THE EFFECT OF CHROMATIC ABERRATION ON VISUAL ACUITY

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SUMMARY

1. Differences of threshold contrast are predicted from optical theory for a grating acuity target in monochromatic and white light. The greatest differences, up to 65%, are predicted for gratings of lower contrast and pitch than those normally used in measurements of visual acuity.

2. Using three subjects, we measured contrast thresholds with 1.5 and 2.5 mm diameter artificial pupils for natural and paralysed accommodation, using a tungsten lamp and wave-lengths of 546 and 578 mm.

3. Excellent agreement is obtained between predicted and measured differences.

4. Results confirm that observed acuity and sensitivity differences between white and monochromatic lights are largely optical in origin, but involve at least two independent colour mechanisms as spectral weighting functions. Stiles's π_4 and π_5 sensitivities afford a much better fit to observed differences than the C.I.E. visibility curve.

INTRODUCTION

The difference in focus of the human eye for colours was first noted by Newton (1704). There is now close agreement on its magnitude (Ivanoff, 1947; Hartridge, 1947; Campbell, 1957); over the effective range of the visible spectrum it amounts to about 2D. In white light the eye can be in focus for only one wave-length; consequently the retinal image formed by the rest of the spectrum will be blurred and this light energy must reduce the contrast of that portion of the image in accurate focus. If the visual nervous system ignored differences in colour when resolving spatial targets, this reduction in contrast should lead to a reduction of visual acuity for objects illuminated with white light as compared with monochromatic light. Does the presence of this substantial amount of chromatic aberration in the eye affect visual acuity?

Reported improvements in acuity range from 24 % (Luckiesh & Moss, 1933) to none whatever (Hartridge, 1947). However, the theoretical

improvement of visual acuity in monochromatic compared with white light has never been calculated from the known chromatic aberration of the eye. Recent advances in optical theory (Hopkins, 1962) have made it possible to relate quantitatively visual performance and optical quality (Campbell & Green, 1965; Campbell & Gubisch, 1966). A comprehensive description of retinal images for white and monochromatic grating targets is now possible and justifies a re-examination of the effects of chromatic aberration.

THEORY

Monochromatic light. The performance of an ideal optical system (limited only by diffraction) possessing a given aperture in monochromatic light can be specified by its modulation transfer function; that is, the curve which describes the loss of contrast caused by the optical system for objects which are sine-wave gratings of varied spatial frequencies. The effect of defocusing the ideal system is a decrease of its modulation transfer function everywhere except at the lowest (i.e. zero) and at the highest spatial frequencies transmitted (Fig. 1). Defocus produces the greatest loss of image contrast at frequencies one half the maximum one transmitted (Hopkins, 1962).

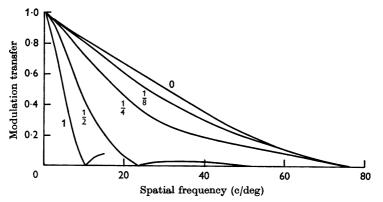


Fig. 1. Theoretical effect of focus on the modulation transferred from image to object by a diffraction-limited optical system. The pupil diameter is 2.5 mm and the light of wave-length 578 nm. Departure from perfect focus is given in dioptres by each corresponding curve.

White light. Just as a white image can be regarded as the superposition of many monochromatic images (Linfoot, 1956), the modulation transfer function for an optical system with chromatic aberration in polychromatic light can be derived by summing several transfer functions for coloured lights, each representing a given amount of defocus and weighted according to the luminance of that colour. If the source of light is a tungsten lamp of known colour temperature, the weighting of each colour is given with good

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accuracy by Planck's law (Moon, 1961). For example, a lamp whose colour temperature is 2500° K radiates twice as much energy at 654 nm than at 555 nm.

