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VARIABLE-RATIO SCHEDULES AS VARIABLE-INTERVAL SCHEDULES WITH LINEAR FEEDBACK LOOPS

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The mathematical theory of linear systems has been used successfully to describe responding on variable-interval (VI) schedules. In the simplest extension of the theory to the variable-ratio (VR) case, VR schedules are treated as if they were VI schedules with linear feedback loops. The assumption entailed by this approach, namely, that VR and VI-plus-linear-feedback schedules are equivalent, was tested by comparing responding on the two types of schedule. Four human subjects' lever pressing produced monetary reinforcers on five VR schedules, and on five VI schedules with linear feedback loops that reproduced the feedback properties of the VR schedules. Pressing was initiated by instructions in 2 subjects, and was shaped by successive approximation in the other 2. The different methods of response initiation did not have differential effects on behavior. For each of the 4 subjects, the VR and the comparable VI-plus-linear-feedback schedules generated similar average response rates and similar response patterns. The subjects' behavior on both types of schedule was similar to that of avian and rodent species on VR schedules. These results indicate that the assumption entailed by the VI-plus-linear-feedback approach to the VR case is valid and, consequently, that the approach is worth pursuing. The results also confute interresponse-time theories of schedule performance, which require interval and ratio contingencies to produce different response rates.

Key words: variable-ratio schedules, feedback, linear system theory, interresponse-time theory, shaping, ratio strain, lever press, humans

McDowell and Kessel (1979) used the mathematical theory of linear systems (Aseltine, 1958) to describe responding on variable-interval (VI) schedules of reinforcement. The linear system theory is a set of mathematical techniques that can be used to calculate the response of a system to a known input, provided the system can be described at least in principle by a linear differential equation. The result of the analysis for the VI case was a mean-value rate equation (cf. McDowell, Bass, & Kessel, 1983), which can be written (McDowell, 1980):

$$R_{\mathrm{OUT}} = \left\{\ln\left[1 + \frac{P_B}{\gamma P_R}\left(e^{1/R_{\mathrm{IN}}} - 1\right)\right]\right\}^{-1}. \quad (1)$$

This equation expresses response rate, R_{OUT}, on a VI schedule as a joint function of reinforcement rate, R_{IN}, reinforcer value, P_R, and response aversiveness, P_B. Gamma is a scalar constant that is characteristic of the organism. The quantities, P_R and P_B, as well as methods of measuring them, have been discussed in detail elsewhere (McDowell, 1980, 1986).

Equation 1 provides an excellent description of VI responding (McDowell, 1980, 1986; McDowell & Kessel, 1979) that is superior to the descriptions provided by seven other mathematical accounts of the VI case (Catania, 1973; Killeen, 1981, 1982; Rachlin, 1978; Staddon, 1977, 1979), including Herrnstein's (1970) matching-based account (McDowell et al., 1983; McDowell & Kessel, 1979; McDowell & Wood, 1984, 1985). Although several properties of Equation 1 remain to be tested (McDowell, 1986), the success of the linear-system description of the VI case indicates that applications of the theory to other cases might also be successful. In the present article we consider how the theory might be applied to responding on variable-ratio (VR) schedules.

McDowell (1979, 1980) pointed out that the simplest mathematical approach to the VR case was to treat it as VI responding, but with the addition of a feedback loop. Of course feedback also occurs on VI schedules, but it is restricted to a small range of low response rates

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and is minimal in comparison to VR feedback, which operates at all response rates (see McDowell, 1980). Feedback approaches to understanding behavior are not new; they have figured prominently in the work of Baum (1973, 1981), Rachlin (1978), and Staddon and Motheral (1978), among others.

Feedback on a VR schedule is determined by the average ratio requirement, $\bar{n}$, that the schedule arranges. If the responses specified by this average requirement are emitted slowly, reinforcers will be delivered slowly, and if they are emitted rapidly, reinforcers will be delivered rapidly. The direct relationship between reinforcement rate and response rate on VR schedules is described by the feedback function,

$$R_{\mathrm{IN}} = (1/\bar{n})R_{\mathrm{OUT}}, \qquad (2)$$

which is a line with intercept equal to zero and slope equal to the reciprocal of the average ratio requirement. For VR schedules, Equation 2 is true by definition because it describes a defining, or necessary, property of these schedules. However, the linear feedback described by Equation 2 can be arranged independently of a ratio contingency because $1/\bar{n}$ is simply a number that can assume any finite value greater than zero.

Because Equation 1 was written to describe responding on simple VI schedules, it is evident that the linear system theory requires the composite of Equation 2 on Equation 1 to describe responding on a VI schedule to which a linear feedback loop has been added. This type of schedule is not difficult to arrange. Reinforcement on a VI-plus-linear-feedback schedule is delivered according to a time-based, or interval, contingency, just like on a simple VI schedule. However, reinforcement rate on this type of schedule increases linearly with response rate according to Equation 2. In other words, throughout the session the effective average interreinforcement interval (or VI value, i.e., $1/R_{\mathrm{IN}}$) varies directly with the average interresponse time ($1/R_{\mathrm{OUT}}$). Higher response rates produce smaller mean VI values (i.e., higher reinforcement rates) and lower response rates produce larger mean VI values (i.e., lower reinforcement rates). According to the linear system theory, responding on this type of schedule must occur in the following manner. An initial response rate produces an initial reinforcement rate according to Equation 2. This reinforcement rate produces a new response rate according to Equation 1, which in turn produces a new reinforcement rate according to Equation 2, and so on, until responding reaches equilibrium, where Equations 1 and 2 are satisfied simultaneously.

In McDowell's (1979, 1980) suggested approach to the VR case, VR schedules are treated mathematically as if they were VI schedules with linear feedback loops. In other words, the two types of schedule are assumed to be equivalent. Implicit in this assumption is the view that the only important difference between VI and VR schedules is the linear feedback that the latter arrange. If this view is correct, then response characteristics that are unique to VR schedules must be due to the linear feedback, rather than to any special property of the response-based ratio contingency per se. It follows that VI-plus-linear-feedback schedules, which arrange linear feedback in the absence of a ratio contingency, should produce response outputs that are indistinguishable from those produced by VR schedules. For example, the high response rates that VR schedules are known to generate (Baum, 1981; Zeiler, 1977) should also be produced by VI schedules with linear feedback loops. As another example, strained, two-valued (i.e., zero or very rapid) responding is sometimes observed on VR schedules when the mean ratio requirement is large (e.g., Ferster & Skinner, 1957). If strained responding is observed on a VR schedule, then it should also be observed on the comparable VI-plus-linear-feedback schedule.

