

A QUANTITATIVE ANALYSIS OF EXTREME CHOICE

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Six homing pigeons were trained on a variety of concurrent variable-interval schedules in a switching-key procedure. Unlike previous work, reinforcer ratios of up to 160 to 1 and concurrent extinction variable-interval schedules were arranged in order to investigate choice when reinforcer-frequency outcomes were extremely different. The data obtained over 11 conditions were initially analyzed according to the generalized matching law, which fitted the data well. The generalized matching law was then fitted only to conditions in which the reinforcer ratios were between 1 to 10 and 10 to 1. The deviations of choice measures from the other four more extreme reinforcer-ratio conditions were significantly more towards equal choice than predicted by this second generalized matching fit. A contingency-discriminability model, which predicts such deviations, described the data more effectively than did the generalized matching law, and also correctly predicted the maintenance of responding on both alternatives when one was associated with extinction.

Key words: concurrent variable-interval schedules, choice, generalized matching, contingency-discriminability model, reinforcer sensitivity, key peck, pigeons

A considerable number of experiments concerned with the quantitative analysis of behavior and time allocation on concurrent variable-interval (VI) schedules of reinforcement have been reported in the last 36 years. This work was summarized by Davison and McCarthy (1988). Initially, it was thought that relative behavior allocation equaled (or matched) the relative reinforcers obtained from the response alternatives (see the summary by Herrnstein, 1970). This relation is termed the strict matching law. Later, consistent deviations from strict matching became obvious, and Baum (1974) replaced the strict matching law by the generalized matching law, in which behavior ratios were a power function of obtained reinforcer ratios. In logarithmic terms, the generalized matching law is written

$$\frac{B_1}{B_2} = c \left(\frac{R_1}{R_2} \right)^a, \quad (1)$$

where B and R denote behavior and reinforcer measures, respectively, and the subscripts denote the two alternatives. The constant a is

called sensitivity to reinforcement (Lobb & Davison, 1975); it measures the change in choice relative to the change in obtained reinforcer ratios. The second constant, c , is called inherent bias; it represents a constant proportional preference for one alternative over the other as the independent variable (here, relative reinforcer rate) is varied. Equation 1 is usually fitted by transforming the equation to a straight line by taking logarithms of both sides of the equation

$$\log \left(\frac{B_1}{B_2} \right) = a \log \left(\frac{R_1}{R_2} \right) + \log c. \quad (1b)$$

The generalized version of the matching law has generally fitted data well. However, recently a number of problems have been reported. These problems concern the constancy, or otherwise, of the value of a . First, both Alsop and Elliffe (1988) and Logue and Chavarro (1987) showed that the value of a was not independent of overall reinforcer rate, but increased with increasing reinforcer rates. Thus, it is not simply the ratio of obtained reinforcers that controls behavior allocation. Second, Logue and Chavarro also reported that sensitivity to reinforcer magnitude and reinforcer delay in the concatenated version of the generalized matching law was not independent of the absolute values of these independent variables. Third, Davison (1988) showed that sensitivity to reinforcer magnitude was affected by overall reinforcer rate. Fourth, Davison and Hogsden (1984)

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showed that the relation between logs of behavior ratios and logs of reinforcer-magnitude ratios was not linear. Finally, Miller, Saunders, and Bourland (1980) showed that sensitivity to relative reinforcer frequency was a function of the physical disparity between the stimuli signaling the alternative responses. Taken together, these results suggest that the sensitivity of choice to changes in the rate, magnitude, or delay of reinforcers is not an organismic constant. They show, rather, that sensitivity varies as a function of the contextual environment. The generalized matching relation, then, does not have a particularly compelling status as a law. However, if sensitivity is taken as a variable, the generalized matching law can have a useful descriptive function.

An alternative to the generalized matching law was offered by Davison and Jenkins (1985). This contingency-discriminability model was designed to account for Miller et al.'s (1980) data. It assumes that changing the disparity between the discriminative stimuli changes the degree to which animals can differentiate between the different responses that led to different consequences. The model described Miller et al.'s data well and provided reasonable measures of the discriminability between the discriminative stimuli and the reinforcer rates that they signal. The model as offered was written

$$\frac{B_1}{B_2} = c \left(\frac{d_r R_1 + R_2}{d_r R_2 + R_1} \right), \quad (2)$$

where the variables B and R are the same as in Equation 1, and d_r is a measure of the discriminability between the alternative response-reinforcer relations. The value of d_r can range from 1 (no discrimination) to infinity (perfect discrimination). Algebraic manipulation of Equation 2 provides an equivalent equation that better indicates the operation of the model:

$$\frac{B_1}{B_2} = c \left(\frac{R_1 - pR_1 + pR_2}{R_2 - pR_2 + pR_1} \right). \quad (3)$$

In Equation 3, p is the inverse of the discriminability measure d_r in Equation 2. It describes the proportional *confusion* (the inverse of discriminability) between the two reinforcer contingencies. The value of p can vary from 0 (no confusion, and strict matching bi-

ased by c) to .5 (complete confusion, and no control by changing reinforcer rates, biased by c). Equation 3 asserts that, under less than perfect contingency discriminability, the reinforcer tally on Alternative 1 loses a proportion of reinforcers to Alternative 2, and a proportion of the tally on Alternative 2 is lost to Alternative 1. Thus, according to Equation 3, the subject's behavior allocation is controlled by the *apparent* reinforcers gained at the alternatives rather than by the reinforcer tallies (R_1 and R_2) measured by the experimenter. Davison and McCarthy (1994) showed that this approach was able to describe performance on two- and three-key concurrent VI schedules when the differences between the colors signaling the alternatives were varied. The model defined by Equation 3 is similar to the model offered by Vaughan and Herrnstein (1987), and it is related to models offered by Wearden (1983) and Burgess and Wearden (1986).

If sensitivity to reinforcement (a in Equations 1 and 1b) is taken as a variable rather than a constant, Equations 1 and 3 provide very similar predictions of choice as a function of alternative reinforcer rates over the range of reinforcer ratios typically studied, even though Equation 3 is nonlinear on log-log coordinates (Davison & Jenkins, 1985). One way to compare these models is to arrange very large differences between the two reinforcer rates on concurrent VI VI schedules, because the differences between the predictions of Equations 1 and 3 increase as the reinforcer ratio becomes more extreme. Equation 3 is ogival, and predicts log behavior ratios that, although continuing to increase with increasing log reinforcer ratios, become progressively less extreme than the predictions of Equations 1 and 1b. The available literature on concurrent VI VI performance contains no systematic variation of log reinforcer ratios greater than -1 or 1 (1 to 10 , or 10 to 1) with which to compare the predictions of the generalized matching law and the contingency-discriminability model. However, the literature does provide a number of reports of performance on concurrent VI extinction schedules, and these are informative. Even when changeover delays (times between changing over and subsequent reinforcer deliveries) are arranged, responding does not seem to cease on the extinction al-

ternative (e.g., Davison & Hunter, 1976). This result is apparently inconsistent with Equations 1 and 1b, which predict exclusive choice, but is consistent with Equation 3 if the two response-reinforcer contingencies are not perfectly discriminable ($p > 0$ in Equation 3). According to Equation 3, if there is any confusion between these contingencies, reinforcers will apparently be available on the extinction alternative. If confusion is complete ($p = .5$), VI and extinction response rates will be equal if there is no bias ($c = 1.0$). More generally, if $R_2 = 0$, then

$$\frac{B_1}{B_2} = c \left(\frac{1-p}{p} \right). \quad (4)$$

In the present experiment, we found the best fitting values of p by fitting Equation 3 to concurrent VI VI data, and used the obtained values to predict response ratios in a concurrent extinction VI condition.

