1,057	181	36.4	4.0	78
3	822	1,163	429	903	7.0	33.0	185
4	1,353	963	742	425	32.0	8.4	207
5	520	1,366	230	986	4.6	35.4	122
6	1,815	2,007	1,433	1,356	10.0	9.8	438
7		787	1,870	542			
8	2,237		795		16.6	3.6	280
	1,079	1,456		1,882	4.2	15.4	320
9	2,056	1,287	1,613	867	15.0	5.0	323

APPENDIX

(Continued)

Condi- tion	Resp	Responses		ime	Rein	forcers	_ Change-
	Left	Right	Left	Right	Left	Right	overs
10	382	1,598	305	2,322	2.0	17.8	119
11	224	155	398	110	32.8	7.2	44
12	34	408	39	347	5.4	34.6	18
13	126	356	225	308	20.0	20.0	56
14	209	65	314	70	35.8	4.2	14
15	88	405	112	333	8.4	31.6	35
16	600	1,702	802	1,576	19.2	19.4	192
17	1,043	1,124	1,367	1,144	31.0	8.8	279
18	308	2,058	468	2,089	3.8	36.6	147
19	859	1,015	1,505	992	34.8	5.0	227
20	478	2,000	558	1,969	7.0	33.0	165
21	730	693	1,930	765	17.4	2.4	196
22	242	269	475	354	20.4	19.6	92
23	275	190	547	233	31.0	9.0	71
24	78	577	124	665	4.4	35.6	32
25	198	179	652	227	35.0	5.0	55
26	91	775	168	627	7.6	32.4	46
27	1,078	1,024	1,684	989	7.4	1.4	264
28	212	1,952	222	2,717	0.2	9.0	70
29	957	707	2,026	748	7.4	1.4	211
30	458	1,462	547	1,885	2.0	7.6	151
31	440	1,334	1,064	1,807	4.8	5.0	174