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THE MEASUREMENT OF VOLUME CHANGES IN HUMAN LIMBS

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The measurement of volume changes in the limbs or in portions of the limbs of man is now recognized as a valuable technique in many physiological and clinical investigations. The various methods employed in the measurement have been recently reviewed (Potter, 1948). Volume plethysmography has proved particularly useful in investigations on vasomotor response and, combined with the venous occlusion procedure, has become the standard method for estimating peripheral blood flow in man.

Although there have been many refinements to the technique of volume plethysmography during the past 20 or 30 years, the current procedure is still based essentially on the system originally devised by Schäfer & Moore (1896) and subsequently adapted to human limb measurements by Hewlett & van Zwaluwenburg (1909). In this system, the limb, or a segment of it, is sealed in a rigid jacket so that volume changes of the enclosed part will cause corresponding volume displacements of the fluid filling the space between the limb and the jacket. The sealed space, filled with either water or air, is connected to a volume recorder by an air- or water-line.

It is apparent, from the extensive literature which now exists on the application of this technique, that it must always suffer from some of the following disadvantages and limitations: (*a*) The apparatus tends to be cumbersome. (*b*) The sealing of the limb within the jacket gives rise to problems, for a perfect water- or air-tight seal must be achieved without appreciable constriction of the tissues and the seal must be arranged so that the volume change of the limb enclosed results in an equivalent volume displacement of the fluid filling the jacket. (*c*) During any continuous record of volume change there must be no relative movement between the limb and its jacket. The subject is, therefore, largely immobilized, and the possibilities of recording volume changes in relation to normal muscular activity are very limited. (*d*) The air- or water-line connecting the subject with recording apparatus is restricted in length and the apparatus must, therefore, remain reasonably near the subject. This adds further to the immobilization of the subject and restricts observations to environmental and other conditions to which the recording apparatus can be exposed. In the case of the liquid-filled plethysmograph, this disadvantage has been recently overcome by the employment of electrical transmission (Cooper & Kerslake, 1951). (*e*) The enclosure of an appreciable part of a limb within a chamber may result in local vasomotor changes, particularly if the fluid filling the chamber is maintained at a constant temperature. The validity of a continuous record of vasomotor change by this technique is therefore dubious. (*f*) In order to avoid the more obvious sources of error, the apparatus becomes complicated and a considerable time is often required to mount the apparatus on to the limb of the subject. There has been much discussion of sources of error in employing the plethysmograph (e.g. Lansdowne & Katz, 1942).

Since these disadvantages seem inherent in any method which attempts to measure directly volume changes in a limb, any further advances in the technique for recording such changes must depend on the development of methods for recording the changes of limb weight or of limb dimensions. Changes of limb weight would be directly proportional to changes of limb volume, and such changes have been recorded in the past (Smirk, 1933). There are, however, obvious technical difficulties in recording vasomotor changes or blood flows by weight measurements.

There remains the possibility of recording limb volume changes indirectly by the measurement of corresponding changes in the limb dimensions. The discussion which follows examines the theoretical and practical aspects of this indirect method.

THEORETICAL CONSIDERATIONS

For physiological purposes, an estimate of limb volume change is usually required. If changes in limb dimension are actually recorded, it must, therefore, be possible to deduce the corresponding volume changes from these records.

When a limb changes in volume, it is reasonable to suppose that there is no significant change in the dimension measured along the axis of the skeletal element of the limb. The whole of the volume change is absorbed in corresponding changes of the transverse sectional area of the limb. At any particular level of the limb along its axis, the percentage change in volume at the level will be equal to the percentage change of transverse sectional area at that level. If the transverse section is truly circular and if the shape of the section remains unaltered during expansion or contraction, it is obvious that the percentage change in area of the section will be twice the percentage change of the circumference of the section for small changes of area. Geometrical considerations show that this relationship still applies to sections of other shapes if the shape remains unaltered on expansion or contraction (see Appendix I).

On theoretical considerations, therefore, it seems feasible to deduce percentage changes in volume of a limb at a particular level along its axis from corresponding measurements of the percentage changes in circumference (girth) of the limb at the same level. Before describing apparatus which has been devised for this measurement of girth, it will be relevant to examine in more detail the theoretical assumptions underlying the direct relationship between changes of volume and changes of girth.