Whichever model of behavior allocation is correct, it is clear that performance under extreme reinforcer-rate differences must provide important confirmatory evidence. The present experiment investigated choice over a range of reinforcer ratios that extended from the usual range to 160 to 1, and included a concurrent extinction VI condition.

METHOD

Subjects

Six homing pigeons, numbered 21 to 26, were maintained at $85\% \pm 15$ g of their free-feeding body weights. They were experimentally naive at the start of the experiment. Water and grit were always available in their home cages, and they were fed an appropriate amount of mixed grain immediately after the daily training sessions to maintain their designated body weights.

Apparatus

The subjects worked in a standard sound-attenuating experimental chamber that was 330 mm high, 300 mm wide, and 330 mm deep. Two pecking keys, 20 mm in diameter, were situated on one wall of the chamber, 130 mm apart and 260 mm above the grid floor. The keys required about 0.1 N for operation. Centered on the same wall, and 100 mm from the floor, was a food magazine that contained

wheat. During reinforcement, the keys were darkened and the food magazine was raised and lit for 3 s. There was no chamber illumination other than the lit keys and the magazine light.

All experimental contingencies were controlled by a remote PC-compatible computer running MED-PC® software, and all data were collected within the program.

Procedure

The pigeons were slowly deprived of food before being trained to eat from the food magazine when it was presented. An auto-shaping procedure was then used to train the pigeons to peck keys transilluminated by red light. When they were reliably pecking, they were trained on VI schedules on the left and right keys, and then on the switching-key concurrent schedule described below.

During the session, the left key was transilluminated with one of two intensities of yellow light, and the right key was illuminated by red light. In the switching-key procedure, pecks on the left (main) key were reinforced on various pairs of VI or extinction schedules that were changed over conditions, and pecks to the right (switching) key changed the intensity of the left keylight and the associated schedule. After a switching-key peck, there was a changeover delay (Herrnstein, 1961) of 3 s, during which time no reinforcers could be obtained for pecking the left key. Effective changeovers could be emitted during the changeover delay. The concurrent schedules on the left key were arranged according to a dependent-scheduling procedure (Stubbs & Pliskoff, 1969). In this procedure, a reinforcer that had been arranged for one alternative stopped the timing of the VI schedule associated with the other alternative until that reinforcer had been obtained. An overall reinforcer rate of two per minute (VI 30 s) was arranged on the main key throughout all conditions while the relative frequency of reinforcers obtained on the schedule associated with the brighter left-key alternative was varied (see Table 1). Sessions commenced in blackout, lasted until 40 reinforcers had been obtained or until 45 min had elapsed, whichever occurred first, and ended in blackout.

A condition remained in effect for all 6 subjects until a stability criterion had been met for each subject individually. The crite-

Table 1

Sequence of experimental conditions, relative reinforcer frequency in the presence of the brighter stimulus, and the number of training sessions in each condition. The overall reinforcer rate arranged was 0.0333 per second (two per minute) throughout.

Condition	Probability of reinforcement on brighter alternative	Sessions
3	.9000	28
4	.0476	22
5	.9756	20
6	.0124	23
7	.9938	30
8	.3333	21
9	.8000	20
10	.1000	22
11	.9000	20
12	.0000	33

tion was this: After 10 training sessions had been completed on a condition, relative response rates from sets of three sessions (starting at 10, 11, and 12) were assessed for a monotonic trend. Experimental conditions were changed for all subjects when a monotonic increase or decrease had been absent on five occasions (not necessarily consecutively) for all of the subjects. Thus, a minimum of 16 sessions was required per experimental condition. The minimum number obtained was 20 (see Table 1).

The data collected were the times of each response and reinforcer event in each experimental session. The data from the last five sessions of a condition were used for analysis, except in Conditions 7 and 12. In these conditions, the data used were those from the last five sessions before all subjects achieved stability, plus an additional 10 sessions (Condition 7) or an additional five sessions (Condition 12). This procedure was used because some subjects had not obtained any reinforcers on the dimmer alternative when the usual number of sessions' data had been collected. It was used in Condition 12 to make certain that performance was stable. In the first two conditions, the intensities of yellow light on the main key were 4.6 cd/m² and 0.75 cd/m². These data are not analyzed here. From Condition 3 on, these intensities were 0.9 and 0.75 cd/m².

RESULTS

The data summed over the last five sessions of each condition are shown in the Appendix. Figure 1 shows log response ratios plotted as a function of log obtained reinforcer ratios for all subjects in Conditions 3 to 11, as appropriate for Equation 1b. Also shown are the best fitting straight lines obtained by least squares linear regression of Equation 1b to the obtained data. The parameters of the fitted lines are given in Table 2. It is evident from Table 2 that Equation 1b described the data well, with high proportions of the data variance accounted for. The fitted lines had lower slopes (a in Equation 1b) than those generally found in concurrent VI VI performance (e.g., Baum, 1979; Taylor & Davison, 1983). Bias values were close to zero. The low sensitivity values could, of course, have resulted simply from using discriminative stimuli that were not maximally disparate, as was done by Miller et al. (1980). In Figure 1, there appears to be little evidence of systematic deviations, towards the ogivality predicted by Equation 3, from the lines of best fit to Equation 1b. It could, therefore, be concluded at this point that Equation 1b was an effective description of the present data. Despite this excellent fit, we conducted further analyses because the effect expected from Equation 3 might be relatively small, depending on the discriminability between the stimuli signaling the alternatives.