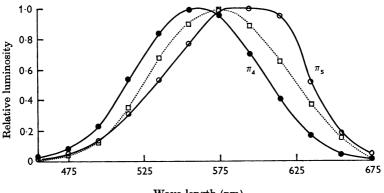
Spectral weighting functions. As colours near the ends of the visible spectrum have a reduced effect upon perception, the separate colour components must be weighted further by the spectral sensitivity of the eye. The standard Commission Internationale de l'Eclairage (C.I.E.) photopic visibility curve would seem at first to be appropriate, but there is some evidence that acuity for a grating of one colour is unaffected by moderately bright backgrounds of another colour. Hence the weighting function should consist of two or more independent parts.

Stiles (1939) demonstrated the presence of three independent colour mechanisms in the human eye; he further showed (Stiles, 1946) how these mechanisms could be used to account accurately for colour sensitivity and discrimination data, such as the C.I.E. curve itself. Von Bahr (1946) also demonstrated the independence of colours experimentally; he measured the grating acuities for two colours, first separately, and then superimposed. He found that acuity for one colour is not affected by a second colour, as long as the eye is in proper focus for the first wave-length. Colour response functions derived by Thomson & Wright (1947) are very similar to those of Stiles, even though their derivations are fundamentally different.

Calculations. Accordingly, we have chosen Stiles's π_4 and π_5 sensitivities as the visual weighting functions to be used for calculating contrast sensitivity differences in white and monochromatic lights. The blue mechanism, π_1 , has not been taken into account as it does not appear to function for the small (0.2°) foveal target we have used (Willmer & Wright, 1945). When the eye is focused for the test wave-length of 546 nm the π_4 sensitivity is used, having a maximum at about 540 nm; when the eye is focused for the test wave-length of 578 nm the π_5 sensitivity is chosen, which has a broad peak near 580 nm. The colour weightings π_4 and π_5 are thus used separately with white light, depending upon the wave-length in optimum focus. This choice is based upon the evidence, given above, that the mechanism not in focus has no influence on the other.

The total weightings given to each colour, including the non-uniform spectral output of the tungsten lamp, are shown in Fig. 2 for the π_4 and π_5 sensitivities. The C.I.E. visibility curve is shown for comparison. Note that all three curves are shifted to the right of their more familiar positions, owing to the tungsten lamp which we used having an increasing output toward longer wave-lengths.

We have calculated the modulation transfer functions for a diffractionlimited optical system having the eye's chromatic difference of focus, taking into account the spectral weightings shown in Fig. 2. The results at 578 nm with two pupil diameters are shown in Fig. 3. Numerical values were obtained for the 'white' transfer functions by using eleven colours between 475 and 675 nm at 20 nm increments in wave-length. It is apparent that the system should perform better in monochromatic light. An improvement is shown in the rending of contrast of up to 15% for a 1.5 mm pupil and 65% for a 2.5 mm pupil.



Wave-length (nm)

Fig. 2. Luminous energy of a tungsten lamp with a colour temperature of 2500° K, weighted by Stiles's π_4 and π_5 colour sensitivities (circles) and C.I.E. photopic luminosity (squares).

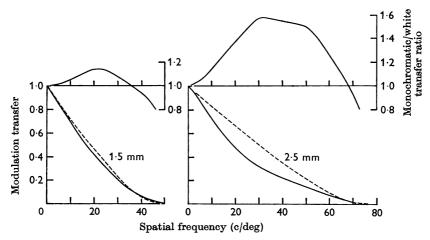


Fig. 3. Modulation transfer functions for a diffraction-limited optical system with a 1.5 mm (left) and 2.5 mm (right) pupil. Dashed curves show theoretical performance in monochromatic light of 578 nm wave-length. Solid curves show theoretical performance in white light weighted by Stiles's π_5 colour sensitivity as in Fig. 2. Ratio of monochromatic to white modulation transfer is given above each pair of curves.