The first step in pursuing the VI-plus-linear-feedback account of the VR case is to test the assumption that the two types of schedule are equivalent. If the assumption holds, then the VI-plus-linear-feedback approach can be pursued further. If it does not hold, then a new approach to the VR case will be required. The purpose of the present experiment was to test the assumption by comparing human subjects' responding on VR schedules to their responding on VI schedules with linear feedback loops. This experiment has five additional benefits. First, it adds to the sparse parametric literature on VR responding (Baum, 1981). Second, it represents the first parametric study of human responding on VR schedules. Third, it introduces a novel method of arranging response-rate feedback. Fourth, it compares two methods of initiating responding in human subjects, namely, instructions and shaping by successive approximation. And fifth, it bears on traditional interresponse-time (IRT) theories (Zeiler, 1977) of schedule performance. According to IRT theories, relatively long IRTs are likely to be reinforced on schedules that arrange interval contingencies, and relatively short IRTs are likely to be reinforced on schedules that arrange ratio contingencies. As a consequence, the former schedules should generate relatively low response rates and the latter should generate relatively high response rates (Ferster & Skinner, 1957; Morse, 1966; Peele, Casey, & Silberberg, 1984; Skinner, 1938; Zeiler, 1979). Clearly, interresponse-time theory requires VI-plus-linear-feedback schedules to generate lower response rates than comparable VR schedules because of the different contingencies that the two types of schedule arrange.

METHOD

Subjects

Four humans aged 31 to 41 years (three male, one female), who were recruited by advertisement, served in the experiment. All subjects were either unemployed or employed part-time while participating, and none were college students. One subject (H31) had previous experience on VI schedules. The other subjects were experimentally naive.

Apparatus

Subjects worked in a small room facing a 54.6-cm (width) by 64.8-cm (height) console that tilted away from the subject at an angle of 23.2° from the vertical. A straight 24.5-cm lever extended horizontally from the center of the panel. A downward force on the lever of approximately 120 N produced an audible click and counted as a response. A 6-digit LED display, an amber (reinforcement) light, a small speaker, a green (session) light, and a row of five blue (schedule) lights were attached to a 17.6-cm (width) by 8.7-cm (height) metal box that was mounted on top of the console. During sessions the room was dimly illuminated by a 7.5-W houselight and continuous white noise masked extraneous sounds. A small viewer mounted in the door permitted one-way observation of the experimental room. The console was controlled and data were recorded by a computer located in an adjoining room.

Procedure

All subjects' lever pressing produced monetary reinforcers in the two phases of the experiment. In one phase the subjects were exposed to VR 15, 30, 60, 120, and 240 schedules. Individual ratio requirements were calculated by Fleshler and Hoffman's (1962) method. In the other phase, the subjects were exposed to five VI-plus-linear-feedback schedules each of which had feedback properties comparable to one of the five VR schedules. Feedback was arranged according to Equation 2 which, when inverted, is written

$$\overline{\mathrm{IRI}} = \bar{n}(\overline{\mathrm{IRT}}). \quad (3)$$

The quantity, $\overline{\mathrm{IRI}}$, is the mean interreinforcement interval, or VI value, and $\overline{\mathrm{IRT}}$ is the mean interresponse time. Feedback comparable to that of a given VR schedule was arranged by setting $\bar{n}$ equal to the average ratio requirement of the comparable VR. Equation 3 and Fleshler and Hoffman's equation were then used to schedule the reinforcers one at a time on-line. Fleshler and Hoffman's equation, which is often used to calculate individual interreinforcement intervals (IRIs) for simple VI schedules, is written

$$\mathrm{IRI}_i = \overline{\mathrm{IRI}}[1 + \ln N + (N - i)\ln(N - i) - (N - i + 1)\ln(N - i + 1)], \quad (4)$$

where $i = \{1, 2, 3, \ldots, N\}$. The index, i, enumerates the individual IRIs and N is the number of distinct IRIs in the schedule. In the present experiment $N = 20$. After each reinforcement, i in Equation 4 was set equal to a randomly selected integer between 1 and 20. After each response (consider, e.g., the jth response), a computer calculated the $\overline{\mathrm{IRT}}$ based on all IRTs since the last reinforcement and multiplied it by the average ratio requirement ($\bar{n}$) of the comparable VR, as Equation 3 requires. This yielded a new "fed back" $\overline{\mathrm{IRI}}$, or mean VI value, which was then used to calculate an individual IRI from Equation 4. This individual IRI was then scheduled. If the next (i.e., the $[j + 1]$th) response occurred after the lapse of the scheduled IRI, then it was reinforced, a new integer value between 1 and 20 was assigned at random to i, and the scheduling process began again. If, however, the next (again, the $[j + 1]$th) response occurred before the lapse of the scheduled IRI, then that IRI was canceled and a new $\overline{IRT}$ was calculated, which yielded a new $\overline{IRI}$ from Equation 3. The new $\overline{IRI}$ was used to calculate a new individual IRI from Equation 4, which was then scheduled. Timing continued during the very brief reinforcer presentations ($\ll 1$ s, as described below) such that the first IRT in a given IRI was the time between the response that produced reinforcement and the first response thereafter. Variable-interval schedules with linear feedback loops arranged in this way have the following properties: (1) the time between reinforcements depends on response rate since the last reinforcement only, just as on VR schedules, (2) faster response rates produce faster reinforcement rates and slower response rates produce slower reinforcement rates (according to Equation 2), just as on simple VR schedules, (3) only interval contingencies are arranged, (4) the scheduled IRI changes with each response, and (5) the first response on a given schedule and the first response after reinforcement can never be reinforced because initial $\overline{IRT}$s are not calculated until these responses have occurred. It is important to recognize the difference between VR and VI-plus-linear-feedback schedules. Although they have the same feedback properties, the two types of schedule arrange different contingencies, that is, they use different criteria to set up reinforcement. Variable-interval-plus-linear-feedback schedules arrange interval contingencies, which means that reinforcement is set up on the basis of elapsed time. Every response on this type of schedule can be reinforced (except as noted above), provided the subject waits until the scheduled interval lapses before responding. This is also possible on simple VI schedules. Variable-ratio schedules, on the other hand, arrange ratio contingencies, which means that reinforcement is set up on the basis of response count. On this type of schedule it is not possible for every response to be reinforced.

Except during the initial training sessions for Subjects H32 and H36, the following procedure was used in both phases of the experiment. All five schedules were presented in each session. The subject worked on one schedule for 10 min, rested for 5 min, worked on the next schedule for 10 min, rested for 5 min, and so on until all schedules were presented. The sequence of schedules was quasi-random within sessions, with the restriction that each schedule appear exactly once per session. One of the blue schedule lights was correlated with each schedule. During work periods the houselight, session light, and the appropriate schedule light were illuminated. During rest periods the subject was required to remain in the experimental room with only the session light illuminated. Subjects H31 and H36 were exposed to the five VR schedules first; the other 2 subjects were exposed to the five VI-plus-linear-feedback schedules first.

For Subjects H31 and H32, reinforcement consisted of the addition of 3.75¢ to the digital display, a brief ($\ll 1$ s) flash of the amber reinforcement light, a brief ($\ll 1$ s) offset of the houselight, and a brief ($\ll 1$ s) sounding of a 1,000-Hz tone. For H36 and H37, reinforcement consisted of the addition of 4.25¢ to the digital display and presentation of the three brief stimuli just mentioned.

