MATCHING AND MAXIMIZING WITH CONCURRENT RATIO-INTERVAL SCHEDULES

LEONARD GREEN, HOWARD RACHLIN, AND JOHN HANSON

WASHINGTON UNIVERSITY AND STATE UNIVERSITY OF NEW YORK AT STONY BROOK

Animals exposed to standard concurrent variable-ratio variable-interval schedules could maximize overall reinforcement rate if, in responding, they showed a strong response bias toward the variable-ratio schedule. Tests with the standard schedules have failed to find such a bias and have been widely cited as evidence against maximization as an explanation of animal choice behavior. However, those experiments were confounded in that the value of leisure (behavior other than the instrumental response) partially offsets the value of reinforcement. The present experiment provides another such test using a concurrent procedure in which the confounding effects of leisure were mostly eliminated while the critical aspects of the concurrent variable-ratio variable-interval contingency were maintained: Responding in one component advanced only its ratio schedule while responding in the other component advanced both ratio schedules. The bias toward the latter component predicted by maximization theory was found.

Key words: maximizing, matching, concurrent schedules, variable-interval schedules, variable-ratio schedules, key peck, pigeons

"Matching" and "maximizing" are names for two currently competing molar explanations of animal choice (see, for example, Baum, 1981; Herrnstein & Heyman, 1979; Prelec, 1982; Rachlin, Battalio, Kagel, & Green, 1981; Staddon & Motheral, 1978). Herrnstein's matching law states that animals allocate time and effort among activities so as to *match* their behavior allocation to the rate of reward contingent on those activities. A general form of Herrnstein's law is:

$$R_x/R_y = b(r_x/r_y)^s, \qquad (1)$$

where R_x and R_y are allocations of responses to activities x and y, r_x and r_y are rates of obtained reinforcement, and b and s are empirical constants ("bias" and "sensitivity," respectively).

Another explanation of choice states that animals allocate time and effort among alternative activities so as to *maximize* the overall value of reward contingent on those activities.

Both matching and maximizing explanations have been extended from symmetrical situations (choice between such activities as pressing one versus another button, or pecking one versus another key) to asymmetrical situations (choice between pressing a button versus not pressing, or pecking a key versus not pecking it). For many of the symmetrical and asymmetrical choice situations studied in the animal laboratory, matching and maximizing make the same predictions (Prelec, 1982; Rachlin, 1978; Staddon & Motheral, 1978). Thus matching may be seen as the fundamental principle of animal choice behavior with maximizing being an incidental finding (Herrnstein & Vaughan, 1980; Prelec, 1982) or maximizing may be seen as fundamental with matching being incidental (Rachlin et al., 1981). This uncertainty holds under concurrent VI VI schedules where both alternatives produce the same reinforcer (e.g., mixed grains). However, when the reinforcers are different (e.g., food and water), the models make different predictions, with the maximizing approach being clearly superior (see, e.g., Rachlin et al., 1981).

Even with the same reinforcer, however, matching and maximizing apparently make different predicitions in one laboratory choice procedure: concurrent variable-ratio variableinterval schedules of reinforcement (concurrent VR VI). With this pair of schedules, the animal chooses between two alternatives: For one alternative rewards depend on number of

The research was supported in part by a grant from the National Science Foundation. The assistance of Dale Nieder and other members of the Psychonomy Cabal is gratefully acknowledged. Reprints may be obtained from Leonard Green, Dept. of Psychology, Washington University, St. Louis, Missouri 63130.