Figure 2 shows a further analysis according to Equation 1b. In this analysis, we fit Equation 1b to the five obtained log reinforcer ratios closest to a log reinforcer ratio of zero in Figure 1. These points all fall within the range of log reinforcer ratios between -1 and $+1$ in Figure 1. If the generalized matching law is as applicable to extreme choice as it is to moderate choice, the four unfitted extreme data points should not deviate in any systematic way from the line fitted to the central data. The fits obtained when Equation 1b was fitted to only the central data are shown in Table 2. The slopes of the fitted line increased for all 6 subjects (but only slightly for Bird 21) compared with the overall generalized matching fit, indicating that the extreme data did not lie on the same fitted line as the central data. Furthermore, 18 of the 24 extreme data points deviated from the fitted

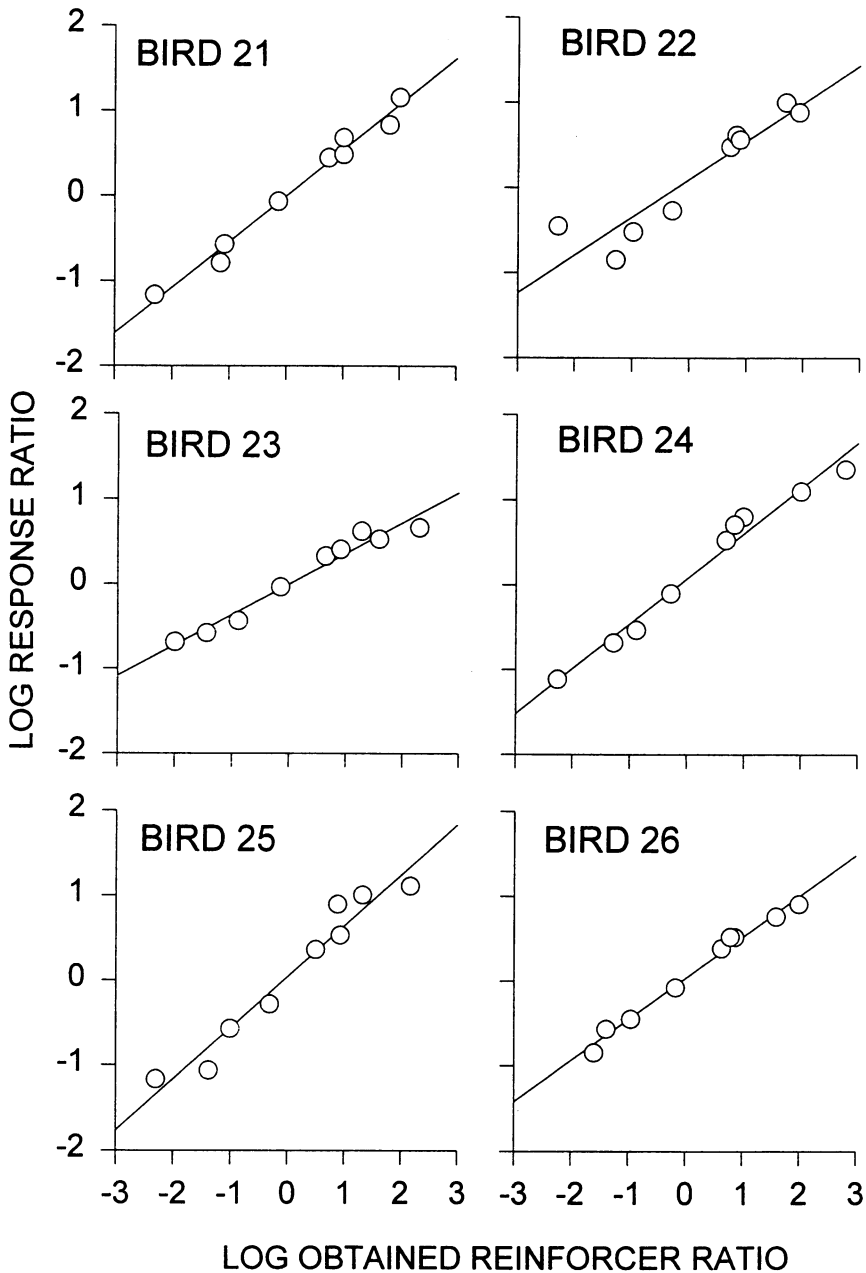


Fig. 1. Log response ratios as a function of obtained log reinforcer ratios for each subject. The straight line is the line of best fit to Equation 1b using linear regression; the parameters of this line are shown in Table 2.

line in the direction of less extreme choice, as predicted by Equation 3. This result is statistically significant at $p = .016$ on a directional binomial test.

A finer analysis, using all the obtained data, is shown in Figure 3. There, the residuals from the fits shown in Figure 2 (predicted

data minus obtained data) are shown as a function of the log obtained reinforcer ratios for all subjects. Negative residuals to the left of the graphs and positive residuals to the right of the graphs indicate a deviation away from the fitted line toward less extreme preference, as predicted by Equation 3. Such de-

Table 2

Results of generalized matching (Equation 1b), generalized matching to central data only, and contingency-discriminability (Equation 3) analyses for each individual subject and for the grouped data. For Equation 3, the logarithm of the bias parameter c is shown to facilitate comparison between the models.

Subject	Slope (a)	SE of slope	Intercept (log c)	SE of estimate	% variance accounted for	p	log c	% variance accounted for
	Generalized matching (Equation 1b)				Contingency discriminability (Equation 3)			
Group	0.49	0.03	0.03	0.11	98	0.12	0.02	99
21	0.54	0.03	0.00	0.11	98	0.09	-0.03	97
22	0.45	0.07	0.10	0.27	87	0.13	0.07	94
23	0.36	0.04	0.00	0.14	94	0.19	-0.01	97
24	0.53	0.03	0.08	0.15	97	0.07	0.09	98
25	0.60	0.05	0.04	0.21	95	0.06	0.00	97
26	0.49	0.02	0.04	0.08	99	0.14	0.04	98
	Generalized matching (Equation 1b) to central data only							
Group	0.60	0.03	0.04	0.04	99			
21	0.55	0.04	0.03	0.08	98			
22	0.63	0.05	0.03	0.09	98			
23	0.48	0.02	0.00	0.03	100			
24	0.71	0.02	0.09	0.03	100			
25	0.69	0.11	0.05	0.18	93			
26	0.55	0.02	0.06	0.03	100			

viations are clear for Birds 22, 23, 24, and 25. Furthermore, least squares linear regressions of the residuals as a function of the obtained reinforcer ratios gave positive slopes for all 6 subjects, which was statistically significant at $p = .016$ on a directional binomial test.

Next, we fitted Equation 3 for all concurrent VI conditions for each individual subject. Because the relation is nonlinear, this was done using the optimizer in Quattro-Pro for Windows®. The data used in these fits were log response ratios, and thus the model was Equation 3 transformed logarithmically. For each individual, Equation 3 was solved for the best fitting values of c and p on a criterion of minimizing the mean square error between obtained log response ratios and the log response ratios predicted by Equation 3. The results are shown in Figure 4, and details of the fits for the group data and for each individual are shown in Table 2 for comparison with the fits to Equation 1b. As Table 2 shows, the percentage of data variance accounted for by the two equations was similar. However, a comparison of Figures 1 and 4 shows that the fits to Equation 3 accounted well for the nonlinearities in the data from the individual

subjects. The group p value of .11 indicates that about 11% of the reinforcers delivered for each alternative were misallocated to the other alternative. Estimates of c were similar between the two analyses.