The first assumption, that the length of the limb remains unaltered during volume changes, seems indisputable when applied to the limb as a whole or to segments of the limb when any restraining influence to expansion is uniformly distributed along the length of the limb. If, however, any measuring device applied to a segment of the limb exerts a greater restraint on the expansion of that segment, compared with the restraint existing for the remainder of the limb, then effectively the length of the measured segment as well as its girth will increase when the volume of the segment increases, so that the percentage increase in volume will no longer be equal to twice the percentage increase in girth. In practice, the measuring device will usually impose some restraint on the limb. If, however, the deformable tissues are considered to be truly elastic under these conditions of restraint, it can be shown (see Appendix II) that the unrestrained percentage girth changes can be accurately deduced from measurements made under restraint if the girth measuring device is suitably calibrated.

The validity of the second assumption necessary in deducing volume changes from girth changes—namely, that the transverse sectional shape remains unaltered during volume changes—cannot be assessed on purely theoretical grounds for the irregular cross-sectional shapes of human limbs. There are also no data for the changes of shape, if any, which occur in human limbs during volume changes. Some indication of the maximum errors likely to arise can be derived from a theoretical consideration of elliptical cross-sections. The relationships between percentage change in volume ($100\delta V/V$) and percentage change in girth ($100\ \delta G/G$) when 'E' is the ratio of the short to the long axis of the elliptical cross-section are as follows:

(i) If, on expansion or contraction, only the long axis of the elliptical cross-section changes, then:

$$\frac{\delta V}{V} \simeq \frac{(9-E^2)}{(9-5E^2)} \frac{\delta G}{G}.$$

(ii) If, on expansion or contraction, only the short axis of the elliptical cross-section changes, then:

$$\frac{\delta V}{V} \simeq \frac{(9-E^2)}{4E^2} \frac{\delta G}{G}.$$

It will be obvious that both (i) and (ii) approximate to $\delta V/V = 2\delta G/G$ as '*E*' approaches unity, that is, as the cross-section becomes almost circular. In this connexion, it is relevant to note that human limbs and digits approximate to a circular cross-section except in the region of joints such as the wrist and the ankle. Volume changes in the region of joints are not commonly required, but it is of interest to note that the ratio of short to long axis of the wrist section is approximately 0·8. If this wrist section is considered to be a true ellipse, then according to formula (i) $\delta V/V \simeq 1{\cdot}44\,\delta G/G$, and according to formula (ii) $\delta V/V \simeq 3{\cdot}27\,\delta G/G$. Considerable errors could arise, therefore, if volume changes at the wrist are deduced from girth changes on the assumption that $\delta V/V = 2\,\delta G/G$ if the entire dimensional change were absorbed in one or other of the axes of the wrist section. Such a unidirectional change is not, however, to be expected in practice—for example, at the wrist the greater part of the volume change must be in the skin and the skin is fairly uniformly distributed in thickness. Marked changes in the shape of the wrist accompanying volume changes are not, therefore, to be expected.

On theoretical grounds, therefore, a fairly accurate derivation of volume changes from observed girth changes in human limbs can be expected.

PRACTICAL CONSIDERATIONS

The precision required for girth measurements, if these are to provide an indirect record of limb volume changes, is of a fairly high order. For example, if a limb segment of 25 cm girth has a blood flow of 1 ml./100 ml./min, the girth will increase only 1/48 mm/sec following venous occlusion. It is necessary to make these small measurements on easily deformable tissues, hence the conditions of measurement must be, as far as is possible, isotonic.

Electrical strain gauges, of the resistance, capacitance and inductance types, have, in recent years, been extensively applied for recording small linear changes of dimension. Almost without exception, however, the gauges have been used for an isometric record. Capacitance and inductance gauges could be adapted for an isotonic record, but relatively complicated electronic amplifiers are required with these types of gauge. Amplification is not essential with resistance-wire gauges, but these cannot be constructed on the isotonic principle.

The apparatus, which is described below, for the precise measurement of changes in girth makes use of the mercury-in-rubber resistance strain gauge (Whitney, 1949). The principle of this gauge is the effect of extension of a small-bore rubber tube on the resistance of a mercury thread completely filling the bore of the tube. The ends of the tube are closed with tapered copper pins, the points being amalgamated before insertion in the tube to ensure good electrical contact between the end of the mercury thread and its respective pin. Leads from the two copper pins enable the gauge to form one arm of a Wheatstone bridge. If the bridge is balanced for a particular degree of extension of the tube, changes of extension plus or minus of the balanced extension will be recorded by the current flowing in the bridge galvanometer. A good linear relationship exists between the bridge output recorded by the galvanometer and the percentage extension or contraction of the gauge.