responses (VR) and for the other alternative rewards depend on time (VI), a response on the VI manipulandum being required only to collect the reward. Although both matching and maximizing predict behavior in accordance with Equation 1 under concurrent VR VI schedules, matching predicts no bias (b = 1)whereas maximizing predicts a strong bias for the VR schedule (if x is the VR schedule, then b > 1 in Equation 1). To see why this prediction follows from maximizing, imagine the case where a person is required to allocate time and work between two jobs, x and y. For job x, pay depends on work output (piecework wages)—the more work on job x, the more pay. For job y, pay depends on the passage of time. (The average rates of pay are fixed but individual payments are made after variable work outputs and times.) Given concurrent choice between the two jobs, there is a nonreciprocating arrangement between them. The critical point is that while working on job x time also accumulates for pay from job y. But output on job y does not count for job x. The optimal strategy, then, would be to work most of the time on job x and occasionally switch to job y to collect any payments that have accumulated. Following this strategy would maximize payment and result in a measure of bias greater than unity (b > 1). The matching law, on the other hand, predicts that as long as the jobs themselves are equivalent and the pay is in the same coin, the method of payment should not matter; bias should equal unity (b = 1).

A study by Herrnstein and Heyman (1979), of concurrent VR VI schedules with pigeons pecking keys for food reward, supports the prediction of matching; response bias was found to be close to unity. But, as pointed out by Rachlin et al. (1981), there is a problem with this study. In order to extend their explanation from symmetrical to asymmetrical choice, both matching and maximizing theories have had to postulate a significant reinforcement value for alternative behavior-for behavior other than the instrumental response or consumption of the reinforcer. Such behavior has been called, in matching accounts (Herrnstein, 1970), "reinforcement for other behavior." In maximizing accounts it has been called "leisure." Originally, such behavior was inferred from (and supposed to vary inversely with) the instrumental response, but it may be directly observed and measured (Staddon & Simmelhag, 1971). Both matching and maximizing explanations of asymmetrical choice infer different amounts of leisure activity with VI and VR schedules. The nature of VI schedules is such that they afford the opportunity to engage in activities other than responding while the temporal aspect of the schedule is being satisfied. This is not true of ratio schedules, which are purely labor intensive. Labor asymmetries are therefore present in situations where ratio and interval schedules are concurrently available. This asymmetry is not taken as important by matching-based accounts of behavior, whereas maximization accounts predict a bias toward the ratio schedule because of it. Measurement of the instrumental response alone may therefore provide a false picture of allocation of time (and even of responses) to the two alternatives (Green, Kagel, & Battalio, 1982). The increased food reward obtained from a bias towards the VR schedule may be compensated by increased leisure obtained from a bias towards the VI schedule.

The purpose of the present experiments was to study choice with the concurrent VR VI contingency but without the analytic problems associated with differential allocations of leisure activity. First, we removed measurement problems. In these experiments, with pigeons as subjects, there were three distinct periods, each signaled by a different set of keylight colors. The pigeon could be pecking on the left key, pecking on the right key, or not pecking. The latter period (which presumably contains all leisure activity) could then be ignored.

Second, we attempted to eliminate the confounding effects associated with different amounts of leisure activity on the two schedules. Instead of presenting concurrent interval and ratio schedules we presented a pair of concurrent ratio schedules. For both alternatives reward was contingent on responding. Time did not count for either schedule. The critical aspect of the concurrent VR VI contingency, from the present perspective, was maintained, however, by counting responses on one key as satisfying the VR requirement on both keys. (In the job example, output on job x would count for job x and job y, but output on job ywould count for job y alone.) This contingency preserves the critical features of the concurrent VR VI contingency as a test between

219

matching and maximizing (see Prelec, 1982, for a similar proposal) but avoids the confounding effects of leisure activity.

METHOD

Subjects

Four female White Carneaux pigeons served as subjects. Three had prior experience pecking keys for access to food; one (Bird 16) was naive. All were maintained at approximately 80% of their free-feeding body weights. Grit and water were freely available in their home cages.

Apparatus

Subjects were studied in a two-key Coulbourn Instruments pigeon chamber housed within a sound- and light-attenuating box. The working space available to the subjects measured 25.4 cm by 27.9 cm by 30.5 cm. Illumination of the chamber was provided by a 7-W white houselight deflected upwards and located near the ceiling, centered on the front wall. A small fan provided ventilation and masked extraneous sounds.

Access to the solenoid-driven food hopper was gained through a 5.0 cm by 5.5 cm opening located centrally below and between the two response keys; the center of the hopper opening was 5.25 cm above the floor. During food deliveries the hopper was illuminated by a 7-W white light.

