CONCURRENT VARIABLE-INTERVAL SCHEDULE PERFORMANCE: FIXED VERSUS MIXED REINFORCER DURATIONS

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Six pigeons were trained on concurrent variable-interval schedules. Two different reinforcer durations were arranged on the left key, and which of these was delivered was determined probabilistically. A single reinforcer duration was arranged on the right key. In Parts ¹ and 3 of the experiment, the probability of the left-key reinforcer durations (1 and 7 seconds in Part 1; 3 and 10 seconds in Part 3) was varied from 0 to 1, keeping the schedules constant and the right-key reinforcer at 3 seconds. Response allocation to the left key fell as the probability of the shorter left-key reinforcer duration was increased. In Part 2, one left-key reinforcer duration was 3 seconds and the other was varied from 0 to 10 seconds, while again the schedules and right-key reinforcer duration (3 seconds) were kept constant. Left-key response allocation increased as the varied reinforcer duration on the left key increased. An extension of the generalized matching law failed to provide a good description of response allocation in these procedures. In Part 4, the left- and right-key reinforcer durations were 3 seconds and the variable-interval schedules were varied. Response allocation was well described by the generalized matching law. Part 5 arranged equal variable-interval schedules on the left and right keys, 3-second reinforcers on the right key, and the left-key reinforcer durations were varied from ¹ to 10 seconds. The relationship between the log response and reinforcer-duration ratios was nonlinear. The effects of varying reinforcer durations in concurrent schedules cannot be described using the generalized matching law with constant parameters. It is suggested more generally that this quantitative relation may not describe performance either when reinforcing situations differ in duration (e.g., with delayed reinforcers) or when response requirements differ in duration.

Key words: concurrent schedules, variable-interval schedules, reinforcer rate, reinforcer duration, generalized matching, sensitivity, key peck, pigeons

The sensitivity of behavior ratios to reinforcer-rate ratios has been the subject of a great deal of research (Baum, 1979; Taylor & Davison, 1983; Wearden & Burgess, 1982). In contrast, much less research has been reported that assessed sensitivity to reinforcer duration or amount ratios. Initially, Catania (1963) reported that performance on concurrent variable-interval (VI) schedules matched or equaled arranged reinforcer-duration ratios. Thus, if B_L and B_R are behavior measures and A_{L} and A_{R} are amount or duration measures:

$$
\frac{B_{\rm L}}{B_{\rm R}} = \frac{A_{\rm L}}{A_{\rm R}}\,. \tag{1}
$$

Catania varied the arranged durations of reinforcers and measured B as response rates. The relationship he found was confirmed by Brownstein (1971), who measured B as time allocation with no explicit response being required to obtain reinforcers.

Todorov (1973) used a procedure in which three pairs of concurrent VI VI schedules, differing in both reinforcer rates and durations, were arranged in each schedule, and he reported that both response- and time-allocation measures were less sensitive to reinforcer-duration ratios than to reinforcer-frequency ratios. He analyzed his data using the concatenated

Part of the work reported here was done by Ian Hogsden toward the requirements of the degree of Master of Science at Auckland University. We thank the New Zealand University Grants Committee for equipment grants to Michael Davison; we also thank the staff and students who helped run this experiment. In addition, we are grateful to Jack McDowell for suggesting Equation 3, and to Peter Killeen for making available to us the BASIC program for multidimensional nonlinear regression. Reprints may be obtained from Michael Davison, Department of Psychology, University of Auckland, Private Bag, Auckland, New Zealand.

generalizing matching law (Baum, 1974; Baum & Rachlin, 1969):

$$
\frac{B_{\rm L}}{B_{\rm R}} = c \left(\frac{N_{\rm L}}{N_{\rm R}} \right)^a \cdot \left(\frac{A_{\rm L}}{A_{\rm R}} \right)^b, \tag{2}
$$

where N refers to the frequency of a reinforcer, c is the bias constant, and a and b are sensitivities to reinforcer-rate and duration ratios, respectively. Using arranged reinforcer durations and obtained reinforcer frequencies, Todorov reported that a averaged 0.9 (range 0.5 to 1.2) for response measures and 0.5 (range 0.3 to 0.6) for time allocation, and that b averaged 0.27 (range 0.2 to 0.4) for response allocation and 0.17 (range 0.1 to 0.3) for time allocation. This result thus failed to support the results of Catania (1963) and of Brownstein (1971) and is also inconsistent with the usual finding that time allocation is more sensitive to reinforcer frequency than is response allocation (Baum, 1979; Taylor & Davison, 1983; Wearden & Burgess, 1982). However, the results reported by Todorov were replicated by Schneider (1973) for response measures. Schneider varied both reinforcer rates and amounts (number of pellets delivered to pigeons) and reported that $a = 0.6$ and $b = 0.34$ in Equation 2. Both Todorov's and Schneider's results are consistent with results reported earlier by Fantino, Squires, Delbrück, and Peterson (1972), who also found that behavior allocation was less sensitive to reinforcer-duration than to reinforcer-rate variations.