Figure 5 shows the residuals from the fits shown in Figure 4 plotted as a function of the log obtained reinforcer ratios. Positive deviations to the left of the graphs and negative deviations to the right indicate deviations towards more extreme choice than predicted by Equation 3, in the direction of Equation 1b. For the four extreme points, 14 of the 24 data points deviated towards more extreme choice ($p > .05$ on a binomial test), and linear regressions showed that 4 of the 6 subjects' deviations had negative slopes (nonsignificant at $p = .05$). Note that Equation 3 was not fitted to the central data points only (as Equation 1b was) because the parameters of the confusion model are mainly determined by the end points.

The data from Condition 12, in which extinction was arranged for one alternative, could not be analyzed in the same quantitative way because the log obtained reinforcer ratio was infinite. The data from this condi-

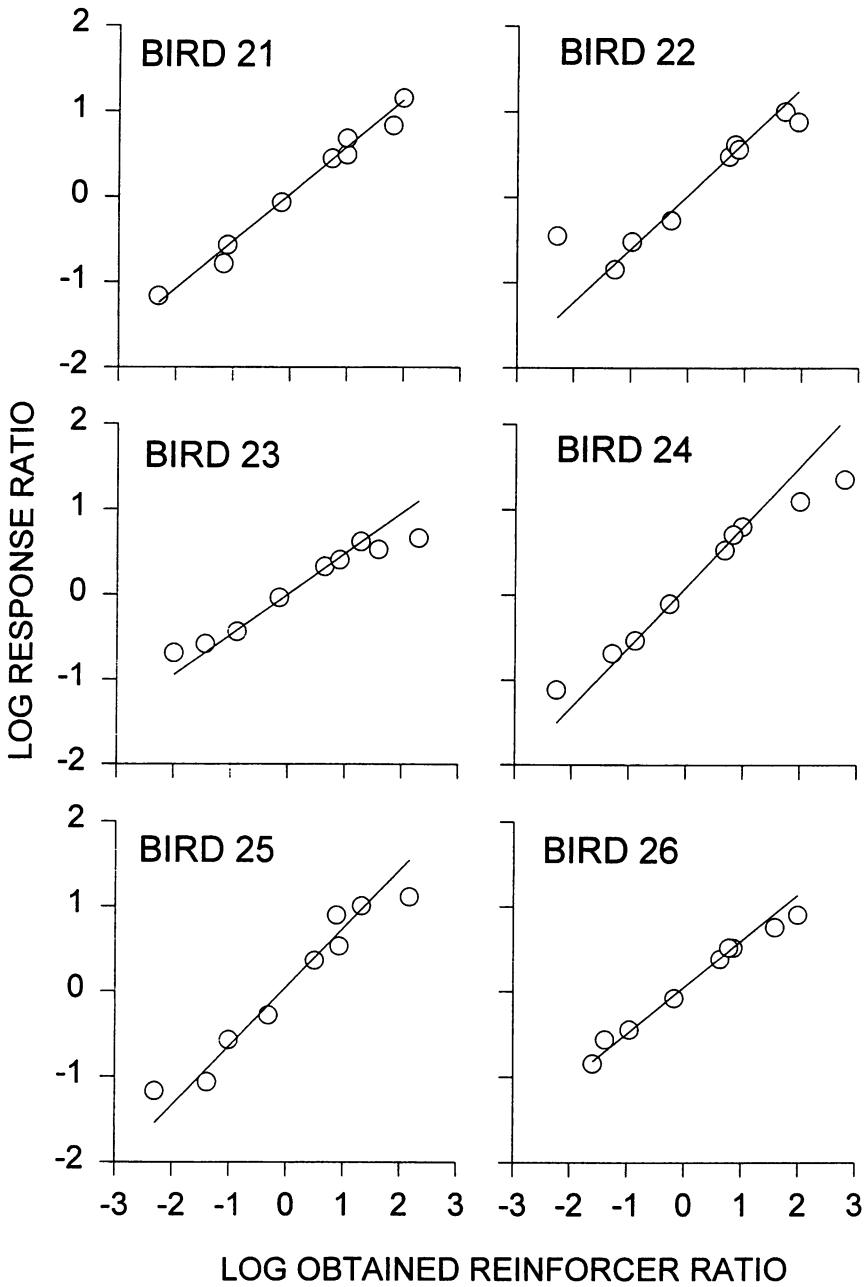


Fig. 2. Log response ratios for each subject as a function of log obtained reinforcer ratios. The straight line is the line of best fit to Equation 1b by least squares linear regression to the five central data points with log reinforcer ratios between -1 and +1. The parameters of the fit are shown in Table 2.

tion, in terms of absolute log response ratios, are shown in Figure 6. For all of the subjects, responding on the alternative associated with extinction was maintained strongly in Condition 12.

DISCUSSION

The results obtained here supported the contingency-discriminability approach over the generalized matching approach in ac-