Equally important is the spatial frequency at which the greatest difference occurs: 25 and 30 c/deg for the smaller and larger apertures respectively. These frequencies are about one half the maximum transmitted at each pupil size, and they are well below the spatial frequencies customarily used in measurements of visual acuity where high-contrast grating targets are employed.

These calculations indicate that the optical difference between white and monochromatic light should be most easily detected as a change in contrast, rather than as a change in maximum resolvable spatial frequency. The difference will be diminished with small pupils by the effects of diffraction. Owing to the increasing imperfection of the eye for large pupils, however (Campbell & Gubisch, 1966), it is impossible to predict accurately what this difference will be for apertures larger than about 3 mm. The colour temperature of the tungsten lamp is not critical; a shift from 2500° K to 2800° K changes the expected ratios of contrast by about $2\frac{9}{0}$.

METHODS

Apparatus. The acuity target consists of adjacent strips of polaroid sheet, each 5 mm wide and forming a square panel 15 cm on a side. The axis of polarization of each strip is perpendicular to that of its neighbour. When viewed through a polaroid analyser the target appears as a grating whose contrast can be varied by changing the angle of the analyser. If contrast C is defined in the usual manner

$$C = rac{L_{ ext{max}} - L_{ ext{min}}}{L_{ ext{max}} + L_{ ext{min}}}$$

in which L_{\max} and L_{\min} are the luminances of the brighter and darker bars, then contrast is related to the angle θ of the analyser by the simple formula

$$C = \sin 2\theta$$

when θ is chosen to be zero where the contrast is zero also.

The target is illuminated from behind by either a high-pressure mercury arc lamp or a 500 W tungsten projection lamp, both run from a stabilized mains supply (Fig. 4). Filters are chosen for the mercury source so that only the yellow 578 nm or the green 546 nm line is transmitted to the target. In front of the mercury and tungsten sources are placed two polarizers whose axes are mutually perpendicular. These two polarized beams are superimposed in a mixing cube and are directed toward a diffusing screen directly behind the target. This diffusing screen removes all polarization from the two beams before they illuminate the polaroid grating.

Procedure. Initially, a rotating polarizer is placed between the mixing cube and the diffusing screen. The light reaching the target alternates between monochromatic and white, allowing a heterochromatic match of the two sources to be made by flicker photometry. An approximate match is made by first placing neutral filters in front of the sources and then an accurate match is obtained by varying the voltage across the tungsten lamp. The colour temperature of the lamp is kept near 2500° K. For a run of twenty consecutive readings, the standard error of this final match was never greater than ± 0.025 density units for any subject.

For measurements of contrast threshold the rotating polarizer is removed and one light source is occluded. A viewing distance between 5.6 and 25 m is selected, corresponding to a

spatial frequency of the target of 10-45 c/deg. An iris diaphragm at the diffusing screen restricts the field of view to 0.2° ; it is adjusted with viewing distance so that this angle remains constant. The analyser is attached to a rotatable mount whose angle can be determined with an accuracy of 20 min of arc.

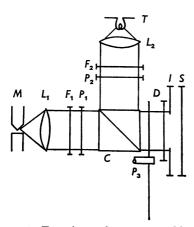




Fig. 4. Experimental apparatus. M, mercury arc lamp; T, tungsten lamp; L_1 , L_2 , collecting lenses; F_1 , F_2 , colour and neutral filters; P_1 , P_2 , fixed polaroids; C, beam-splitting cube; P_3 , rotating polaroid; D, diffusing screen; I, iris diaphragm; S, acuity target; P_4 , analysing polaroid; L_3 , refracting lens; A, artificial pupil; E, eye of observer.

Two pairs of readings at a time are taken with each source of light, and the sources are alternated during a session to minimize the effects of luminance drifts and changes of threshold criterion. The analyser is rotated in one direction until the target attains a just perceptible contrast and that angle is recorded. The analyser is then rotated in the opposite direction, diminishing the contrast and finally increasing it until threshold is reached again. The algebraic difference between these two angles is 2θ and its determination does not require a knowledge of the angle at which contrast is zero. This method of recording angular differences also minimizes the effects of any mechanical drift in the angle of the analyser mount.