All subjects signed a contract before the start of the experiment in which they agreed to participate for 60 sessions or until they were released, whichever occurred first. The contract also stated that their earnings would depend on their performance, that they would be paid at the end of the experiment (although small advances were arranged for some subjects), and that they would be subject to a penalty for missing more than two sessions (forfeiture of one session's average pay per additional session missed) or for early withdrawal from the experiment (forfeiture of one session's average pay per session remaining in the contract). The penalties were approved by the Emory University Human Subjects Committee and meet APA guidelines regarding informed consent. Subjects also were advised that they might be observed during sessions through the viewer in the door. To ensure that subjects did not have timepieces in the experimental room, they were told that metal jewelry might interfere with the operation of the equipment, and they were asked to leave such items with the experimenter.

For H31 and H37, lever pressing was initiated by instructions. At the start of the first session these 2 subjects were read the following instructions:

> This is a situation in which you can earn money. The green light will be on for the entire session, about 70 minutes. You earn money simply by pressing this lever. You can tell whether or not you have pressed hard enough by listening for the click from inside the machine. Look at these blue lights. When the houselight and a blue light are on, it means that you are able to earn money. At the beginning of the session, one of the lights will come on and it will remain on for about 10 minutes. Throughout this time you will be able to earn money. Sometimes when you press the lever this yellow light will flash and a tone will sound. Each time this happens, 3.75 cents [4.25 cents for H37] will be added to the total. The total amount you have earned will be shown on this counter. After 10 minutes, all lights will go off for about five minutes. During this time, you are to stay in this room and rest. Do not read during this period or leave the room. After the rest period, another blue light will come on and you can earn more money. Then there will be another rest period and so on until all five lights have been presented. At the end of the session, I will take a reading from the counter and give you a receipt for the money you have earned. You will be paid in a lump sum at the end of the experiment.

Questions at the first and all subsequent sessions were answered by rereading relevant portions of the instructions.

For H32 and H36, lever pressing was shaped by successive approximation. Subject H32 required two training sessions; H36 required four. At the start of the first training session both subjects were read the following instructions:

> This is a situation in which you can earn money. The green light will be on for the entire session, about 70 minutes. The total amount of money you have earned will be shown on this counter. At the end of the session I will take a reading from the counter and give you a receipt for the money you have earned. You will be paid in a lump sum at the end of the experiment. Please do not ask for additional information about what you are to do.

The subjects' behavior was then observed through the viewer in the door and lever pressing was shaped by hand. Both subjects pressed the lever within 10 min of the start of the session. For each subject, each of the first 25 lever presses was reinforced and then five VR schedules were arranged, each of which remained in effect for 10 reinforcers. The VR schedules had mean ratio requirements ($\bar{n}$) of 5, 10, 15, 25, and 50 responses, and they were presented in order of increasing $\bar{n}$. The schedules were not accompanied by discriminative stimuli, and transitions between them were unsignaled. Subject H32's second training session consisted of the procedure and schedules of Phase 1, except that the schedules were presented in order of increasing $\bar{n}$. After these two training sessions, Phase 1 began for H32. In H36's second and third training sessions, various VR schedules were presented in order of increasing $\bar{n}$. The schedules gradually increased to larger $\bar{n}$s and finally culminated in a VR 150. The VR schedules were not accompanied by discriminative stimuli, and transitions between them were unsignaled. The fourth training session for H36 consisted of the procedure and schedules of Phase 1, except that the schedules were presented in order of increasing $\bar{n}$. After these four training sessions, Phase 1 began for H36.

All subjects participated in two sessions per day. Sessions in each phase continued until a subject's response rates across sessions showed no trends at any schedule value. There was no break between phases and there were no stimulus changes or additional instructions at the start of Phase 2. Some of the procedural details of the experiment, including the total number of sessions in each phase, are summarized in Table 1.

RESULTS

All subjects satisfied the terms of their contracts such that no penalties were incurred.

Each subject's reinforcement and response rates were averaged over the last five sessions on the VR and the VI-plus-linear-feedback schedules. The average rates are listed in Table 2, along with the standard errors of the response-rate means, and the obtained number of responses per reinforcement for the last five sessions on each schedule. On most of the VR and VI-plus-linear-feedback schedules, the obtained number of responses per reinforcement agreed fairly closely with the scheduled $\bar{n}$. The disagreements for H36 at $\bar{n} = 240$ on both the VR and the VI-plus-linear-feedback schedules, and for H37 at $\bar{n} = 240$ on the VI-plus-linear-feedback schedule, were due to sampling error. Only a few response-

Table 1

Reinforcer magnitude, method of response initiation, type of schedule (VI+ = VI-plus-linear-feedback schedule), and total number of sessions for each subject in the two phases of the experiment. For each subject, reinforcer magnitude was the same in both phases.

Subject	Reinforcer magnitude (¢/reinforcer)	Responding initiated by	Schedule type (Phase 1, Phase 2)	Number of sessions (Phase 1, Phase 2)
H31	3.75	instructions	VR, VI+	12, 12
H32	3.75	shaping	VI+, VR	14, 10
H36	4.25	shaping	VR, VI+	20, 9
H37	4.25	instructions	VI+, VR	18, 8

per-reinforcement units accumulated during the last five sessions on these schedules (cf. H37's closer agreement at $\bar{n}$ = 240 on the VR schedule).

The average response rates listed in Table 2 are plotted against $\bar{n}$ in Figure 1. Filled circles represent data from the VR schedules, and unfilled circles represent data from the VI-plus-linear-feedback schedules. Error bars (± 1 standard error) also appear, unless they were equal to or smaller than the diameter of the data point.

The data in Table 2 and Figure 1 show that response rate decreased with $\bar{n}$ for all subjects on both the VR and the VI-plus-linear-feedback schedules. The decline in response rate was much greater for H36 and H37 than for the other 2 subjects. The session-to-session variability in response rate at

Table 2

Obtained number of responses per reinforcement (rsp/rft), mean obtained reinforcement rate (rft/hr = reinforcements per hour), mean response rate (rsp/min = responses per minute), and the standard error (*SE*) of the mean response rate for each of the 4 subjects in the VR and the VI-plus-linear-feedback phases of the experiment. The quantity, $\bar{n}$, is the reciprocal of the slope of the linear feedback function (Equation 2). For the VR schedules, $\bar{n}$ also represents the average ratio requirement. All quantities in the table were calculated from unrounded data.