These consistent results were, however, questioned by Keller and Gollub (1977). In their first experiment, they varied both reinforcer durations and rates. From their data, a can be calculated at 0.47 (range 0.25 to 0.59), and b as 0.61 (range 0.23 to 0.87). The value of a was certainly lower than that usually found for reinforcer rate (range 0.5 to 1.5, Baum, 1979), and the value of b was larger than the value of a. These results agree with those of Schneider and Todorov only in that strong undermatching (a or $b < 1$) was obtained. In Keller and Gollub's second experiment, they exposed each pigeon to only two conditionsfirst, equal reinforcer rates and magnitudes, and second, unequal reinforcer rates and/or magnitudes. Response ratios equaled the products of the rate and amount ratios, implying that $a = b = 1$. They argued that exposing pigeons to sequences of reinforcer-rate and duration ratios might decrease the sensitivity of response allocation to both independent variables. However, two points must be taken into account in this experiment. First, although the composite (across birds) sensitivity in Experiment 2 was 0.98, the standard deviation of this measure was high (0.3), giving a large range of possible real values. Second, arithmetic VI schedules were used in Experiment 1, and constant-probability schedules were used in Experiment 2. It is now known (Taylor & Davison, 1983) that the latter usually give greater sensitivities to at least reinforcer-frequency ratios.

Taken together, the research discussed above supports the form of Equation 2 that was first proposed by Baum and Rachlin (1969). In it, behavior ratios equal the product sum of power functions of the ratios of the controlling independent variables. This product-sum rule also has received support in the analysis of the joint effects of reinforcer frequencies and response forces (Hunter & Davison, 1982). If the product-sum rule is correct, what does it predict for the case in which one concurrentschedule component provides two different reinforcer durations? If, for example, the left key of ^a concurrent VI VI schedule provided different-duration reinforcers (A_{L1}, A_{L2}) different numbers of times per session (N_{L1}, N_{L2}) , while the right key produced a single reinforcer duration (A_R) at the rate of N_R times per session, what extension of the generalized matching law might describe the resulting preference? If, as some previous research has suggested, both a and b in Equation 2 were 1.0, the extension is straightforward:

$$
\frac{B_{\rm L}}{B_{\rm R}} = c \left(\frac{N_{\rm L1} A_{\rm L1} + N_{\rm L2} A_{\rm L2}}{N_{\rm R} A_{\rm R}} \right).
$$

However, if generalized, rather than strict, matching occurs to both reinforcer frequencies and durations, the correct form of the extension is much more difficult to construct logically. One possibility is:

$$
\frac{B_{\rm L}}{B_{\rm R}} = c \, \frac{(N_{\rm L1} + N_{\rm L2})^a \cdot (pA_{\rm L1} + qA_{\rm L2})^b}{N_{\rm R}^a A_{\rm R}^b} \,. \tag{3}
$$

The variable p is the relative frequency of A_{L1} reinforcers, and q is the complement of p . Any equation that is a candidate for describing behavior allocation in concurrent mixed- versus fixed-reinforcer-duration schedules must reduce to Equation 2 when the mixed reinforcer durations are identical $(A_{L1} = A_{L2})$. Equation 3 does reduce to Equation 2 for equal left-key durations, but it provides unlikely predictions when, say, A_{L2} is reduced to 0 s. Under these conditions, Equation 3 predicts that behavior allocation is sensitive to the frequency of the 0-s reinforcers.