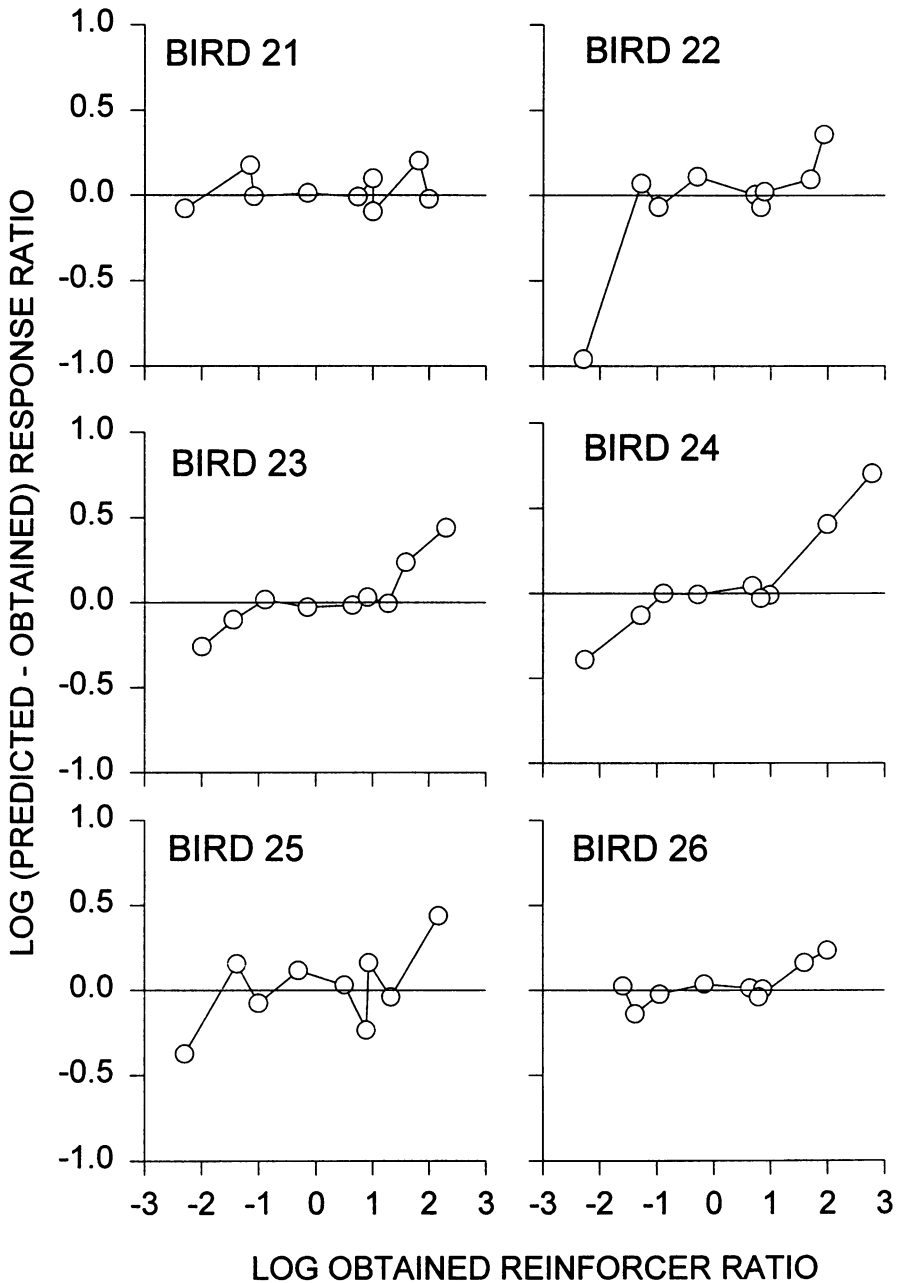


Fig. 3. The deviation of the obtained log response ratios from those predicted by the fitted lines in Figure 2 as a function of the obtained log reinforcer ratio.

counting for choice, particularly extreme choice. In fitting a straight line (e.g., Equation 1b), systematic deviations of extreme data points from the line should not appear when only central data are used to fit the line. However, such deviations were found here.

Log choice measures, across the range studied here, were thus not linearly related to log obtained reinforcer ratios. Equation 3 naturally predicts nonlinearity and data deviations from Equation 1b in the direction we found. Hence, qualitatively, Equation 3 is a descrip-

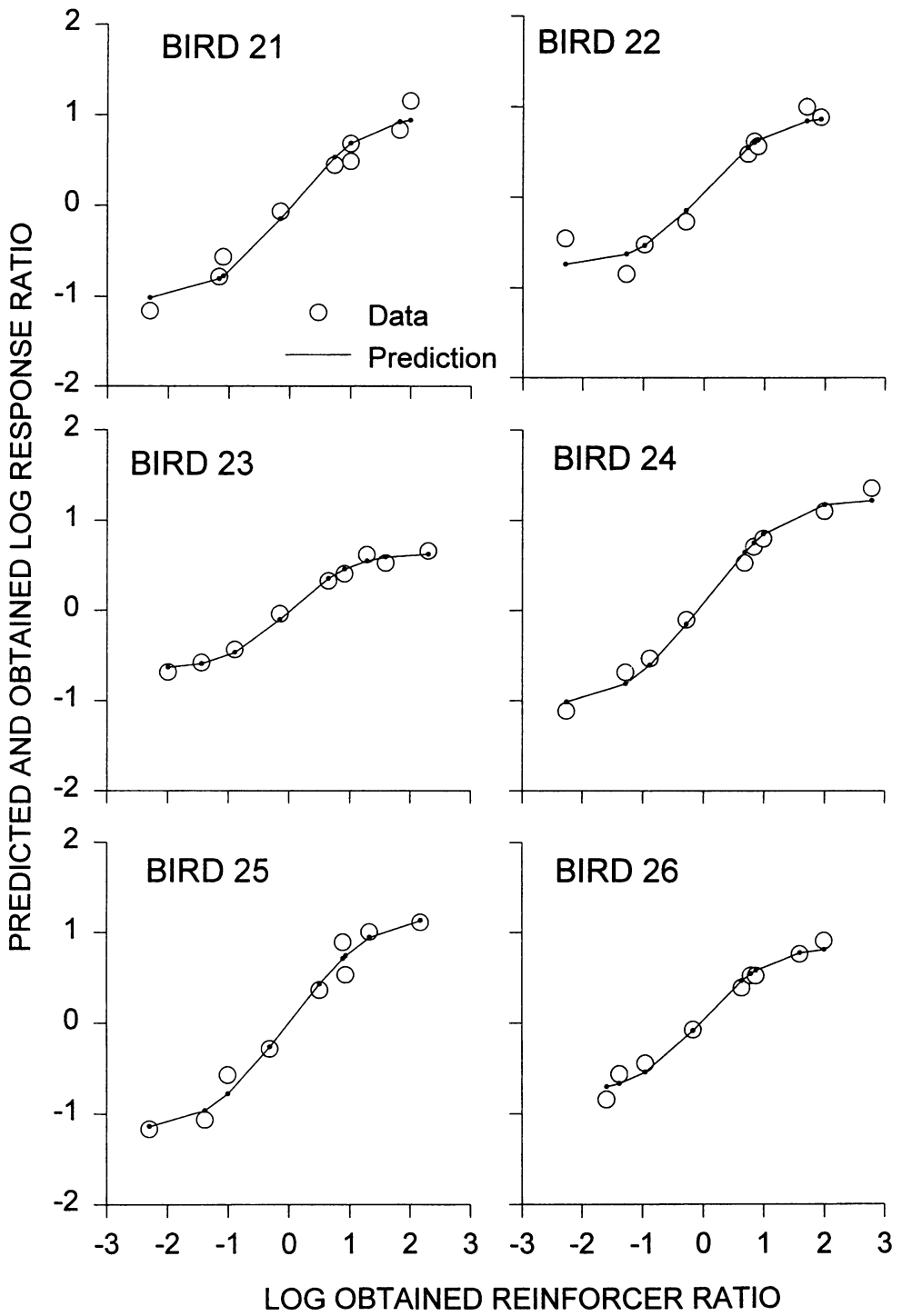


Fig. 4. Log response ratios as a function of obtained log reinforcer ratios, also showing the predictions of Equation 3 when this equation had been fitted to the data. The parameters of the fit are shown in Table 2.