	VR			VI-plus-linear-feedback		
$\bar{n}$	rsp/rft	rft/hr	rsp/min ±*SE*	rsp/rft	rft/hr	rsp/min ±*SE*
			Subject H31			
15	14.8	493.2	121.9 ± 1.1	15.8	468.0	123.5 ± 0.6
30	28.2	255.6	120.1 ± 1.4	30.2	244.8	123.1 ± 0.8
60	52.5	135.6	118.6 ± 1.8	57.8	126.0	121.3 ± 1.0
120	122.1	56.4	114.8 ± 1.9	126.6	56.4	119.0 ± 1.3
240	196.4	34.8	113.9 ± 2.3	236.6	30.0	118.3 ± 1.4
			Subject H32			
15	15.0	495.6	124.0 ± 0.9	15.6	480.0	124.5 ± 2.1
30	30.8	230.4	118.2 ± 0.9	31.0	238.8	123.3 ± 0.8
60	62.8	111.6	116.8 ± 1.7	64.0	111.6	119.0 ± 0.7
120	135.5	50.4	113.8 ± 1.6	123.2	57.6	118.3 ± 0.7
240	277.0	24.0	110.8 ± 1.6	233.0	30.0	116.5 ± 0.8
			Subject H36			
15	15.1	496.8	124.7 ± 4.5	15.7	530.4	138.4 ± 1.1
30	29.9	223.2	111.2 ± 8.4	30.0	267.6	134.0 ± 5.0
60	58.6	116.4	113.6 ± 5.5	58.0	122.4	118.3 ± 12.3
120	120.6	54.0	108.6 ± 9.5	108.8	56.4	102.3 ± 13.8
240	679.0	2.4	27.2 ± 24.7	170.7	14.4	41.0 ± 17.9
			Subject H37			
15	15.0	584.4	146.0 ± 2.4	15.5	554.4	143.1 ± 4.3
30	30.4	289.2	146.4 ± 5.9	30.5	278.4	141.6 ± 5.0
60	58.0	145.2	140.4 ± 4.9	57.8	124.8	120.1 ± 15.5
120	113.3	52.8	99.7 ± 18.1	121.6	60.0	121.6 ± 7.0
240	211.5	7.2	25.4 ± 25.4	415.3	7.2	49.8 ± 26.3

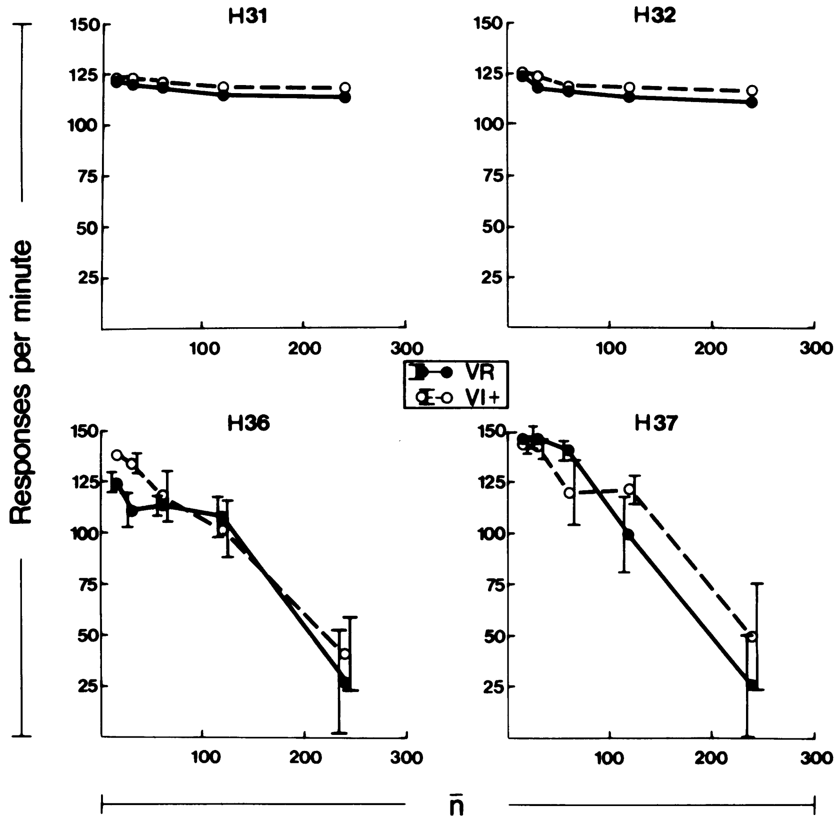

Fig. 1. Response rate on VR (filled circles) and VI-plus-linear-feedback (VI+; unfilled circles) schedules as a function of the reciprocal of the slope of the linear feedback function ($\bar{n}$). For the VR schedules, $\bar{n}$ is also the mean ratio requirement. Error bars, which were omitted when equal to or less than the diameter of the data point, represent ±1 standard error.

most $\bar{n}$s (indicated by the error bars) was also greater for these 2 subjects.

As shown in Figure 1, the VR schedules and the VI-plus-linear-feedback schedules for a given subject produced similar rates of responding at a given value of $\bar{n}$. For H31 and H32, response rates on the VR and the VI-plus-linear-feedback schedules at a given $\bar{n}$ were nearly identical. For H36 and H37, differences between rates were usually accompanied by large, overlapping error bars. Thus, as is clear from the figure, the two types of schedule produced similar response rate versus $\bar{n}$ functions for each subject.

Similarities in behavior on the VR and the VI-plus-linear-feedback schedules also extended to the details of responding. Representative cumulative records for each subject are reproduced in Figures 2 through 5. Each of the four records in a figure shows a subject's final 50 min of responding on a schedule.

Figures 2 and 3 show H31's and H32's responding on the two types of schedule at $\bar{n}$ = 15 and $\bar{n} = 240$. Their performances were similar at the other $\bar{n}$s. Both subjects responded on all schedules at high, steady rates that decreased gradually as $\bar{n}$ increased. The pattern of responding on all schedules was nearly invariant from session to session. Subject H31 did not pause during the final 50 min on any schedule. Subject H32's final 50 min on both the VR and the VI-plus-linear-feedback schedules showed infrequent, brief pauses that ended in abrupt transitions to the response rate that prevailed before the pause. These pauses can be seen in all four records in Figure 3. At each $\bar{n}$, H31's and H32's cumulative records on the VR and the VI-plus-linear-feedback schedules were indistinguishable, as the examples in Figures 2 and 3 show.

Reproduced in Figures 4 and 5 are H36's and H37's cumulative records of responding