Spherical and astigmatic refractive errors of the subjects are corrected to within ± 0.125 D for the viewing distance used. The pupil and accommodation of the observing eye are paralysed with cyclopentolate hydrochloride 1 %.

RESULTS

Effect of focus on thresholds. To find the correct refraction for maximum sensitivity with each of the monochromatic targets used, we measured threshold contrast at 30 c/deg for a range of spherical spectacle lenses while the subject's accommodation was paralysed. The results of these measurements for subject R.W.G. with white and monochromatic light (578 nm) are shown in Fig. 5. The ordinate is contrast sensitivity, the reciprocal of threshold contrast expressed as a decimal fraction between zero and one. The optimum lens correction is about +0.63 D. A clearly increased sensitivity in monochromatic light is also shown; with the

optimum lens, the yellow grating requires about 40 % less contrast to be seen than the white one. The ratio of the sensitivities is about 1.6, which is very close to the theoretical value 1.58 shown in Table 1. The optimum lens correction was found to be about +0.25D for white light and about +0.63D for the yellow light. This is to be expected from the magnitude of chromatic difference of focus in the eye, as the mean wave-length of the white light is shorter than that of the yellow.

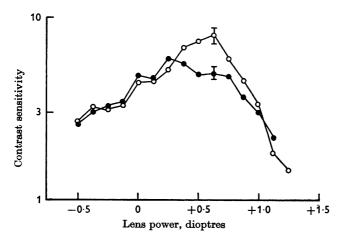


Fig. 5. The effect of focus on contrast sensitivity for subject R.W.G. with a 2.5 mm pupil at 30 c/deg. Accommodation is paralysed. Open circles: monochromatic light, 578 nm. Filled circles, white light. Luminance of either source is 64 cd/m^3 . Vertical bars indicate $\pm 1 \text{ s.e.}$

TABLE 1. Theoretical and measured ratios of monochromatic/white contrast sensitivity at 30 c/deg, averaged among three subjects. The measurements given are means \pm s.e. of means. Stiles's π -mechanisms were used to compute upper theoretical ratio; the response functions of Thomson & Wright (1947) were used to generate ratios beneath

	Yellow (578 nm)/white		Green (546 nm)/white	
	Measured	Theoretical	Measured	Theoretical
Paralysed accommodation	1.55 ± 0.113	${ 1 \cdot 58 \\ 1 \cdot 56 }$	$1{\cdot}57\pm0{\cdot}147$	${1.65 \\ 1.57}$
Natural accommodation	1.31 ± 0.086	_	$1{\cdot}35\pm0{\cdot}122$	

Near their respective optima, contrast sensitivity in yellow light is decreased much more by a small defocus than in white light. That is, the white light affords a greater depth of focus although sacrificing the higher maximum sensitivity of yellow. This increased depth of focus in white light has also been found by Campbell (1957).

Effect of grating pitch on thresholds. Contrast sensitivities over a wide range of spatial frequencies are shown in Fig. 6. The subject R.W.G. used natural accommodation to fixate the target but his natural pupil was always larger than the artificial ones used. The results can be fitted with straight lines in semi-log co-ordinates. The slopes are identical to those found by Campbell & Green (1965) who used the different method of generating a test grating on the screen of an oscilloscope.

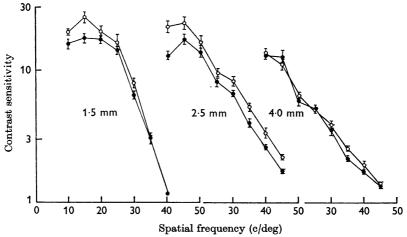


Fig. 6. The variation of contrast sensitivity with spatial frequency in monochromatic yellow (open circles) and white (filled circles) light. The subject is R.W.G., with natural accommodation and artificial pupils of 1.5, 2.5 and 4.0 mm diameter. Retinal illumination is kept constant at 100 td for all three pupil sizes.