An equation which does have the required properties is:

$$
\frac{B_{\rm L}}{B_{\rm R}} = c \, \frac{(N_{\rm L1}A_{\rm L1}^{\ \ 4} + N_{\rm L2}A_{\rm L2}^{\ \ 2})^a}{N_{\rm R}^{\ \ a}A_{\rm R}^{\ \ a}d} \,. \tag{4}
$$

Equation 4 reduces to generalized matching to the ratio of reinforcer frequencies (with a sensitivity *a*) when all reinforcer durations are equal, and to generalized matching to the ratio of reinforcer durations (with a sensitivity ad) when the numbers of reinforcers on the two keys are equal. The dual sensitivity for reinforcer duration is not a problem as in previous research that varied reinforcer durations as the combined sensitivity would simply be estimated as b.

In the first three parts of the experiment reported here, two "states" on the left key were defined, and these (which were not under discriminative control) generally terminated in different reinforcer durations. In Parts ¹ and 3, the probabilities of the states (and reinforcer durations) on the left key were varied. In Part 2 the probability of the left-key states was kept constant, and the reinforcer durations in one left-key state were varied. Parts ¹ and 3 were thus equivalent to varying the reinforcer schedules for the two reinforcer durations on the left key, whereas Part 2 kept these schedules constant and varied the relative left-key reinforcer durations. Because of difficulties in interpreting the results of Parts ¹ to 3, Part 4 varied reinforcer rates on the left and right keys with constant and equal reinforcer durations, and Part 5 varied left- and right-key reinforcer durations with constant and equal reinforcer rates.

METHOD

Subjects

Six homing pigeons, numbered 21 to 26, were maintained at $85\% \pm 15$ g of their freefeeding body weights. The birds had extensive experience on concurrent schedules with reinforcer-rate variations prior to this experiment.

Apparatus

The sound-attenuating chamber, in which noise was masked by an exhaust fan, was situated remote from solid-state control equipment. The chamber was ³⁵ cm high, 30 cm wide, and 35 cm deep and contained two response keys ² cm in diameter, ¹¹ cm apart, and ²⁵ cm from the grid floor. A food hopper, containing wheat, was situated centrally between the keys and 10 cm from the floor. During reinforcer delivery, the hopper was raised, the grain was illuminated, and the keylights were extinguished (there was no houselight). Pecks on lighted keys exceeding about 0.1 N were counted and were followed by feedback clicks.

Procedure

Experimental sessions were conducted seven days per week. They commenced in blackout and terminated in blackout after a fixed number of reinforcers (Appendix 1) had been delivered or after 45 min had elapsed, whichever came first. Each experimental condition was in effect until a stability criterion was satisfied five, not necessarily consecutive, times by all birds. The criterion was that the median proportion of responses on the two keys over five sessions did not differ from the median proportion from the preceding five sessions by more than 0.05.

During the experimental sessions (except during reinforcer delivery) both keys were white, and pecks on the keys were reinforced according to concurrent VI VI schedules (Table 1). Each schedule comprised an irregular sequence of the first 12 intervals of an arithmetic progression in which the smallest interval was one twelfth the average interval. Pairs of schedules were arranged nonindependently (Stubbs & Pliskoff, 1969), such that when a reinforcer was arranged by one schedule, the timers for both schedules stopped until that reinforcer was collected. A changeover delay (Herrnstein, 1961) of 3-s duration was also arranged. This ensured that responses could not produce a reinforcer, even though one was arranged by a schedule, until 3 ^s had elapsed since the subject first responded on that key after responding on the other key.

The schedule on the right key was a stan-

dard VI schedule throughout all parts of this experiment. The schedule on the left key was somewhat unusual. Two states were defined within the left-key VI schedule, and these were chosen by interrogating a probability gate after each left-key reinforcer was obtained. These states were not in any way discriminatively signaled to the subject. In many of the experimental conditions, the two left-key states were correlated with different reinforcer durations (but always with the same reinforcer schedule). For example, in Condition 8 (Table 1), left-state ¹ had a probability of 0.5 and ended in a 1-s duration reinforcer. The alternative state (left-state 2) ended in a 7-s reinforcer. Thus, with a left-key schedule of VI 120 s, left-key 1-s reinforcers were delivered on VI 240 s, and left-key 7-s reinforcers were also delivered on VI 240 s. Changing the probability of left-state ¹ changed the relative probability of the two left-key reinforcers. It should be clear that standard concurrent VI VI schedules were arranged if the probability of leftstate ¹ was either 0 or 1, or if the two left-key reinforcers were the same duration.