The response keys were located to the left and right of center, 25 cm from the floor and 3.5 cm from the left and right walls. Each key was 2.54 cm in diameter and required a minimum force of .25 N to operate and produce a feedback click.

All scheduling and data collection were controlled by solid-state programming equipment located in an adjacent room.

Procedure

Experimental Conditions: The subjects were studied on several concurrent variableratio variable-ratio (concurrent VR VR) schedules of reinforcement. However, unlike the standard concurrent VR VR procedure in which the VR schedules are independent of each other, several differences were instituted in the present procedure.

Responding on either of the two response

keys changed the color of the key from amber to red if the left key had been pecked or from amber to green if the right key had been pecked. The peck started a 1-sec timer that kept the key illuminated (either red or green) for 1 sec since the last effective peck. If a subject pecked once, the key would change color for 1 sec and then return to amber. Once the key changed color, it would remain red or green until 1 sec had passed without a response. Any responses during this time reset the 1-sec interval. If the bird changed over from pecking one key to the other, say left to right, the left key immediately returned to amber and the right key would simultaneously change to green. This procedure served to signal the subject when it was effectively working on a particular VR schedule. When either key was amber, the animal was not working on that key and therefore could not receive food from that alternative. The right key was green only when the pigeon was pecking that key, and the left key red only when the pigeon was pecking the left key.

Pecks on the right key advanced its variableratio programmer. Pecks on the left key operated another, independent, variable-ratio programmer. In addition, however, pecks on the left key also advanced the right VR programmer. Thus, pecks on the left key changed the color of the left key and advanced the VR programmers for both the left and right keys. Pecks to the right key changed the color on the right key but had the singular effect of advancing only the right VR programmer. (A more symmetrical procedure was studied by Shull and Pliskoff, 1971. There, pecks on each key advanced the programmers for both keys.)

The present concurrent VR VR procedure is similar to a concurrent VR VI procedure. Under a concurrent VR VI, the VR programmer advances only when the subject is responding on the VR key while the VI timer runs continuously, even while the pigeon is pecking on the VR key. Similarly in our procedure, responses on the right key advance only the right VR programmer while responses on the left key advance both VR programmers. Henceforth, we will call the left key the VR key and the right key the "VR" key. Thus, the left, VR, key is equivalent to the VR key under a standard concurrent VR VI schedule, and the right, "VR" key is equivalent to the VI key on the concurrent VR VI. As is the case under the standard concurrent VR VI, probability of reinforcement from the "VR" key increases as the pigeon is responding on the VR key, but responding on the "VR" key does not affect the probability of reinforcement on the VR key. Our procedure thus maintains this critical aspect of the concurrent VR VI while removing other imbalances and confounding effects.

When responding on the left key set up a reinforcer from the left it was delivered immediately; the same was true when responding on the right key set up a reinforcer from the right. When responding on the left set up a reinforcer on the right, however, it could be collected only when the bird switched over and pecked the right key. No changeover delay was used.

Reinforcement consisted of 3-sec access to mixed grains, during which time the keys and chamber were darked. Subjects were studied daily unless they exceeded their 85% body weights. Each daily session ended after 60 reinforcers. Each concurrent VR "VR" schedule condition was in effect for 17 days. Data recorded were pecks to the left and right keys; number of reinforcers from the left and right keys; time spent with the left key red, time spent with the right key green, time spent with both keys amber; and the number of changeovers.

Each subject was studied under seven concurrent VR "VR" schedule conditions in the following order: concurrent VR 15 "VR" 30, concurrent VR 90 "VR" 30, concurrent VR 60 "VR" 90, concurrent VR 60 "VR" 30, concurrent VR 60 "VR" 60, concurrent VR 60 "VR" 15 and concurrent VR 30 "VR" 90. Following completion of these conditions, replications of some conditions were performed.