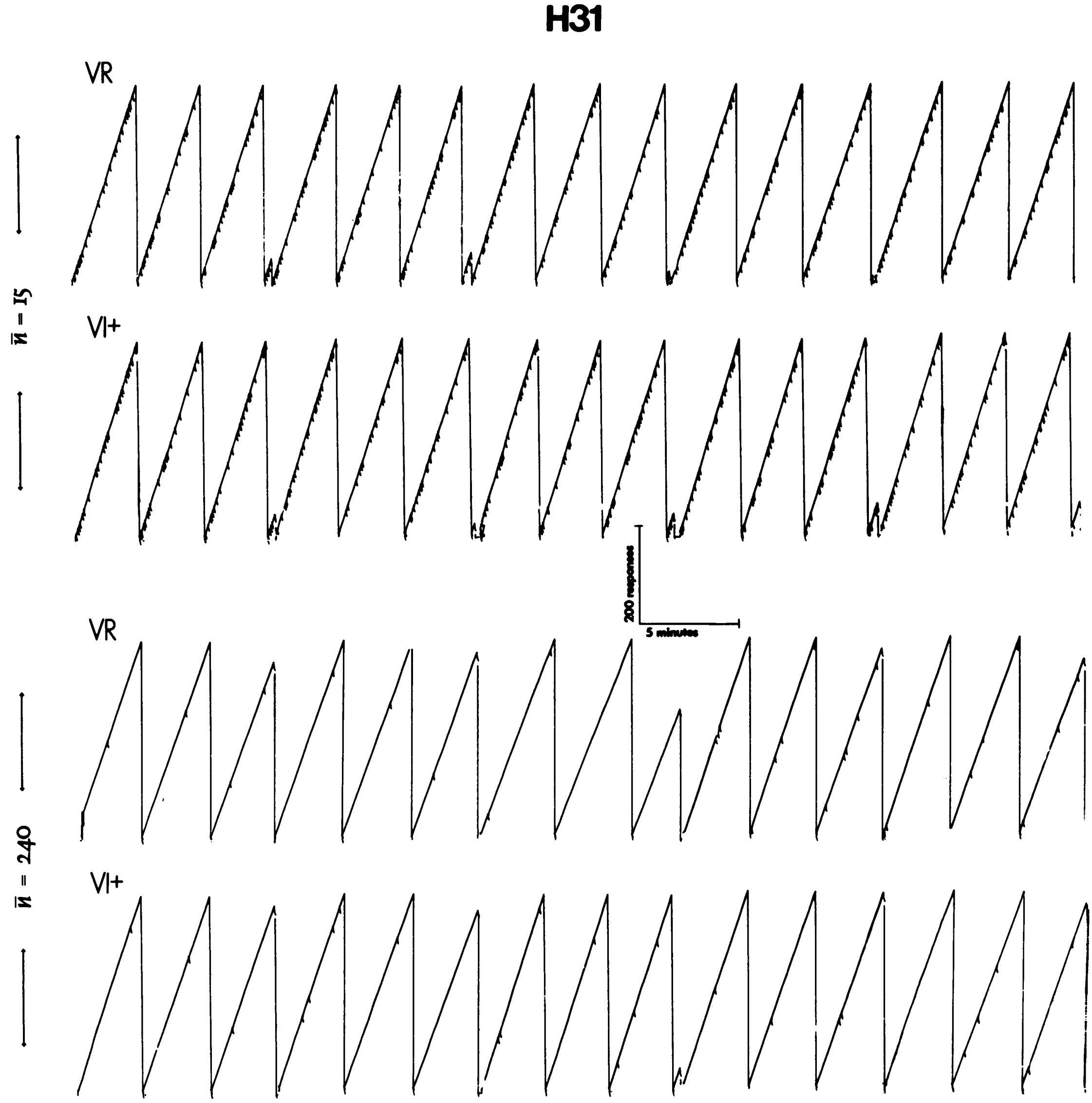

Fig. 2. Cumulative records of H31's final 50 min of responding on VR and VI-plus-linear-feedback schedules (VI+) at $\bar{n}$ = 15 (top two records) and $\bar{n}$ = 240 (bottom two records). The pen reset every 10 min, and within 10-min periods it reset every 400 responses. Downward deflections of the pen indicate reinforcer deliveries.

at $\bar{n}$ = 15 and $\bar{n}$ = 240 on the VR and the VI-plus-linear-feedback schedules. At $\bar{n}$ = 15, H36 and H37 responded on both types of schedule at high steady rates. The pattern of responding was fairly constant from session to session. Subject H36 occasionally showed pauses and rate fluctuations. Very infrequently, H37 showed brief pauses that ended in abrupt returns to the prevailing response rate. The performances of these 2 subjects at $\bar{n}$ = 30, 60, and 120 were similar, except that the frequency and duration of the pausing increased somewhat at the larger $\bar{n}$s. At $\bar{n}$ = 240, H36 and H37 showed strained responding on both types of schedule. Response rates were either zero or high and steady, with occasional periods of fluctuating, intermediate rates. At each $\bar{n}$, H36's and H37's cumulative records on the VR and the VI-plus-linear-feedback schedules were very similar, as the examples in Figures 4 and 5 show.

DISCUSSION

For each subject, the VR and the VI-plus-linear-feedback schedules produced similar