The sensitivity difference between white and yellow lights shown for a 2.5 mm pupil in Fig. 5 is diminished here, being only about 25 %. For a 4.0 mm pupil, the difference is altogether absent, or at most only marginally significant. Spherical aberration, which increases with pupil diameter, could account for the smaller difference. The monochromatic/white sensitivity ratio is also smaller with a 1.5 mm pupil than with one 2.5 mm, especially at higher spatial frequencies. The steeper slopes of the 1.5 mm results are caused by diffraction through the small pupil; both the greater slopes and the decreased difference between yellow and white sensitivities agree with the performance predicted in Fig. 3.

For comparison, the contrast sensitivities of subject F.W.C. with natural accommodation and a 2.5 mm pupil are shown in Fig. 7. The differences between yellow and white sensitivities are somewhat larger than the previous subject's, averaging 45% over the range of frequencies used. Nevertheless, the equivalent visual acuities (unity contrast grating) in the two lights used would differ by only 7% (see Discussion). The monochromatic/white ratio 1.45 for subject F.W.C. is nearer to the theoretical value of 1.58 for paralysed accommodation than the ratio 1.27 in Fig. 6 found for the younger subject R.W.G. with natural accommodation. This

result suggests that the larger amplitude of accommodation occurring in youth might obscure part of the difference between white and monochromatic sensitivities.

Contrast thresholds at 30 c/deg. To provide a more precise comparison of theory and measurement, we obtained ten readings of threshold contrast (from each of three subjects) with a 2.5 mm pupil and a 30 c/deg grating.

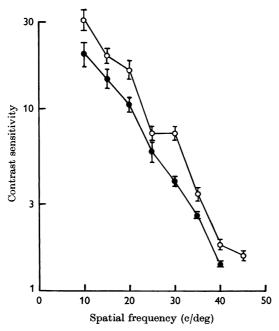


Fig. 7. Variation of contrast sensitivity with spatial frequency for subject F.W. C., with a 2.5 mm artificial pupil and natural accommodation. Open circles: yellow light, 578 nm wave-length. Filled circles: white light from a tungsten lamp. Retinal illumination from either source is 100 td.

Yellow, green and white gratings were presented for natural and paralysed accommodation. The averaged contrast sensitivity ratios for each individual are displayed in Fig. 8 and the grand averages among all three for each category are given in Table 1. The relation suggested above between age and departure from the theoretical sensitivity ratio for natural accommodation is illustrated for yellow and white targets. However, it does not occur for green and white gratings, so we must conclude that the regular progression of open circles on the left of Fig. 8 is coincidental.

Nevertheless, there is one influence of age evident in Fig. 8 which holds for green as well as yellow lights: the sensitivity ratio decreases consistently for each next older subject with paralysed accommodation. This trend would be expected if there were a regular decrease in the quality of the dioptrics of the older subjects. For the theoretical ratio was derived assuming perfect optical quality, and a lower quality produces a lower ratio. It can be seen from Fig. 5 that simple defocus, a form of optical defect, eventually removes all measurable differences between monochromatic and white contrast sensitivities. Although the correlation of sensitivity ratio with age is reasonable, its statistical significance in the results shown in Fig. 8 is marginal. The greatest separation between ratios is no more than 2 s.E. of the respective means.