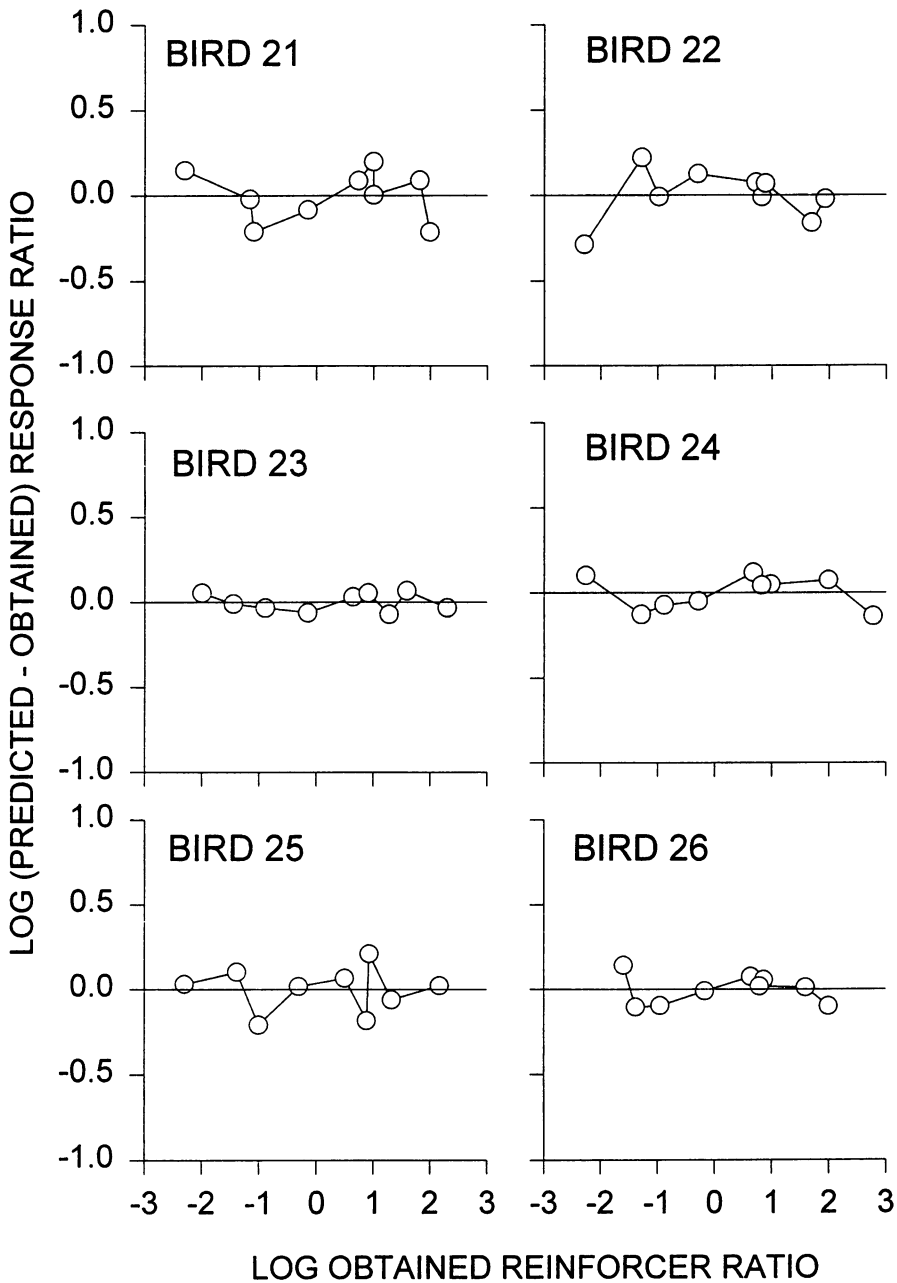


Fig. 5. The deviation of the obtained log response ratios from those predicted by the fitted lines in Figure 4 as a function of the obtained log reinforcer ratio.

tion that is more likely to be correct. It is more difficult to conclude that Equation 3 is quantitatively better than Equation 1b. As Figure 1 and Table 2 show, when fit to all the data, Equation 1b fits well on a criterion of variance accounted for, and in this fit (but

not in the fit to only the central data), the nonlinearities are not startling. This is because, in Figure 1, the nonlinearities predicted by Equation 3 occur for both the central data (towards more extreme choice) and for the extreme data (towards less extreme

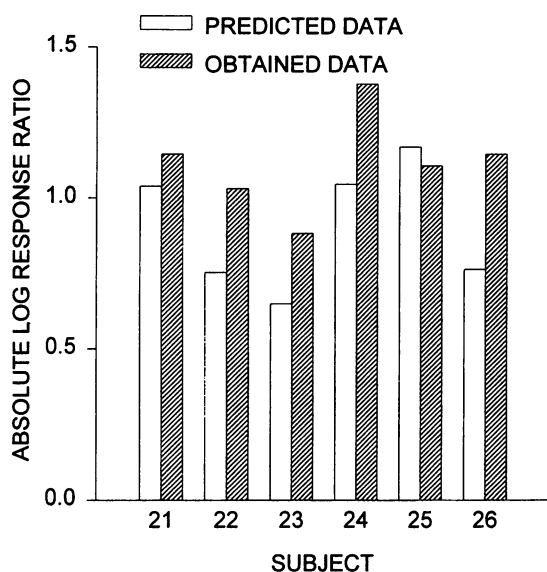


Fig. 6. Absolute values of log response ratios obtained in Condition 12 (concurrent extinction VI schedule) and the values predicted for this condition from Equation 4.

choice). The fitted straight line tends to obscure such deviations, but they become clear, and perhaps even overenhanced, when the nonlinear model is fitted (Figure 4). Further, the p , or confusion, parameters obtained from fitting Equation 3 had sensible values, as did the c , or bias, parameters (Table 2). We suggested earlier that the generalized matching relation was a useful description of choice, even though it had some theoretical problems. We believe that the present research suggests that the generalized matching approach is, however, a poor description of extreme choice, and that a better and more accurate descriptor of choice across a wide range of values is offered by Equation 3. We argue further that Equation 3, and the associated notion of contingency discriminability, is also an effective molar theory of choice because it specifies how the effect of a reinforcer differential on choice is modulated by a discriminative-stimulus differential to produce a quantitative value of choice. The concept of contingency discriminability, or its inverse, contingency confusion, defines the operations that affect the value of this measure in a way that sensitivity to reinforcement (a in Equations 1 and 1b) does not.