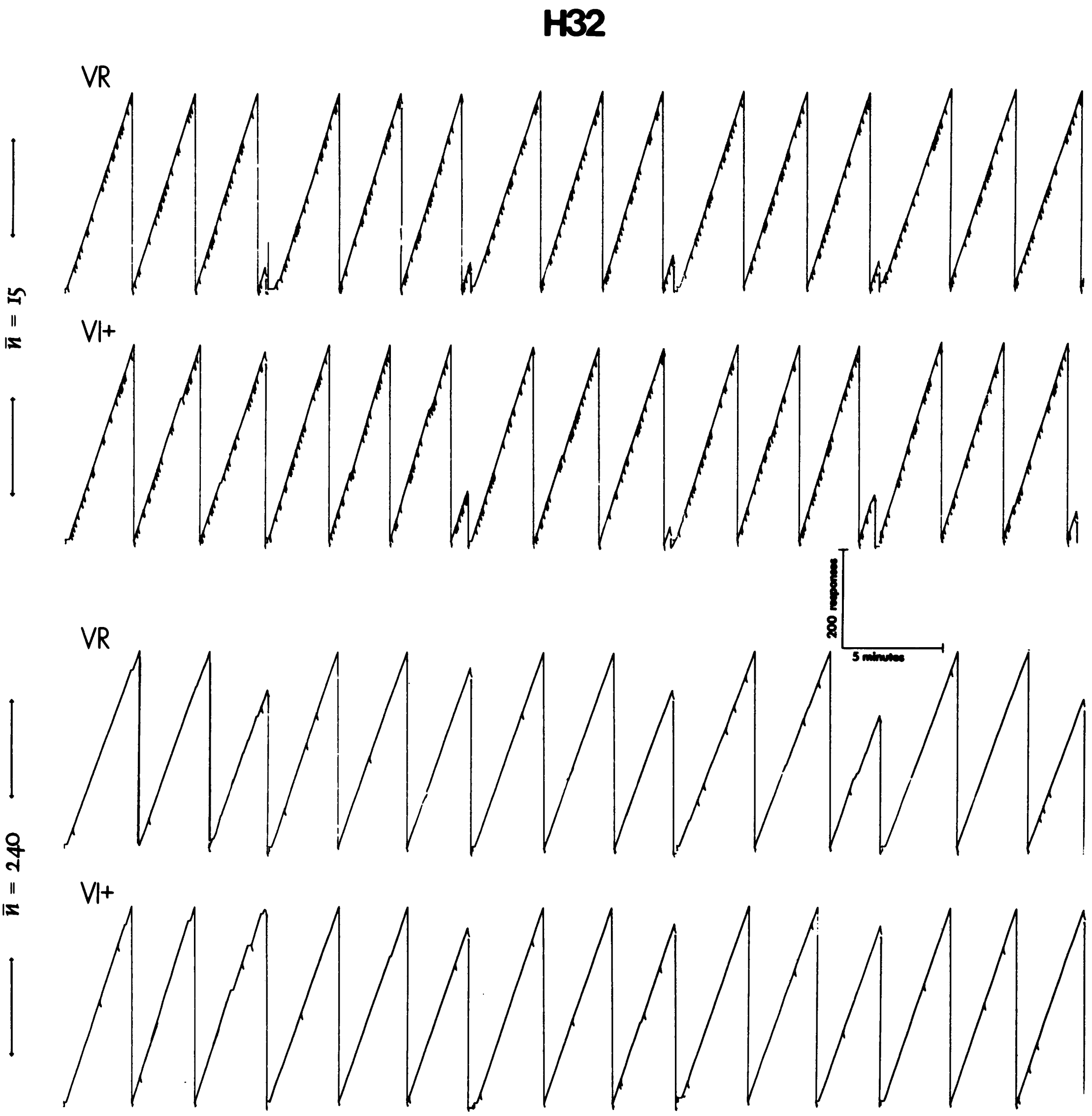

Fig. 3. Cumulative records of H32's final 50 min of responding on VR and VI-plus-linear-feedback schedules (VI+) at $\bar{n} = 15$ (top two records) and $\bar{n} = 240$ (bottom two records). The pen reset every 10 min, and within 10-min periods it reset every 400 responses. Downward deflections of the pen indicate reinforcer deliveries.

average response rates at each value of $\bar{n}$, similar response rate versus $\bar{n}$ functions, and similar patterns of responding. The behavior of H36 and H37 differed from that of the other 2 subjects in three ways: They showed a greater decline in response rate with $\bar{n}$, more session-to-session variability in response rate, and strained responding at $\bar{n} = 240$ on both the VR and the VI-plus-linear-feedback schedules. These between-subject differences cannot be accounted for by the order of presentation of the two types of schedule, or by the method of initiating responding. The only procedural detail that could have been responsible is the magnitude of the reinforcer, which was slightly larger for H36 and H37 than for the other 2 subjects. It is also possible that these differences simply reflect between-subject variability. Ferster and Skinner (1957) and Brandauer (1958) both reported considerable between-subject variability in the behavior of pigeons on VR schedules. For example, in both experiments, sustained responding was observed in some birds but not in others on VR schedules with high mean ratio requirements ($\bar{n} = 360$, 400, and 600).

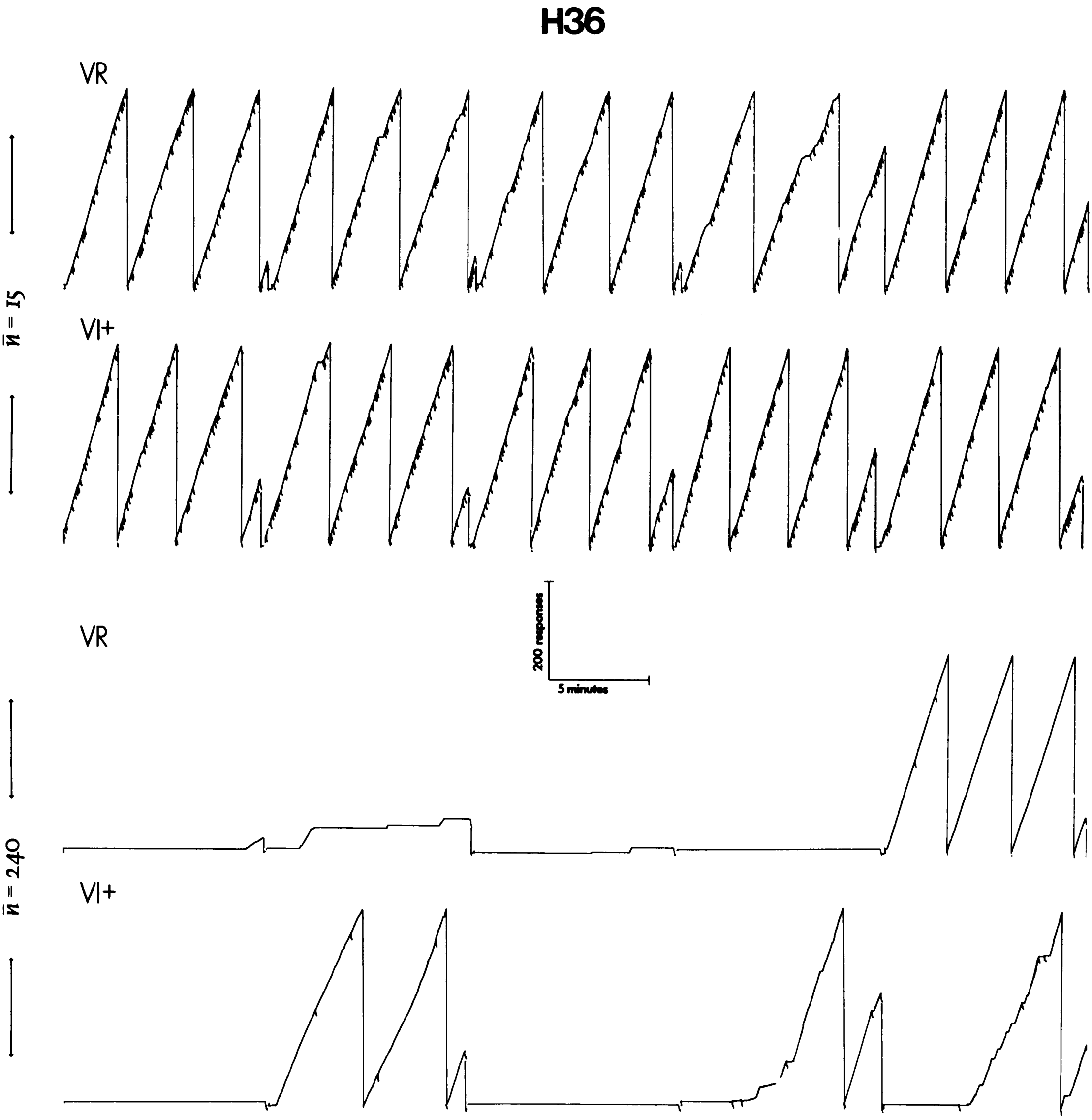

Fig. 4. Cumulative records of H36's final 50 min of responding on VR and VI-plus-linear-feedback schedules (VI+) at $\bar{n}$ = 15 (top two records) and $\bar{n}$ = 240 (bottom two records). The pen reset every 10 min, and within 10-min periods it reset every 400 responses. Downward deflections of the pen indicate reinforcer deliveries.

An additional finding of the present experiment is that the two methods used to initiate responding, namely, instructions and shaping by successive approximation, did not have differential effects on behavior (cf. Matthews, Shimoff, Catania, & Sagvolden, 1977; Shimoff, Catania, & Matthews, 1981).