In Part ¹ of this experiment, the reinforcer duration on the right key was 3 ^s and, with the exception of Conditions ¹ and 9 in which 3-s reinforcers were arranged in both left-key states, the left-key reinforcer durations were ¹ and ⁷ s. The probability of occurrence of the left-key states was varied from 0 to ¹ in Conditions ² to ⁸ (Table 1). When the probability of left-state ¹ was 0, the schedules were concurrent VI 120 ^s (7-s reinforcer) VI 120 ^s (3-s reinforcer). When the probability was 1, the schedules were concurrent VI 120 ^s (1 s) VI 120 ^s (3 s).

In Part 2, the probability of left-state ¹ was always 0.5, and the reinforcer durations in leftkey state 2 were varied from 0 (no reinforcer delivered, although a response was required when the VI schedule arranged a reinforcer) to 10 s. The schedules were concurrent VI 60 ^s (left) VI 120 ^s (right), and the right-key reinforcer duration was 3 s. Part 3 replicated the procedure of Part 1, again using concurrent VI 120-s VI 120-s schedules but with left-key reinforcer durations of 3 and 10 s. The rightkey reinforcer duration was again 3 s.

Parts 4 and 5 were control conditions. Reinforcer durations of 3 ^s were arranged on both left and right keys in Part 4, and the VI schedules on the left and right keys were varied

Table ¹

Sequence of experimental conditions, VI schedule and reinforcer duration values, and numbers of sessions training in each experimental condition in Parts ¹ to 5. The probability value given in parentheses shows how the experimental condition was actually arranged.

	Reinforcer durations(s)			Probability	
Condi-	Left State 1 State 2		Right	left state 1	Ses- sions
tion					
	Part 1: Conc VI 120 s VI 120 s				
ı	3	3	3	.5	24
2	1	1	3	1(.5)	23
3			3	0(.5)	39
$\overline{4}$	ı	7	3	.1	31
5		7	3	.7	21
6		7	3	.3	28
7	1	7	3	.9	22
8	ı	7	3	$\mathbf{.5}$	35
9	3	3	3	\cdot 5	23
	Part 2: Conc VI 60 s (left) VI 120 s (right)				
10	3	3	3	.5	23
11	3	5	3	.5	18
12	3	7	3	.5	19
13	3	$\mathbf{1}$	3	.5	21
14	3	2	3	.5	31
15	3	$\bf{0}$	3	$\ddot{5}$	20
16	3	10	3	.5	28
	Part 3: Conc VI 120 s VI 120 s				
17	3	3	3	1(0)	25
18	10	10	3	0(1)	24
19	3	10	3	.7	15
20	3	10	3	.3	19
21	3	10	3	.5	20
22	3	10	3	.1	27
23	3	10	3	9.	22

Part 4: Variation of reinforcer-rate ratio with 3-s reinforcers (probability of left-state $1 = .5$)

Condition	Left key VI schedule	Right key VI schedule	Sessions
24	120	120	30
25	60	240	31
26	240	60	28
27	45	360	32
28	360	45	28
29	60	60	36

Part 5: Variation of reinforcer duration. Conc VI 120s VI 120 s (probability of left-state $I = .5$)

over six experimental conditions. (In practice, the same programming arrangement was used in Parts 4 and 5 as was used in Parts ¹ to 3.

The left-key reinforcer durations were equal, and the probability of their occurrence was set at 0.5). In Part 5, concurrent VI 120-s VI 120-s schedules were arranged, the reinforcer duration on the right key was 3 s, and the reinforcer duration on the left key was varied from ¹ to 10 ^s over six experimental conditions. (Again, in practice, the left-key reinforcer durations were equal, and they were selected with a probability of .5).

In each condition of the experiment, the numbers of responses on the two keys, the time spent responding on the two keys, measured from the first response on one key to the first response on the other, and the numbers of reinforcers obtained on the left key in states ¹ and 2 and on the right key were collected. As the subjects had extensive experience on concurrent schedules, they were placed directly on Condition ¹ (Table 1).

RESULTS AND DISCUSSION

The data from all five parts of this experiment, averaged over the last five (except for one condition) sessions of each experimental condition, are shown in Appendix 1. For Parts 4 and 5, reinforcer numbers are shown in both left-state ¹ and left-state 2 because these conditions were programmed in the same way as Parts 1 to $\overline{3}$ (probability of left-state $1 =$.5), but with equal-duration reinforcers and schedules in both left-key states. Although both time-allocation and response-allocation data are given in the Appendix, only response data have been used in the detailed analyses. Time-allocation data followed response-allocation data closely and thus do not warrant a separate analysis. The time data are reported so that they are available for subsequent reanalyses.