The experimental procedure provides a good test between matching and maximizing because it seems intuitively that, with a strong bias toward the VR schedule, reinforcement will be maximized. However, since the optimum allocation of behavior cannot be derived analytically, we resorted to a simulation to demonstrate this. We simulated a pigeon in the experiment, let the simulated pigeon vary its allocation of pecks to the two keys from one extreme to the other, and measured how much reinforcement was obtained at each allocation. We then determined the allocation

that provided maximum reinforcement and compared it with the corresponding allocation made by the actual pigeons.

Stat Bird Conditions: The equipment controlling the stimuli and scheduling in the previous experiment was programmed to perform this task in the following manner. A pulse, occurring every second, was routed through a probability generator which then delivered that pulse to the circuitry reading left-key pecks with probability p or to the circuitry reading right-key pecks with probability 1 - p. The statistical procedure was studied under the same concurrent VR "VR" values as in the experiment: concurrent VR 15 "VR" 30, concurrent VR 90 "VR" 30, concurrent VR 60 "VR" 90, concurrent VR 60 "VR" 30, concurrent VR 60 "VR" 60, concurrent VR 60 "VR" 15, and concurrent VR 30 "VR" 90. Under each of these schedules, probability values, p, of a left response ranged from .05 to .95 in .05 increments. The simulated responding continued until 600 food deliveries would have occurred.

Using this procedure, we were able to determine the distribution of food deliveries between the two components under a particular set of concurrent VR "VR" schedules when the responses were distributed in a particular way. It was also possible to determine which distribution of responses would lead to matching and which to maximizing under our procedures and to compare the actual performance of our subjects to these statistical results.

Control Procedure: The subjects and apparatus were the same as in the experimental conditions. A series of standard concurrent VR VR schedule conditions were studied. Responding on each key advanced only the variable-ratio programmers associated with that key. That is, responding on the right key again advanced only the right variable-ratio programmer, whereas responding on the left key, unlike the experimental conditions, now advanced only the left variable-ratio programmer. Key color changes (from amber to green on the right key and from amber to red on the left key) and all other contingencies and procedures were as before.

Each subject was studied under the same seven previous concurrent VR "VR" schedule conditions. All subjects initially were studied on a concurrent VR 90 VR 30 followed by a concurrent VR 30 VR 90 schedule. Bird 23 then received the following order of conditions: concurrent VR 15 VR 30, concurrent VR 60 VR 60, concurrent VR 60 VR 30, concurrent VR 60 VR 90, and concurrent VR 60 VR 15. Birds 24, 5, and 16 received the following order of conditions: concurrent VR 60 VR 30, concurrent VR 60 VR 90, concurrent VR 60 VR 60, concurrent VR 15 VR 30, and concurrent VR 60 VR 15.

RESULTS

Experimental Conditions

Figure 1 shows R_{VR}/R_{VR} , the ratio of the number of pecks on the left, VR, key (the key that operated both programmers) to the number of pecks on the right, "VR", key (the key that operated only the right programmer) as a function of r_{VR}/r_{VR} , the ratio of the number of reinforcers obtained by pecking the left key to the number of reinforcers obtained from pecking the right key. Response and reinforcement ratios were calculated each day for each subject. The points of Figure 1 are medians of the last 5 days at each condition. The filled-in points are the replications of various conditions. On three occasions the median $r_{VR}/r_{VR''}$ was zero (Pigeon 23 at VR 60 "VR" 15, Pigeon 5 at VR 90 "VR" 30, and Pigeon 5 at a replication of VR 60 "VR" 15); in these cases the birds obtained all their reinforcers from the "VR" key. These points are not plotted or included in the calculations. The overall best fitting line ($y = .73 \times +2.7$) provides estimates of the constants of Equation 1 of s = .73 and b = 2.7. There was considerable variability within and between subjects (overall $r^2 = .600$), but in Figure 1 all points, except one, are above the solid line (b = 1)showing strong bias towards the VR (as opposed to the "VR") key. The value of 2.7 for bias means that, on the average, when reinforcers were about equal for the two alternatives, the pigeons pecked 2.7 times on the VR key for each peck on the "VR" key.