The results from Condition 12 replicated previous results from concurrent extinction

VI schedules (Davison & Hunter, 1976) in that the subjects continued responding to the extinction alternative, in many cases quite strongly. Generalized matching (Equations 1 and 1b) predicts the cessation of responding to the extinction alternative, whatever the values of a and c . The maintenance of responding to the extinction alternative in Condition 12 shows that Equations 1 and 1b cannot accurately describe the present data. Equation 4, derived from Equation 3, does predict that responding should be maintained on the extinction alternative, and the amount of such responding should be predictable from the values of p and c obtained from the fits of Equation 3 to the individual-subject concurrent VI data. These predictions were calculated, and are shown in Figure 6 alongside the obtained absolute log response ratios. Five of the 6 subjects showed a larger preference to the VI alternative than was predicted by the fits to Equation 3 (that is, the p values obtained from the fits to Equation 3 were generally overestimated). Although the differences between the obtained and predicted data were not significantly different on a sign test, these results could indicate a quantitative problem with Equation 3. It is worth mentioning that Equation 3, in a relative form rather than a ratio form, could have been fitted to all conditions of this experiment including Condition 12, with a consequently more accurate estimate of p inclusive of extinction conditions. This was not done, because we wished to investigate how well concurrent extinction VI performance could be predicted from concurrent VI VI performance. In summary, the fact that responding was strongly maintained in extinction argues against Equations 1 and 1b and for a different quantitative approach. The amount of responding maintained in extinction did not clearly indicate whether or not Equation 3 was the appropriate alternative approach.

Can the generalized matching relation be saved? One argument that might be offered against our interpretation of the present data comes from results reported by Todorov, Castro, Hanna, Bittencourt de Sa, and Barreto (1983). They showed that sensitivity to relative rates of reinforcers progressively decreased with an increasing number of exposures to different concurrent VI VI schedules. As Table 1 shows, we progressively increased

the reinforcer-rate differential on the two alternatives up to Condition 7. If the effect that Todorov et al. described was operating here, then sensitivity to reinforcement should have fallen, as it did. However, an analysis of trends in point estimates of sensitivity carried out over *all* successive conditions ($N = 6$ subjects, $k = 9$ conditions) showed a statistically non-significant trend ($z = 0.09$, $p > .05$), and this trend was also nonsignificant for each individual. As a more direct comparison, Condition 11 was a direct replication of Condition 3, done after considerable exposure to other conditions. The values of point estimates of sensitivity were smaller in Condition 11 than in Condition 3 for 3 of the 6 subjects, and were therefore not significantly different on a binomial test. Thus, there was no strong evidence in the present data that sensitivity falls with increasing exposure to concurrent VI VI schedules as described by Todorov et al. It may be that, in fact, Todorov et al.'s finding of decreased preference with increasing exposure to experimental conditions is partly accounted for by the present finding of decreasing sensitivity with increasing reinforcer differentials, because Todorov et al. did arrange the more disparate reinforcer-rate conditions later in their training sequence.

It has generally been argued that changeover delays are important in obtaining high sensitivities to reinforcement (Herrnstein, 1961), but, on the other hand, quite a number of experimenters have reported typical high-sensitivity values with no changeover delay at all (see Davison & McCarthy, 1988, for a review). It was because of this lack of clarity that we used a substantial (3-s) changeover delay in the present experiment. The arrangement of a changeover delay could, perhaps, be used to explain the present results. A changeover delay might force subjects at least to spend time, and perhaps to emit responses, on the lower reinforcer-rate alternative, when that alternative had been contacted. The dependent-scheduling procedure used here could possibly have amplified such an effect. However, we explicitly arranged the procedure so that subjects could exit from either alternative before the changeover delay had elapsed, so they were not forced to spend more than a minimal amount of time on the lower reinforcer-rate alternative.

We need to confront the possibility that the

present results could have been caused by a failure to continue conditions to stability, especially at extreme reinforcer ratios. Our defense against this possibility is twofold. First, assuming that the change in behavior allocation following a transition between conditions would be a negative or positive hyperbola or exponential curve, we assessed stability using a procedure that was sensitive to trends, of whatever size, and not to absolute changes in relative responses. We did not accept stability for an individual subject until a monotonic trend across three sessions had been absent on five occasions, and we did not accept stability in the condition until the performance of each of the subjects had met this individual criterion. This constitutes a sensitive stability criterion, one that should be no easier to satisfy at extreme choice than at equal choice. Second, there is ample evidence (e.g., Davison & Hunter, 1979; Hunter & Davison, 1985; Shull & Pliskoff, 1967) that stability on concurrent VI VI schedules is achieved in three to seven 45- to 60-min sessions. Although these results did not provide information on the effects of large disparities in reinforcer rates, Mazur (1992) showed that the rate of approach to asymptotic performance on concurrent VI VI schedules in transition was not systematically related to the difference in the VI schedule reinforcer rates. The numbers of training sessions arranged in the present experiment (Table 1) were thus considerably in excess of the number that previous reports have suggested are necessary.

Since its introduction by Davison and Jenkins (1985), a number of models either identical to or similar to the contingency-discriminability model have been proposed. Vaughan and Herrnstein (1987) suggested Equation 3 to account for performance on concurrent VI VI schedules in which the alternatives were signaled by slides containing trees and those containing no trees. As they stated, Equation 3 "seems a more reasonable way [than Equation 1b] to formalize the undermatching due to stimulus confusions" (p. 13). Burgess and Wearden (1986) discussed a modification of Herrnstein's (1970) equation that assumed a confusion between contingent and noncontingent reinforcer rates when some reinforcers were delivered independently of responding. Their model is concep-

tually identical to Equation 3. It is evident, then, that some researchers have focused on discriminative stimuli as the source of confusion, whereas others have focused on confusion about reinforcers. However, if behavioral control by a contingency is concerned with both the antecedent stimuli and the consequential reinforcers, then logically the confusion must be between two or more complete three-term stimulus-behavior-reinforcer contingencies. In other words, an inability to discriminate the stimuli will cause confusion about reinforcer differentials, and an inability to discriminate reinforcer differentials will, via a partial failure of differential reinforcement, cause confusion about stimuli.

Subsequent to Davison and Jenkins's (1985) paper, the contingency-discriminability model has been used successfully in analyzing stimulus disparity effects in two-alternative concurrent VI performance (Alsop & Davison, 1991) and in three-alternative concurrent VI performance (Davison & McCarthy, 1994). The concurrent-schedule model has also been extended to signal-detection performance in which sample-stimulus discriminability and contingency discriminability play separable parts (Alsop, 1991; Davison, 1991; McCarthy & Davison, 1991). Although all these papers have shown Equation 3 to be viable, in the sense of providing an adequate fit to the data, they were unable to test the generalized matching law (Equations 1 and 1b) differentially against the contingency-discriminability model (Equation 3), because both provide equally good fits to conventional, nonextreme data (Davison & Jenkins, 1985). The present research provided the first differential test. As Davison and Jenkins showed, Equation 3 fits standard, nonextreme, concurrent-schedule data as well as Equations 1 and 1b do. Davison and Jenkins also argued that the confusion measure of Equation 3, unlike sensitivity (a) in Equations 1 and 1b, clearly defines the independent variables (stimulus disparity, free delivery of reinforcers) that change the parameter's value. Given the problems with generalized matching outlined in the introduction, and the finding here that the contingency-discriminability model has also passed its first critical test, we submit that the model deserves serious consideration as an alternative to generalized matching.