The actual voltage output of the strain-gauge bridge for a given percentage change in length of the gauge is dependent not only on the voltage applied to the bridge circuit but also on the type of galvanometer used for recording the output. Since the resistance of the strain gauge is usually low (2–4 Ω for a two-strand gauge) it is desirable, for the usual considerations which arise when recording with galvanometers, to use a galvanometer of comparatively low coil resistance and to damp the galvanometer electromagnetically with the resistance of the bridge circuit itself. No separate damping resistance is, therefore, required across the terminals of the galvanometer, and the resistance of the bridge circuit is suitably matched to the galvanometer by adjusting the resistance of the two ratio arms of the bridge. Since the necessary damping resistance is usually greater than the galvanometer coil resistance, the resistance of each ratio arm of the bridge will be much higher than that of the gauge and compensating arms of the bridge. For example, if a standard galvanometer of 10 Ω coil resistance and of 5 c/s natural frequency is used with a 2 Ω strain gauge, it is necessary to adjust each of the ratio arms of the bridge to about 80 Ω. Under these circumstances a bridge output of about 10 μV is obtained per 0·01 % change in the length of the gauge and per 1 V applied to the bridge, and cyclical changes of length up to a frequency of about 3 per sec can be reliably recorded. This output is quite adequate to drive galvanometers of only moderate current sensitivity, since up to 2 V can, if necessary, be applied to a 2 Ω gauge bridge circuit. For most physiological purposes it is necessary to attenuate the maximum sensitivity available and this can be conveniently done, without affecting the electrical matching of the galvanometer to its circuit, by varying the voltage applied to the bridge. A suitable circuit, incorporating these principles, is described below.

A practical form of the gauge for measuring changes in limb girth is illustrated in Fig. 1. This two-strand gauge measures a 1 cm-long segment of the limb. Different limb girths are accommodated by using mercury-in-rubber loops of different length, each loop being itself adjustable over about 2 cm of girth by use of the adjusting nut (Fig. 1 (ix)) provided on the gauge mounting. If it is desired to sample a limb segment longer than 1 cm, either several two-strand gauges, each with its own bridge and galvanometer circuit, can be employed, mounted at intervals along the limb segment, or a multi-strand gauge, constructed on the same lines as those indicated in Fig. 1, but with several mercury-in-rubber loops, can be used.

A suitable bridge circuit for use with the gauges is shown in Fig. 2. The system of temperature compensation for the gauges may require some explanation. The resistance of mercury has a comparatively high thermal coefficient, so that a temperature change of 1° C in the mercury filling of the gauge produces approximately the same change in resistance as would be produced