Table 1 shows values of s, b, and amount of variance accounted for (r^2) for individual subjects. The sensitivity (s) for Subject 24 is very low but the sensitivities of the other subjects are typical of those found in studies with concurrent schedules (de Villiers, 1977).

Table 2 shows absolute rates of response for each pigeon in the various experimental con-

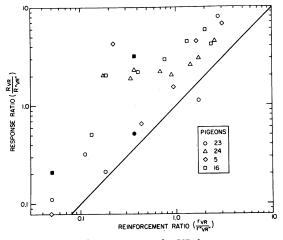


Fig. 1. Ratio of responses on the VR key to responses on the "VR" key as a function of the ratio of reinforcers obtained by pecking the VR key to reinforcers obtained from pecking the "VR" key. The filled-in points are replications of various conditions. (Based on the median of the last 5 days of each condition and plotted on logarithmic axes.) The solid line represents unbiased matching.

ditions. When the calculations were repeated with times (that the key was red or green), the results were similar to those with responses.

Simulation

Figure 2 shows overall simulated rate of reinforcement $(r_{VR} + r_{"VR"})$ as a function of p(the proportion of responses made by the simulated bird on the VR key) for each pair of VR "VR" conditions. The filled-in points are maxima of the functions. Each function has a maximum somewhere in the range $.5 \le p <$ 1.0.

Figure 3 shows relative rate of reinforcement, $r_{VR}/(r_{VR} + r_{..VR''})$, as a function of p for each condition. The straight diagonal line is the locus of points at which relative rate of reinforcement equals relative rate of response. Where the functions cross this diagonal line (the filled-in points), the simulated pigeon

Table 1

Values of s, b, and amount of variance accounted for by the best fitting line for each subject across all experimental conditions.

Subject	\$	Ь	r²	
23	.72	1.9	.48	
24	.26	2.7	.62	
5	.93	2.6	.66	
16	.85	3.1	.95	

conc VR "VR" 30-90	Birds							
	23		24		5		16	
	VR 54.0	"VR" 47.4	VR 60.7	"VR" 13.4	VR 119.2	"VR" 18.0	VR 71.1	"VR' 16.9
15-30	66.6	8.4	39.4	15.0	83.6	18.6	73.7	12.7
60-90	59.4	23.6	47.9	23.4	66.5	42.9	75.8	17.1
60-60	23.0 (44.9)	111.2 (86.6)	54. 3	24.4	57.0	87.1	63.9	21.6
60-30	24.8	82.8	55.6	29.5	10.2	134.5	52.3 (65.5)	23.9 (20.3
90-30	10.4	91.4	39.8	17.2	0.0	130.6	31.7	62.3
60-15	0.0	175.3	52.2	25.4	94.9 (6.5)	22.3 (135.3)	54.1 (14.5)	26.5 (67.8

 Table 2

 Median absolute rates of response (pecks/min of total session time) for each pigeon on each experimental condition. (Replications are in parentheses.)

would be matching (with s = b = 1.0 in Equation 1).

The simulation thus demonstrates that both maximizing (Figure 2) and matching (Figure 3) are possible under our procedure. The simulation also demonstrates that the behavior implied by the maximizing theory differs from that implied by matching (see Figure 4). Figure 4 (the points for which are taken from Figures 2 and 3) shows that matching and maximizing both predict a decrease in VR responding relative to "VR" responding as the ratio VR/"VR" increases. Matching predicts that

Fig. 2. Overall rate of reinforcement in the simulation as a function of allocation of responding to the two keys for each condition. The filled-in points are maxima.

the shift will be rapid, with no responding at all on the VR key for those conditions tested where VR > "VR." Maximizing predicts that the shift will be slow and less extensive. Only at the most extreme value of VR/"VR" (VR 60 "VR" 15) does the predicted equilibrium point reach as low as .5. The difference between maximizing and matching predictions is evidence that the procedure of this experiment can provide a good test between the two models.