Fig. 1. Results from Part 1. Relative left-key responses (left-key responses divided by total responses) emitted as a function of the probability of delivery of a 1-s reinforcer (vs. a 7-s reinforcer) on the left key. The schedules were concurrent VI 120 ^s VI 120 s, and the right-key reinforcer duration was 3 s. In Conditions ¹ and 9, 3-s reinforcers were delivered on both keys. The data were pooled over the last five sessions of each experimental condition.

Part ¹ of this experiment had VI 120-s schedules on both keys and varied the probability of 1- and 7-s reinforcers on the left key in Conditions 2 to 8. Condition ¹ was a simple concurrent VI 120-s VI 120-s schedule with 3-s reinforcers, and Condition 9 replicated this baseline condition. Figure ¹ shows the relative frequency of left-key responses (number of leftkey responses divided by total responses) as the probability of I-s reinforcers on the left key was increased from 0 to 1. Figure ¹ also shows the baseline data from Conditions ¹ and 9. This replication was satisfactory for all except Birds 24 and 25, for which the Condition 9 data moved in opposite directions, but in both cases the Condition 9 data were closer to the expected (no-bias) data value of .5. We thus put little faith in the Condition ¹ data for Birds 24 and 25.

As the probability of left-state ¹ (l-s) reinforcers was increased relative to left-state 2 (7-s) reinforcers, we would expect a decrease in relative response allocation. This decrease can easily be seen in Figure 1, and it was confirmed by a nonparametric trend test (Ferguson, 1965)

at $p < .05$ ($\mathbf{S} = 106$, $N = 6$, $k = 7$, $z = 6.4$). Individually, the data points for all birds fell, two of them (Birds 23 and 24) without reversal. At $p = 0$ (left-key 7-s vs. right-key 3-s reinforcers), the mean relative response frequency was 0.54, and at $p = 1$ (left 1-s vs. right 3-s reinforcers), it averaged 0.24. It is evident from the data of Conditions ¹ and 9 that all subjects were biased to the right key (though Bird 24 did show a left-key bias in Condition 1). Averaged over all birds, the relative response allocation was 0.40 (log $c = -0.18$) in Condition 1 and 0.39 (log $c = -0.19$) in Condition 9. The reason for this bias is unknown.

It is appropriate to deal with Part 3 of this experiment at this point. Part 3 was a systematic replication of Part ¹ with 3- and 10-s reinforcers on the left key. The relative response-allocation data are shown in Figure 2 for all birds. Again, this measure decreased as the probability of left-state ¹ (3-s reinforcers) was increased. The decrease was significant at $p < .05$ ($\Sigma S = 56$, $N = 6$, $k = 7$, $z = 3.4$), but it is much less evident to the eye than the decrease in Figure 1. The Condition 17 repli-

Fig. 2. Results from Part 3. Relative left-key responses (left-key responses divided by total responses) emitted as a function of the probability of delivery of a $3\text{-}s$ reinforcer (vs. a $10\text{-}s$ reinforcer) on the left key. The schedules were concurrent VI 120 s VI 120 s, and the right-key reinforcer duration was 3 s. In Conditions 1 and 9, 3-s reinforcers were delivered on both keys. The data were averaged over the last five sessions of each experimental condition except in Condition 17 (see Appendix).

cation of Conditions ¹ and 9 (Figure 2) was not particularly good (mean relative response frequency 0.46, log $c = -0.07$, the relativeresponse frequencies being too large (toward the left key) for Birds 22, 23, and 25. Perhaps bias had been affected by exposure to longer reinforcer durations or to varying reinforcer durations (Essock & Reese, 1974) on the left key. When all left-key reinforcers were ¹⁰ ^s (Condition 18), the average relative response allocation was 0.57.

In Part 2, a VI 120-s (3-s reinforcer) schedule was arranged on the right key, the probability of left-state ¹ was 0.5, and the reinforcer in left-state ¹ was ³ s. The reinforcer schedule on the left key was VI 60 s. Over seven conditions, the reinforcer duration in left-state 2 was varied. Figure 5 shows the relative response allocation for each bird. An increasing trend was expected from Equation 4 as the left-state

2 reinforcer duration was increased, and a generally increasing trend, with many reversals, can be seen. On ^a nonparametric trend test, this increase was significant at $p < .05$ $(\Sigma S = 66, N = 6, k = 7, z = 4)$, but only Birds 21, 24, and 25 showed an individually significant increase. In Condition 15, concurrent VI 120-s (3-s reinforcer) schedules were effectively arranged, and relative response allocation was 0.5 overall (log $c = 0$), though three of the six birds continued to show a right-key bias.