Certain burst–pause patterns of responding could have been differentially reinforced on the VI-plus-linear-feedback schedules in this experiment. For example, a pattern consisting of a few rapid responses after reinforcement, followed by a pause, would have been differentially reinforced. No burst–pause patterns of responding were apparent in any subject's cumulative records, although a fine-grained analysis could not be carried out because individual response times were not recorded.

The response rates on the VR and the VI-plus-linear-feedback schedules in this experiment can be compared with response rates of human subjects on ordinary VI schedules. Four series of VI schedules from McDowell and Wood's (1984) study of monetarily rein-

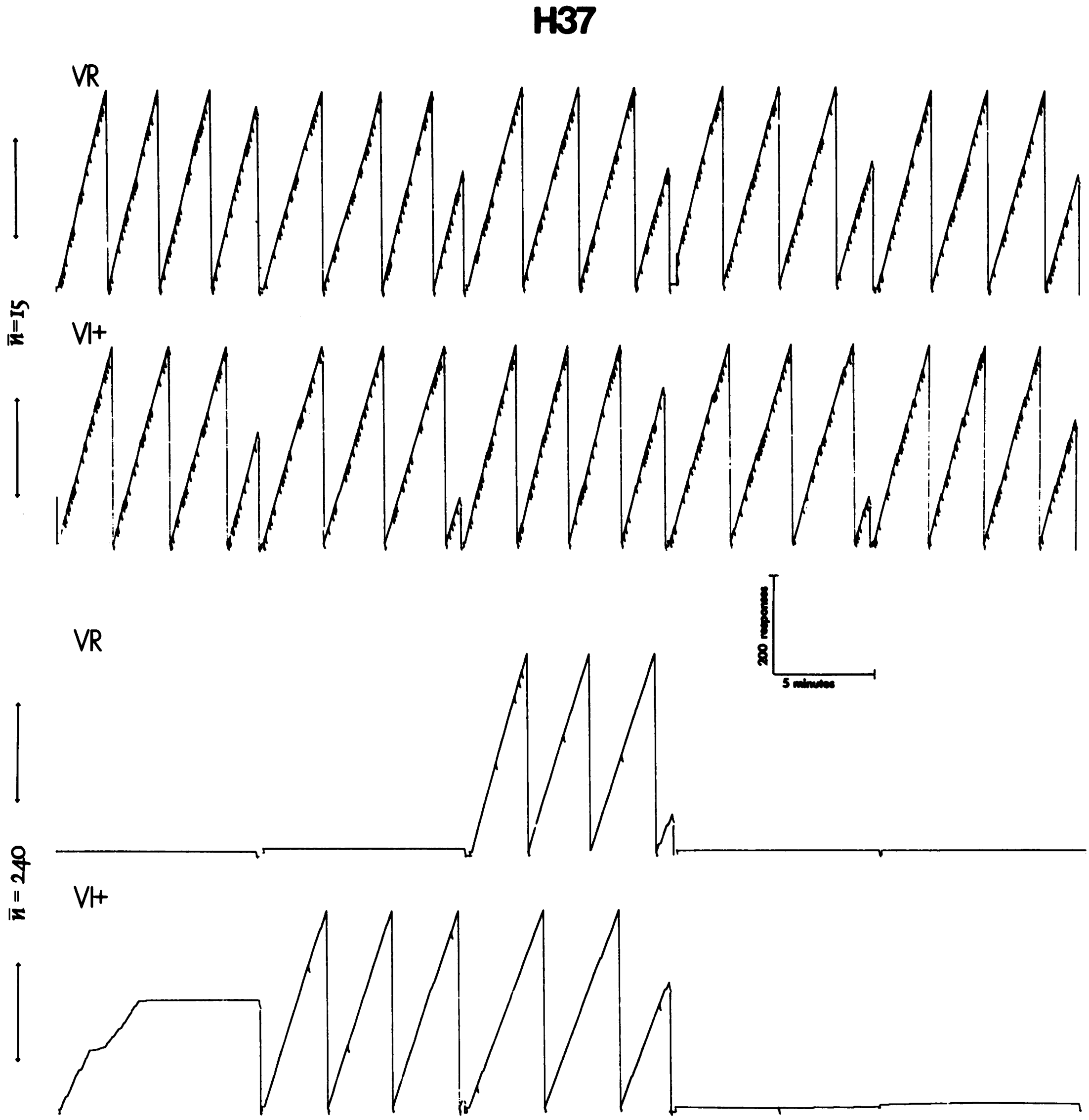

Fig. 5. Cumulative records of H37's final 50 min of responding on VR and VI-plus-linear-feedback schedules (VI+) at $\bar{n}$ = 15 (top two records) and $\bar{n}$ = 240 (bottom two records). The pen reset every 10 min, and within 10-min periods it reset every 400 responses. Downward deflections of the pen indicate reinforcer deliveries.

forced lever pressing in humans entailed force requirements and reinforcer magnitudes comparable to those used in the present experiment. These series were from H18, H19, H20, and H23, where reinforcer magnitude was 4¢. The median asymptotic response rate (i.e., the median estimate across subjects of Herrnstein's [1970] *k*) on these series was 69.4 responses/min. The median of the eight highest response rates from the present experiment (one per type of schedule per subject) was 124.6 responses/min. The latter median, which exceeds 2 responses/s, was about 80% greater than the median asymptotic response rate on the VI schedules. This result is consistent with earlier findings of higher response rates on VR than on VI schedules in human subjects (Matthews et al., 1977).

The behavior of the human subjects in the present experiment was similar to that of avian and rodent species on VR schedules. For example, response rates of pigeons (Brandauer, 1958 [excluding fixed-ratio 1 schedules]; Green, Kagel, & Battalio, 1982) and rats

(Mazur, 1983) on VR schedules have been found to decrease from the low to the high end of the $\bar{n}$ range. Similarly, high, steady rates of responding on VR schedules have been reported in both pigeons (Brandauer, 1958; Ferster & Skinner, 1957) and rats (Mazur, 1983). Higher response rates on VR than on VI schedules have also been reported in pigeons (Catania, Matthews, Silverman, & Yohalem, 1977; Ferster & Skinner, 1957; Zuriff, 1970) and rats (Mazur, 1983). In addition, many of the response details observed in the behavior of the human subjects in the present experiment have also been reported in the behavior of other species. For example, Ferster and Skinner observed the following details in pigeons' key pecking on VR schedules: (1) brief pauses that ended abruptly in returns to the prevailing rate of responding, (2) occasional periods of fluctuating, intermediate response rates in otherwise steady, high-rate records, (3) some between-session variability in overall response rate, and (4) behavior at high mean ratio values ($\bar{n} = 360$) that consisted of zero or very rapid response rates with occasional periods of fluctuating, intermediate rates (i.e., strained responding). Brandauer (1958) reported similar details of responding in his pigeons' key pecking, including an increased frequency of pausing at larger values of $\bar{n}$. Mazur (1983) reported brief, abruptly ending pauses in his rats' lever pressing, and an increased frequency of pausing at larger values of $\bar{n}$. In addition, as noted above, both Ferster and Skinner, and Brandauer reported considerable between-subject variability in the behavior of their pigeons. These similarities in behavior across species are especially noteworthy in view of the different experimental procedures that were used with the different subject species. For example, consummatory reinforcers were used in the experiments with pigeons and rats, and it is unlikely that these subjects had significant uncontrolled extraexperimental histories of reinforcement and punishment. By contrast, nonconsummatory reinforcers were used in the present experiment, and the subjects' uncontrolled extraexperimental histories were extensive.