There is one area in which Equation 3, the contingency-discriminability model, is less effective than Equations 1 and 1b, the generalized matching relation. The former model has not, as yet, been concatenated to deal quantitatively with controlling variables other than reinforcer rate. When individual reinforcers rather than the rates of identical reinforcers are different, it is unclear how to conceptualize them in an extended version of Equation 3. For example, if different reinforcer magnitudes are arranged for different operants, these may not only have different reinforcer values, they may also have different discriminabilities. Thus, varying reinforcer magnitudes may confound variations of both reinforcer values and reinforcer discriminabilities. This, perhaps, is evidenced by the differential outcomes effect in conditional discriminations (Trapold, 1970), with the apparent increase in signaling-stimulus discriminability actually resulting from an increase in contingency discriminability. It will be particularly difficult to deal with both the psychophysical measurement of reinforcement value and the psychophysical measurement of contingency detection in a single quantitative model of choice. This is left for further research to determine.

Finally, we think it important that extreme choice be adequately described and predicted by choice theories for practical, applied reasons. Many of the practical applications of such models are, explicitly, in conditions of extreme choice. Davison (1992) has argued that theories of behavior allocation, such as Equation 1, should allow a practitioner to calculate how much in the way of resources needs to be allocated to changing behavior in a particular situation. Such a calculation should allow the avoidance of both the allocation of insufficient resources to produce the desired change and the allocation of more resources than are necessary. Applying the generalized matching approach to such problems, as discussed by Davison, would be efficient (in terms of amount of behavior change obtained for resources used) in the 10 to 1 through 1 to 10 reinforcer-ratio range, but outside that range, generalized matching could be seriously in error. It would also be very seriously in error if the settings in which the behavior occurred were less than optimally discriminable, which may occur fre-

quently. According to the present approach (using Equation 3), it would be quite easy to find situations in which relatively extreme predictions from the two approaches differed by 100% or more. For quantitative applied work on extreme behavior allocations, then, Equation 3 should be used. However, because Equation 3 is nonlinear, more baseline manipulations of reinforcer rates are required to obtain its parameters prior to any therapeutic intervention.

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APPENDIX

Data for each subject summed over the final five sessions of each experimental condition, except in the case of Conditions 7 (the final 15 sessions) and 12 (the final 10 sessions), after stability had been achieved. The data from the latter two conditions have been divided by 3 and 2, respectively, to make these data commensurate with the data from the other conditions.

Subject	Condition	Responses		Time(s)		Reinforcers		Change-overs	
		Bright	Dim	Bright	Dim	Bright	Dim		
21	3	7,260	2,374	4,099	1,289	182.00	18.00	335	
	4	1,784	10,933	751	4,828	13.00	187.00	454	
	5	8,342	1,230	4,594	593	197.00	3.00	237	
	6	600	8,745	335	4,754	1.00	199.00	130	
	7	7,528	529	4,685	272	198.00	2.00	196	
	8	3,255	3,794	2,531	2,846	83.00	117.00	525	
	9	8,618	3,097	4,436	1,418	169.00	31.00	506	
	10	3,633	13,470	1,454	5,005	15.00	185.00	358	
	11	10,748	2,250	4,771	952	182.00	18.00	380	
	12	684	9,535	340	5,143	0	200.00	157	
	22	3	6,288	1,520	5,789	1,585	174.00	26.00	96
		4	899	6,312	913	6,775	10.00	190.00	36
5		4,654	463	4,960	552	198.00	4.00	29	
6		1,706	4,850	2,222	5,253	1.00	193.00	16	
7		5,585	726	5,570	819	197.67	2.33	16	
8		3,973	7,395	2,852	5,215	67.00	133.00	93	
9		7,103	2,362	5,464	1,738	168.00	32.00	95	
10		1,993	6,612	1,479	4,607	19.00	181.00	85	
11		8,645	2,356	6,343	1,749	177.00	23.00	56	
12		386	4,135	469	5,044	0	200.00	3	
23		3	9,792	2,355	4,663	1,235	190.00	10.00	114
		4	3,237	12,312	1,081	3,990	7.00	193.00	183
	5	11,765	3,522	4,430	1,323	195.00	5.00	90	
	6	2,448	11,879	823	4,102	2.00	198.00	98	
	7	10,289	2,259	4,827	1,081	199.00	1.00	62	
	8	6,655	7,288	2,963	2,926	83.00	117.00	306	
	9	10,025	4,736	4,384	2,189	163.00	37.00	164	
	10	4,404	11,997	2,097	5,388	23.00	178.00	209	
	11	10,284	4,044	4,490	1,732	178.00	22.00	151	
	12	1,445	11,034	633	5,180	0	200.00	29	
	24	3	6,483	1,029	5,906	1,394	154.00	16.00	165
		4	1,697	8,219	2,460	4,239	9.00	173.00	299
5		9,010	716	4,782	414	198.00	2.00	193	
6		506	6,590	1,317	4,326	1.00	182.00	119	
7		10,258	450	5,483	256	200.00	0.33	65	
8		4,950	6,255	3,517	3,561	66.00	127.00	271	
9		8,457	2,511	4,702	1,513	166.00	35.00	261	
10		2,371	8,082	1,541	4,369	23.00	177.00	171	
11		8,850	1,736	4,677	1,146	174.00	26.00	198	
12		349	8,317	214	5,176	0	200.00	175	
25		3	14,604	1,858	5,850	569	177.00	23.00	63
		4	1,188	13,720	652	5,880	8.00	192.00	89
	5	11,722	1,154	5,443	621	191.00	9.00	104	
	6	726	10,677	307	4,891	1.00	199.00	113	
	7	12,835	990	5,489	332	198.67	1.33	62	
	8	6,166	11,767	2,504	4,352	66.00	134.00	150	
	9	11,532	4,995	4,358	1,649	152.00	48.00	266	
	10	3,714	13,818	1,199	5,181	18.00	182.00	165	
	11	11,828	3,472	4,911	1,031	180.00	21.00	166	
	12	974	12,432	355	5,222	0	200.00	47	

APPENDIX

(Continued)

Subject	Condition	Responses		Time(s)		Reinforcers		Change-overs
		Bright	Dim	Bright	Dim	Bright	Dim	
26	3	8,845	2,658	4,416	1,170	177.00	24.00	149
	4	2,745	10,002	1,246	4,431	8.00	192.00	187
	5	12,745	2,194	5,564	976	196.00	5.00	10
	6	1,681	11,673	824	5,008	5.00	195.00	37
	7	13,138	1,615	5,311	731	198.00	2.00	25
	8	8,069	9,567	3,773	4,357	81.00	119.00	121
	9	11,674	4,771	5,317	2,355	156.00	36.00	167
	10	3,662	10,211	1,567	4,987	20.00	180.00	134
	11	12,145	3,630	4,955	1,343	172.00	28.00	265
	12	1,275	11,326	524	4,959	0	200.00	56