The mean data for each subject from each of the first 23 experimental conditions (as shown in Appendix \overline{I}) were then fitted to Equation 4. This was accomplished by means of a randomleap multidimensional nonlinear curve-fitting routine using the logarithmic transform of Equation 4. The estimated values of a , b , and $log c$, and the proportion of the data variance accounted for, are shown in Table 2. Although

Fig. 3. Results from Part 2. Relative left-key responses (left-key responses divided by total responses) emitted as a function of the reinforcer duration in left-key State 2. The reinforcer in left-key State ¹ was ³ s, and the probability of left-key State ¹ was .5. The schedules were concurrent VI 60 ^s (left) VI 120 ^s (right), and the right-key reinforcer duration was 3 s. Data were averaged over the last five sessions of each experimental condition.

Table 2

The values of the parameters of Equation 4 estimated by nonlinear regression from the 23 conditions comprising Parts ¹ to 4. Mean data from each experimental condition were used. The value of b in Equation 2 is ad . VAC is the proportion of the data variance accounted for by the fit.

Bird	a	d	log c	VAC
21	.06	6.14	$-.33$.25
22	.45	.98	$-.01$.23
23	.54	.85	$-.25$.38
24	1.27	.37	$-.24$.30
25	.83	.58	.02	.37
26	.57	.34	$-.01$.27
Mean	.58	.62	$-.08$.37

most parameter estimates appear to fall in the expected range, the fits were uniformly poor, with a maximum of only 38% of the data variance accounted for.

The basis for the poor fits of the model to the data from Parts 1, 2, and 3 of this experiment could be simply the relatively small data variance obtained over Conditions ¹ to 23 (Figures 1, 2, and 3). On the other hand, the model (Equation 4) could be incorrect. One way of choosing between these alternative explanations is to obtain precise estimates of both a and d in Equation 4 from standard concurrent VI VI schedule manipulations, and to use these estimates to predict performance in Parts 1 to 3. If the data did not deviate systematically from such predictions, the problem in fitting Equation 4 to the data from Parts ¹ to ³ could be that of too much error variance in combination with too little systematic variance. (Note that the Equation 4 model cannot fit the data of Parts ¹ to 3 better than the fits shown in Table 2. The model might, however, successfully describe data obtained using much larger reinforcer-duration differences on the left key.)

Parts 4 and 5 of the experiment were conducted to obtain direct estimates of a and d. Part 4 consisted of a standard variation of left and right reinforcer rates on concurrent VI VI schedules. These conditions (24 to 29) were arranged by giving 3-s reinforcers in both left-key states and on the right key and by setting the left-state ¹ probability gate at 0.5. The Part 4 data should be describable by Equation 2 with the reinforcer-duration ratio set at 1. Thus, least squares linear regressions were carried out on the logarithmically transformed data (Baum, 1974) to obtain estimates of a and c .

Figure 4 shows the obtained Part 4 data for each bird, with the obtained linear equations and the standard deviations of the parameter estimates. The fits of the straight lines to the data were excellent, as evidenced by the small standard deviations of the slopes and intercepts, and by an average mean-square error of 0.015. The values of a averaged 0.87 (range 0.72 to 1.05), well within the usual range for concurrent arithmetic VI schedules (Taylor & Davison, 1983). The average value of bias (log c) was -0.08 , and the overall bias was toward the same key as shown in the analyses of Conditions 1, 9, 15, and 17. Similar analyses of time-allocation data gave an average a value of 0.98 (range 0.8 to 1.12) and individual a values for time allocation were greater than those for response allocation for all birds (Baum, 1979; Taylor & Davison, 1983). Timeallocation fits were also good (average $MSE =$ 0.017), and the average time-allocation bias was -0.03. Part 4 thus showed absolutely standard concurrent VI VI schedule performance.