Contrast sensitivity ratios averaged among all three subjects are given in Table 1. The experimental results are below theoretical expectations by

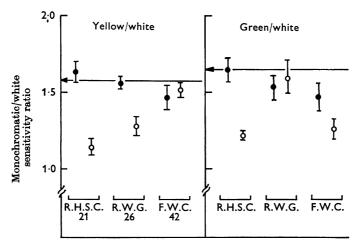


Fig. 8. Ratios of monochromatic to white contrast sensitivities for three observers. Filled circles: paralysed accommodation. Open circles: natural accommodation. Each point is the mean of ten observations at 30 c/deg; the vertical bars indicate ± 1 s.E. The two horizontal arrows point to the theoretical ratios expected for paralysed accommodation. Age of each observer is given below his initials.

less than one half the S.E. of the means. No theoretical ratio is given for natural accommodation, as we have not undertaken to determine the wave-length which is in best focus when the tungsten lamp is the illuminant. We interpret the excellent agreement between measured and theoretical ratios as verifying the optical relation between visual sensitivities in monochromatic and white light. Further, the agreement supports the use of Stiles's foveal colour mechanisms as the appropriate spectral weighting functions.

DISCUSSION

Comparison with previous experiments. Earlier workers usually obtained their results as differences in visual acuity for gratings of unity contrast. With a given source of light, the spatial frequency of the grating was

increased until it was just visible. Acuity was then measured as the reciprocal of the distance (in min of arc) between centres of adjacent dark and light bars.

The maximum resolvable spatial frequencies for gratings of unity contrast in white and yellow light can be estimated from Fig. 6 by extrapolating the results down to the abscissa, where contrast sensitivity is equal to 1.0. This sensitivity would arise from a grating whose contrast is 1.0 at threshold. Bearing in mind that the conventional measure of visual acuity for high-contrast gratings would be 2.0 for a spatial frequency of 60 c/deg, we observe that acuity differences estimated from Fig. 6 will be very small. With a 2.5 mm pupil, the maximum resolvable frequencies for yellow and white lights are 55 and 52 c/deg respectively, corresponding to visual acuities of 1.83 and 1.73. Thus a 25% difference in threshold contrast is equivalent to an acuity difference of only 6%.

This is the key to the success of our method: that a large change in the contrast threshold of a grating may cause only a very small change in the conventional measure of acuity for that grating. It follows from the optical theory illustrated in Fig. 3 that the greatest changes in modulation transfer occur not at the highest spatial frequencies resolved but at frequencies one half or less than the highest. The results in Figs. 6 and 7 demonstrate the usefuleness of manipulating the contrast of a grating as well as its spatial frequency.

It should now be apparent why acuity differences were not found at all by Hartridge (1947) and why they were observed only under conditions of diminished contrast by Shlaer, Smith & Chase (1942). Because of his negative results, Hartridge was led to postulate a 'neural antichromatic response' which suppressed the perception of chromatic fringes and their effect on visual acuity. From his earlier calculations, Hartridge (1922) knew that such fringes must be present in the retinal image, although in fact he underestimated the effect of colour on the image quality (Gubisch, 1967).

The method of Luckiesh & Moss (1933) was similar to our own; the light sources used were tungsten at a colour temperature of 2410° K and a sodium-vapour discharge at 589 nm. However, their subjects viewed a grating of unity contrast binocularly and with natural accommodation and pupils. They measured acuities with both sources for luminances between 0.064 and 64 cd/m², and found an acuity improvement in the monochromatic light decreasing from 24% at the lowest to 8% at the highest luminance. The figure of 8% compares favourably with the estimated acuity from our own data of 6–7% at the same luminance.

The increased advantage of monochromatic over white light at low luminances observed by Luckiesh & Moss (1933) arises from the purely optical nature of the differences and the simple shape of the sensitivity curves. So long as the theoretical ratio of modulation transfer functions remains constant, the algebraic difference between maximum resolvable frequencies with the two kinds of lights will also remain constant, regardless of the actual contrast thresholds or spatial frequencies involved. For example, the maximum resolvable frequencies estimated for a 2.5 mmpupil from Fig. 6 are 52 and 55 c/deg. The difference between them is 3 c/deg, determined only by the slope of the lines and the sensitivity ratios. As luminance decreases, acuity drops and the maximum resolvable frequencies become lower also; but the difference will remain at 3 c/deg. At the lowest luminance used by Luckiesh & Moss (1933), the theoretical improvement calculated on this assumption is 18%, somewhat smaller than the 24% actually found. However, the retinal illumination was no greater than 3 td at that point and a photopic weighting function may no longer be valid.