Actual results for individual subjects (crosses) and medians across subjects (dotted line) are shown in Figure 4 for comparison with the simulation. Unfortunately, the simulation can tell us only that the present pro-

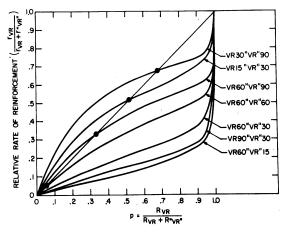


Fig. 3. Relative rate of reinforcement in the simulation as a function of allocation of responding to the two keys for each condition. The filled-in points indicate matching of relative rate of reinforcement to relative rate of responding.

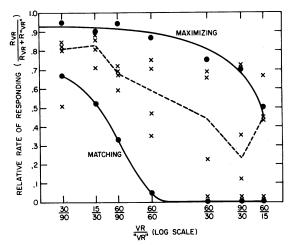


Fig. 4. Relative rates of response that yield maximization and matching in the simulation as a function of experimental condition (ratio of VR to "VR"). The solid points are from the simulation (transposed from Figures 2 and 3). The crosses are relative rates of response of actual individual pigeons. The dashed line is the median of the crosses.

cedure is potentially a good test and that matching and maximizing both predict the general direction of choice variation that occurred. The simplifying assumptions in the simulation of a constant overall rate of response (one per second), a fixed interresponse time, and a random distribution of responses to the two keys did not reflect the much more complex behavior of the pigeons. The actual equilibrium points generally fall between the predictions of matching and maximizing and do not decide between the two theories. However, the shift in responding is considerably slower and less extensive than matching predicts, with the overall shape of the function more closely resembling the maximizing function. Assuming (contrary to fact) that the simulation provides exact predictions of matching and maximizing, Figure 4 would indicate that the bias towards the VR key shown in Figure 1 was greater than predicted by matching but not as great as predicted by maximizing.

Control Conditions

The purpose of the control conditions was to demonstrate that the graded (nonexclusive) preference shown in Figure 1 was a function of the unusual contingencies of the experiment and was not due to the change of key colors, the pigeons' insensitivity, or to the particular schedules chosen. The behavior ex-

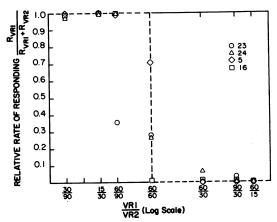


Fig. 5. Relative rate of responding as a function of control condition (ratio of VR1 to VR2).

pected with "normal" concurrent VR VR schedules is exclusive preference for the shorter VR (Herrnstein & Loveland, 1975).

Figure 5 shows for each pigeon the median relative rate of response (of the last five days) at each condition. Except for Pigeon 23 at concurrent VR 60 VR 90, all pigeons showed virtually exclusive preference for the shorter of the two concurrent VR schedules. (When both keys were equal [VR 60], preference is expected to be unstable and can lie anywhere on the vertical part of the dotted line.)

Table 3 shows absolute rates of responding for each pigeon in the various control conditions.

DISCUSSION

A response bias close to unity found with standard concurrent VR VI schedules (Herrnstein & Heyman, 1979) seemed to support a matching explanation of choice and to disconfirm the prediction of a maximizing explanation of choice that a large bias should be found in favor of the VR schedule. Maximizing could explain a bias of unity only by supposing that a bias greater than unity due to the value of the food reward was counteracted by a bias in the opposite direction due to the value of leisure. Green et al. (1982), in fact, showed that a maximization model that accounts for leisure by correcting for the differential rates of responding on the VR and VI schedules did predict Herrnstein and Heyman's results.

To further test this explanation the present experiment duplicated the essential conditions

Fa ble 2	5
-----------------	---

Median absolute rates of responding (responses/min) for each pigeon on each control condition.