In Part 5 (Conditions 30 to 34), the left- and right-key schedules were both VI 120 s, the right-key reinforcer duration was 3 s, and the left-key reinforcer duration was varied from ¹ to 10 s. Figure 5 shows log response ratios as a function of log (arranged) reinforcer-duration ratios. However, no equations of fitted lines are shown, for the data obviously were not linear. Rather, each subject (except perhaps, Bird 24) showed a positively accelerated increase in log response ratio with increasing log duration ratio. At the left of each graph is Condition 31 (1-s vs. 3-s reinforcers), and to the right is Condition 32 (10-s vs. 3-s reinforcers). The data from Condition 34 (7-s vs. 3-s reinforcers), or possibly Condition 33 (5-s vs. 3-s reinforcers), generally did not fall on the overall trend. We offer no explanation for this and cannot rule it out as a real effect. Another single-condition estimate of bias was obtained from Condition 30 (3-s reinforcers on the left and right keys), which gave a mean value of $log c$ of -0.13 . Time-allocation data from Part 5 showed a pattern similar to that of the response-allocation data. Overall, the data from Part 5 show rather clearly that a constant $$ value in Equation 2, and thus in Equation 4, cannot explain the relative behavior change produced by varying reinforcer durations on one key of a concurrent VI VI schedule.

It could be suggested that the present re-

Fig. 4. Results from Part 4. The logarithm of the ratio of left-to-right key responses as a function of the logarithm of the ratio of corresponding frequencies of reinforcement. Reinforcer durations were ³ s. The data shown are from each of the last five sessions of each experimental condition. Equations of the least squares regression lines are shown, with the standard deviations of the slope and intercept estimates beneath them.

Fig. 5. Results from Part 5. The logarithm of the ratio of left-to-right key responses as a function of the logarithm of the ratio of left-to-right key reinforcer durations arranged. The schedules were concurrent VI 120 ^s VI 120 s. The data shown are from each of the last five sessions of each experimental condition.

sults, and those of Part 5 particularly, might be understood better in terms of the amounts the animals ate, rather than in terms of the durations of availability of food. Epstein (1981), using a food magazine very similar in design to that used here, showed that the amount eaten by pigeons was a negatively accelerated function of reinforcer duration. Epstein's result, however, implies the exact opposite of the results shown in Figure 5. The function to the left of the x-axis zero point should increase in slope with increasing (negative) distance from zero, and the function to

the right of zero should decrease in slope with increasing distance from zero. The effect or value of reinforcer durations thus may not be linearly related to either arranged durations or to times spent eating.

GENERAL DISCUSSION

This experiment was designed to provide data that would allow an already-accepted model (the concatenated generalized matching law; Baum, 1979; Hunter & Davison, 1982) to be extended to a procedure in which mixed reinforcer durations were delivered on concurrent VI VI schedules. The model proposed for this extension, Equation 4, appeared to have all the properties that were needed. A generalized form of Equation 4 (i.e., with exponents) was used because of previous reports that such a form was necessary with both reinforcer rate (Baum, 1979) and reinforcer duration or amount (Schneider, 1973; Todorov, 1973). The generalized version also adds the bias parameter, c , in Equation 2. A second integral part of Equation 4 is the rule by which different reinforcer durations for the same response are combined. This aspect of the equation was determined by the requirement that the equation reduces to Equation 2 when either the two reinforcer durations were the same or when one was 0 ^s in duration. There were thus strong empirical and logical reasons for using Equation 4. However, Equation 4 did not accurately describe the data obtained from the first 23 experimental conditions.

Parts 4 and 5 took the two components of Equation 4 (reinforcer-rate ratios and reinforcer duration) and assessed their contributions separately. The data from Part 4 (Figure 4) for reinforcer-rate variation with 3-s reinforcers showed absolutely standard concurrent VI VI performance (Taylor & Davison, 1983). Averaged across birds, a was 0.87. The problems in Parts ¹ to 3 did not arise from abnormal a values or from any unusual relationship between response ratio and reinforcer-frequency ratios for these subjects.

Part 5 kept the reinforcer schedules constant and varied the reinforcer duration on one key. The data (Figure 5) showed that the resulting performance could not be described by a constant b value; b was some function of the varied left-key reinforcer-duration values, and it appeared to increase (to

give more extreme response ratios) as the varied value increased. This result, then, is consistent with the conclusion of Essock and Reese (1974) that longer reinforcer durations are weighted more heavily than shorter durations. Both of these results disagree with the suggestion by Staddon and Innis (1966) that variable reinforcer durations are arithmetically averaged.