Martin & Pearse (1947) compared acuities with a tungsten source whose colour temperature was 2680° K and a red light formed with a broadband ruby glass filter. Their subjects, who had natural accommodation and pupils, viewed the grating target binocularly. From 0.2 to 10 cd/m^2 , measured acuity improvements fell from 10 to 2%. It is impossible to calculate accurately the expected improvement for natural accommodation without more information, but the use of broad colour filters instead of a monochromatic source would inevitably lower the theoretical difference.

Spectral weighting functions. The reasons for employing two independent colour sensitivity functions in our calculations has already been given. Would another choice of colour weighting yield different theoretical ratios? For yellow light, the C.I.E. curve (Fig. 2) leads to an expected sensitivity ratio of 1.58 at 30 c/deg. This is identical to the number obtained using Stiles's π_5 results. With green light, however, the C.I.E. weight gives rise to a theoretical ratio of 1.89, which is quite different from the ratio of 1.65 obtained using Stiles's π_4 curve or the ratio 1.57 from the corresponding function of Thomson & Wright (1947). None of our data indicates that the higher ratio 1.89 is attained experimentally, hence a single weighting function must be abandoned as unrealistic. The identity of ratios calculated from the C.I.E. and π_5 curves arises from the fact that the C.I.E. curve, while narrower than the π_5 , is also more remote from the point of optimum focus for the wave-length used.

Observations with achromatizing lenses. Attempts have been made to measure visual acuity when a target is viewed through a lens which compensates for the chromatic difference of focus of the eye (Helmholtz, 1909; Hartridge, 1947). No improvement in resolving power was observed, and this finding seems to contradict the present conclusions. We have repeated

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this experiment using an achromatizing lens designed by Thomson & Wright (1947). Campbell (1957) has shown that this lens corrects completely the chromatic difference of focus for monochromatic lights viewed one at a time.

The contrast threshold for a grating with a spatial frequency of 30 c/deg was measured by subject F.W.C. through a 2.5 mm pupil. The achromatizing lens was centred close to the cornea. Accommodation was paralysed and the eye was refracted so that the contrast of the grating was maximal. Contrast thresholds were determined alternately with and without the lens. The ratio of these contrasts was 1.04, which for ten readings is not significantly different from unity (0.1 < P < 0.2).

How can this paradoxical negative finding be explained? This lens was designed to correct for chromatic difference of focus and this it does. However, such a lens does not necessarily eliminate chromatic differences in size or position of the image (Ditchburn, 1966). These differences are not apparent when the light source is monochromatic. In white light, an achromatizing lens can actually introduce coloured fringes not present originally (Hartridge, 1950). To investigate the performance of the Thomson & Wright lens (1947), we viewed a bright point source of white light through the lens and a purple filter. The filter absorbs light from the yellow and green portions of the spectrum but transmits red and blue light.

We found it virtually impossible to centre the achromatizing lens with sufficient precision so that the blue and red images of the point source coincided, although each was in sharp focus. That is, the lens corrects for chromatic difference of focus but introduces prismatic displacements of the colours in the image. This practical difficulty of centring the lens on the visual axis must degrade the contrast of any image viewed in white light, and it could account for the failure to observe an improvement of contrast sensitivity or visual acuity when it is used.

We are grateful for the cooperation of Mr R. H. S. Carpenter as a subject. R. W. G was supported by a United States Churchill Foundation Scholarship and a National Science Foundation Graduate Fellowship. The achromatizing lens was kindly lent to us by Professor W. D. Wright.

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