Interresponse-time theories of schedule performance (Zeiler, 1977) are not supported by the results of the present experiment. According to one version of IRT theory (Ferster & Skinner, 1957; Morse, 1966; Skinner, 1938), behavior interacts with interval and ratio contingencies to produce the characteristic VI and VR performances. More specifically, because responses tend to be emitted in clusters, reinforcement on ratio schedules is likely to occur after short IRTs whereas reinforcement on interval schedules is likely to occur after long IRTs. This differential reinforcement is said to produce the relatively high response rates on VR schedules and the relatively low response rates on VI schedules. A second version of IRT theory (e.g., Peele et al., 1984) ignores properties of responding and simply asserts that VI schedules differentially reinforce long IRTs because the probability of reinforcement on a VI schedule increases with IRT duration (up to the limiting value of unity). By a similar argument, variable-ratio schedules are said not to differentially reinforce any class of IRTs because the probability of reinforcement on a VR schedule is unrelated to IRT duration. Clearly, this version of IRT theory also requires response rates on VI schedules to be lower than those on VR schedules. A third version of IRT theory (e.g., Zeiler, 1979) entails two opposing processes. One is the tendency of reinforcement to "strengthen" behavior, that is, to increase its rate of occurrence. This tendency is present for both VI and VR responding. On VI schedules, however, the strengthening effect of reinforcement is partially counteracted by the differential reinforcement of long IRTs. On VR schedules, the strengthening effect operates unopposed. Again, the result is lower response rates on VI than on VR schedules. All three versions of IRT theory require a relatively low rate of responding to develop whenever a time-based, interval contingency is in effect, and a relatively high rate of responding to develop whenever a response-based, ratio contingency is in effect. The critical factor in generating the response-rate difference is the type of contingency (i.e., interval vs. ratio). According to IRT theories, a relatively low rate of responding should develop on a VI-plus-linear-feedback schedule (as compared to a VR schedule) because it arranges interval contingencies, which ensure higher probabilities of reinforcement for longer IRTs. The present results are contrary to this prediction and, consequently, confute all three versions of IRT theory. Zeiler (1979) has noted that data from differential-reinforcement-of-low-rate schedules also fail to support the theory.

Interresponse-time theory has also been

used to account for the empirical observation that behavior can be maintained better by a lean VI schedule than by a VR schedule with a large mean ratio requirement, all else being equal (Zeiler, 1977). Interval, but not ratio, contingencies are said to be capable of "regenerating" responding (Zeiler, 1977, 1979) because they arrange higher probabilities of reinforcement for longer IRTs. As responding weakens on a lean VI schedule (i.e., as the IRT increases), reinforcement becomes more probable. The next response is likely to be reinforced, and so responding may be maintained, at least at a low rate. But as responding weakens (i.e., as the IRT increases) on a VR schedule with a large mean ratio requirement, reinforcement does not become more probable. Hence, responding may become strained or it may cease altogether. The results of the present experiment are not in agreement with this account. Subjects H36 and H37 showed strained responding at $\bar{n} = 240$ on both the VR and VI-plus-linear-feedback schedules (Figures 4 and 5), even though the interval contingencies arranged by the latter schedule ensured higher probabilities of reinforcement for longer IRTs. According to IRT theory, these interval contingencies should have "regenerated" responding, that is, produced a roughly steady, if low, response rate.

The present results also appear to be inconsistent with findings recently reported by Peele et al. (1984, Experiment 3), who studied the responding of pigeons on a two-component multiple schedule. One component of the schedule was a tandem VI VR and the other was a tandem VR VI. The ratio schedule in each component was a VR 100. The IRI of the VI schedule in each component was yoked to the duration of the VR in the other component such that the durations of the two multiple-schedule components were equal. According to Peele et al., the feedback functions in the two components were identical. Of course the components differed in that one ended in a VR and the other ended in a VI. Given identical feedback loops, a feedback account predicts equal rates of responding in the two components, whereas IRT theory predicts a higher response rate in the tandem VI VR component. Peele et al. found a higher response rate in the tandem VI VR component, and they concluded that IRT theory was supported. But this conclusion is questionable because of a confound introduced by the yoking procedure. The between-component yoking ensured that reinforcement in one component depended not only on responding in that component, but also on responding in the other component. For example, rapid responding on the VR in one component reduced the duration of the VI in the other. Hence, reinforcement in each component was not a function of responding in that component only, but was a joint function of responding in both components. Because the within-component feedback was confounded with the between-component feedback, Peele et al.'s results are difficult to interpret. Even if the between-component feedback is ignored, however, there is a second problem, namely, that the feedback functions in the two tandem components actually may have been different. The data in their Table 5 show that similar reinforcement rates were produced by different response rates. If the two feedback functions were identical, this could happen only if responding fell in a region where the slope of the feedback function was nearly zero. But if the two feedback functions were different, similar reinforcement rates would be produced, as a rule, by different response rates. Because the exact forms and parameter values of the feedback functions cannot be determined from Peele et al.'s data, the possibility that the two feedback functions were different cannot be ruled out.

Contrary to IRT theories, the results of the present experiment suggest that the principal difference between VI and VR schedules is the linear feedback that the latter arrange. In other words, the assumption entailed by the VI-plus-linear-feedback account of the VR case is supported. This indicates that the VI-plus-linear-feedback approach is worth further study. The next step, which is currently in progress, is to work out the formal details of the theory. This entails finding the simultaneous solution sets of Equations 1 and 2, and determining their properties. The theory must be able to describe the form of the response rate versus $\bar{n}$ function, and to account for various known properties of VR responding, such as ratio strain and high response rates.

As a final point, we note that the method of arranging feedback used here has additional applications. For example, just as linear feedback can be added to a VI schedule, it can be removed from a VR schedule. This is accomplished by solving Equation 2 for $\bar{n}$ and setting $1/R_{IN}$ equal to the mean IRI of a comparison VI. The resulting equation is then used to calculate $\bar{n}$ and the current ratio requirement after each response. More generally, it is worth noting that the present method of arranging feedback is not restricted to linear relationships. Indeed, feedback functions of any form whatsoever can be arranged simply by writing the desired equation in place of Equation 2. In addition, the operation of the feedback loop itself can be studied by varying the width of the feedback window, which is the amount of time or number of responses used to calculate the response rate upon which the loop operates. The present method can also be used in the absence of molecular response-reinforcement contingencies. For example, a feedback loop of any desired form can be added to a variable-time schedule to produce an environment that entails only the molar rate contingency arranged by the feedback function itself (cf. Baum, 1973).

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