How may the data from Part ⁵ be modeled? The model that appears most satisfactory assumes that the value of a reinforcer, V , is an exponential function of reinforcer duration:

$$
\frac{B_{\rm L}}{B_{\rm R}} = c \left(\frac{\text{Exp}(\alpha A_{\rm L})}{\text{Exp}(\alpha A_{\rm R})} \right),
$$

where α is a constant. This model accounted for 91%, of the group-data variance from Part 5, with $\alpha = 0.25$. For the individual subjects, the variance accounted for ranged from 80% to 92%, and the α values ranged from 0.23 to 0.3. The relatively high percentages of variance accounted for, and the consistency of the α values, are suggestive. But when this model is extended to take frequencies of outcomes into account:

$$
\frac{B_{\rm L}}{B_{\rm R}} = \frac{[N_{\rm L1} \text{Exp}(\alpha A_{\rm L1}) + N_{\rm L2} \text{Exp}(\alpha A_{\rm L2})]^a}{N_{\rm R}{}^a \text{Exp}(\alpha A_{\rm R})}
$$

it fits no better than the Equation 4 fits shown in Table 2. Thus, not only is the relationship between log response ratios and log reinforcerduration ratios nonlinear for equal reinforcer frequencies, but there may be some further interaction between reinforcer frequency and duration not addressed in the above Equation or in Equation 4.

Why has the nonlinearity between log response ratios and log reinforcer-duration ratios not previously been reported? Largely, it seems, because reinforcer durations have not been extensively manipulated before. Catania (1963) reported only three data points for three birds, and Brownstein (1971) reported three or four data points for three birds. Any inconstancy in b would be difficult to discover in such data. Also, both these authors used a constant sum of reinforcer durations between the keys (Catania, 9 s, and Brownstein, 6 s), which may have some bearing on the constancy of b. A variable sum was used here and by Schneider (1973) and Todorov (1973). Schneider did not report a sufficient number of reinforcer-amount ratios at constant reinforcer-rate ratios to permit an analysis like that done here. Todorov reported six different reinforcer-duration ratios, but these comprised three sets of inverse pairs (e.g., 2 vs. 4 ^s and 4 vs. 2 s). The obtained data are thus three data points and their inverses, and thus are not useful for demonstrating inconstancy in b.

The concatenated generalized matching law (Equation 2), with its implication of constant $a, b,$ and c parameter values, appears inapplicable to variation in reinforcer duration. This finding limits the generality of the concatenated generalized matching law, which has also been shown not to apply with a constant power to delay of reinforcement (Williams & Fantino, 1978), and hence to concurrent-chain terminal links to reinforcement (Davison, 1983; Duncan & Fantino, 1970). The delayed reinforcer or terminal-link schedule results are qualitatively similar to the results reported here. Taking these two results together, it may be the case that constant-sensitivity generalized matching does not occur when reinforcing conditions differ in temporal extent between two alternatives. This suggestion may also apply to the concatenation of response-related independent variables in the generalized matching law. Hunter and Davison (1982) varied response requirements by varying the force with which keys had to be pecked. A constant force-ratio exponent was demonstrated over many absolute force levels and force ratios, and this variation did not affect sensitivity (a) to reinforcer-frequency ratios. But Beautrais and Davison (1977) varied response requirements by varying the fixed-ratio (FR) components of second-order VI(FR:S) schedules and found that sensitivity to reinforcer-frequency ratios was consistently affected by changes in the FR values on the two keys. Thus response-requirement and reinforcer-frequency sensitivities were not independent. The difference between the two experimental procedures for varying response requirements is that the former does not produce different response-time requirements, but the latter does.

The generalized matching law and related formulations may thus be severely limited in their ability to describe behavior, especially when behavioral or reinforcing events differ in duration. This latter possibility, supported by the present data, suggests that applications of the generalized matching law to, for instance, data on commitment and self-control (Green & Snyderman, 1980) will fail, for neither reinforcer delay nor duration ratios will, in general, have a constant exponent. It is unclear whether an extension of the generalized matching law (e.g., Davison, 1982, 1983) can accommodate the apparent parameter inconsistencies, or whether a more radical approach is required. But it is clear that a quantitative law of effect (de Villiers, 1977) must describe findings such as those obtained here.

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Received March 22,1983

Final acceptance December 13,1983

APPENDIX

Numbers of responses emitted, time (min) spent responding, and the number of reinforcers obtained averaged over the last five sessions of each condition. Because of a data loss, the data from Condition 17 are averaged over only two or three sessions. L/Sl and L/S2 refer to the two left-key states.