conc VR VR	Birds							
	23		24		5		16	
	VR 1	VR 2	VR 1	VR 2	VR 1	VR 2	VR 1	VR 2
30-90	144.5	2.0	56.2	1.6	153.5	0.8	92.9	0.7
15-30	141.5	0.0	67.1	0.2	186.2	0.0	105.9	0.0
60-90	51.9	94.4	77.7	0.1	184.2	1.6	112.9	0.0
60-60	32.6	83.4	20.0	54.7	118.2	48.2	1.7	129.2
60-30	0.0	182.5	5.1	69.9	0.0	164.1	1.7	129.5
90-30	4.8	152.7	0.0	72.3	0.0	156.5	1.1	98.1
60-15	0.0	189.7	0.0	74.5	0.0	156.1	0.1	120.9

of concurrent VR VI schedules while eliminating the unmeasured and uncontrolled allocation of leisure in the standard experimental arrangement. That is, under both the present procedure and the standard concurrent VR VI, responding on the VR key satisfies the requirement for both schedules; as the subject remains on the VR schedule, the alternative schedule ("VR" in this procedure, VI in the standard procedure) continues to advance. Responding on the "VR" key (the VI in the standard procedure) satisfies the requirements for that schedule only. Time spent in other behavior (i.e., leisure) was eliminated as a confounding effect through the measurement procedure used (see introduction and procedure). When this was done, a large bias of choice in favor of the VR schedule was indeed found. A simulation of the present contingencies indicated, however, that this bias might still be less than expected with various simplifying assumptions regarding rate and distribution of responding.

The present experiment does not, therefore, provide unequivocal support for maximizing as opposed to matching. It does, however, negate the use of results with standard concurrent VR VI schedules as evidence against maximizing, results that have been cited (Herrnstein & Heyman, 1979; Herrnstein & Vaughan, 1980; Prelec, 1982) as the strongest, sometimes the only, such evidence available.

REFERENCES

- Baum, W. M. Optimization and the matching law as accounts of instrumental behavior. Journal of the Experimental Analysis of Behavior, 1981, 36, 387-403.
- de Villiers, P. Choice in concurrent schedules and a quantitative formulation of the law of effect. In

W. K. Honig & J. E. R. Staddon (Eds.), Handbook of operant behavior. Englewood Cliffs, N.J.: Prentice-Hall, 1977.

- Green, L., Kagel, J. H., & Battalio, R. C. Ratio schedules of reinforcement and their relationship to economic theories of labor supply. In M. L. Commons, R. J. Herrnstein, & H. Rachlin (Eds.), Quantitative analyses of behavior (Vol. 2): Matching and maximizing accounts. Cambridge, Mass.: Ballinger, 1982.
- Herrnstein, R. J. On the law of effect. Journal of the Experimental Analysis of Behavior, 1970, 13, 243-266.
- Herrnstein, R. J., & Heyman, G. M. Is matching compatible with reinforcement maximization on concurrent variable interval, variable ratio? Journal of the Experimental Analysis of Behavior, 1979, 31, 209-223.
- Herrnstein, R. J., & Loveland, D. H. Maximizing and matching on concurrent ratio schedules. Journal of the Experimental Analysis of Behavior, 1975, 24, 107-116.
- Herrnstein, R. J., & Vaughan, W., Jr. Melioration and behavioral allocation. In J. E. R. Staddon (Ed.), *Limits to action: The allocation of individual be*havior. New York: Academic Press, 1980.
- Prelec, D. Matching, maximizing, and the hyperbolic reinforcement feedback function. *Psychological Re*view, 1982, 89, 189-230.
- Rachlin, H. A molar theory of reinforcement schedules. Journal of the Experimental Analysis of Behavior, 1978, 30, 345-360.
- Rachlin, H., Battalio, R., Kagel, J., & Green, L. Maximization theory in behavioral psychology. The Behavioral and Brain Sciences, 1981, 4, 371-388.
- Shull, R. L., & Pliskoff, S. S. Changeover behavior under pairs of fixed-ratio and variable-ratio schedules of reinforcement. Journal of the Experimental Analysis of Behavior, 1971, 16, 75-79.
- Staddon, J. E. R., & Motheral, S. On matching and maximizing in operant choice experiments. *Psy*chological Review, 1978, 85, 436-445.
- Staddon, J. E. R., & Simmelhag, V. L. The "superstition" experiment: A reexamination of its implications for the principles of adapative behavior. Psychological Review, 1971, 78, 3-43.

Received April 4, 1983 Final acceptance June 29, 1983