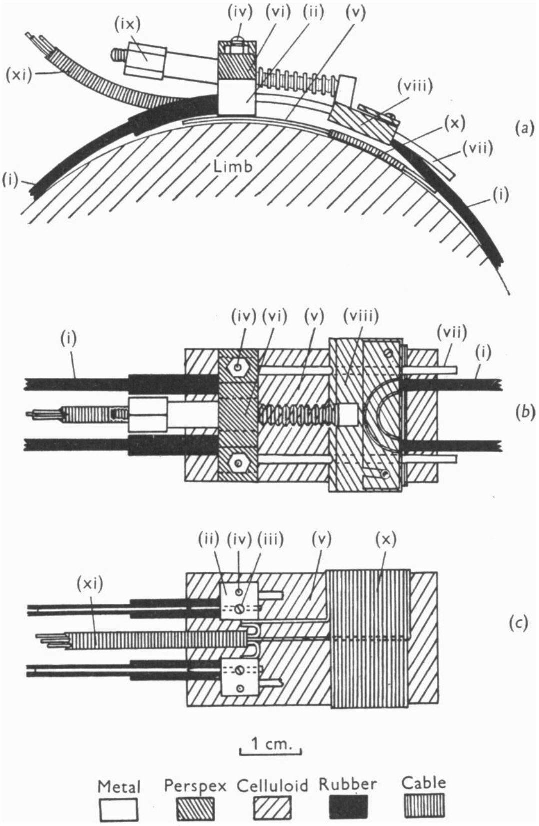

Fig. 1. Constructional details of two-strand mercury-in-rubber gauge. (*a*) Lateral view of gauge mounting viewed along axis of limb to which gauge is fitted. (*b*) Plan view of gauge mounting. (*c*) Plan view of gauge mounting partly dismantled to show electrical connexions. The plug in each end of the mercury-filled gauge rubber (i) is secured in a hole bored in the terminal block (ii) by means of a set screw (iii). The two terminal blocks are each secured to the curved celluloid base (v) by means of a countersunk screw (iv), the ends of the screws protruding above the blocks providing attachments for the Perspex bridge (vi). Each block also carries a curved guide rail (vii) for the sliding Perspex block (viii). A semicircular groove in (viii) accommodates the looped end of the gauge rubber, the rubber being kept in the groove by means of the hinged flap. The adjustment screw has a hinged attachment to the sliding block and passes through the Perspex bridge, any backlash of the adjusting nut (ix) being absorbed by the compression spring mounted on the screw between the bridge and the sliding block. The copper winding for thermal compensation (x) is mounted on the celluloid base, and the three electrical leads from the winding and the two terminal blocks are taken from the gauge by the three-core flexible lead (xi).

by a 0·05 % change in the length of the gauge. If limb girths are to be accurately recorded under varying conditions of ambient temperature, it is obviously necessary to compensate the gauge thermally. This compensation is conveniently provided by mounting a copper-wire winding on the gauge (Fig. 1 (x) and Fig. 2, R_2). This copper winding is subject to the same temperature changes as is the mercury filling the gauge and, if R_3, R_4 and R_5 have low thermal coefficients and are not subject to large temperature changes, then a consideration of the thermal coefficients of mercury and of copper will show that changes in the gauge resistance due to temperature changes will be compensated by equivalent changes of resistance in R_2.

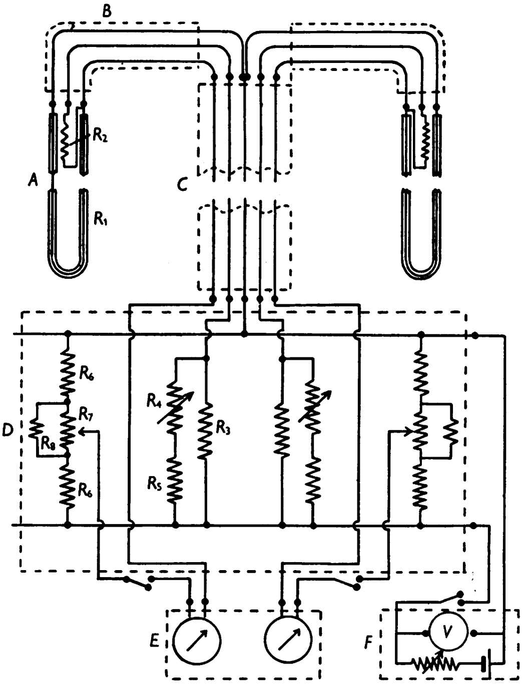

Fig. 2. Circuit diagram for two-channel recording from a pair of two-strand gauges. Components are lettered for the left-hand channel only. Separate units of the system are blocked off by the interrupted lines. The gauge *A* with its flexible lead *B* is plugged into one end of the long, multicore lead *C* which connects the subject with the recording circuit, and which may be common to several gauges. The bridge unit *D*, the recording galvanometer *E* and the battery power supply *F* are also common to the gauges in use. R_1, mercury-in-rubber strain gauge; R_2, copper thermal compensation winding, resistance $0{\cdot}28R_1$; R_3, constantan portion of bridge-balancing resistance, value $0{\cdot}76R_1$; R_4R_5, adjusting shunt for R_3: R_4 variable (maximum about $20R_1$), R_5 fixed, about $10R_1$; R_6, ratio arm of bridge (each ratio arm should be about twice the critical damping resistance for the galvanometer); R_7R_8, apex potentiometer calibrated for gauge tension: R_7 wire-wound 5Ω, R_8 $0{\cdot}5\Omega$ constantan shunt.

The rubber tube used in constructing the gauges requires some comment. The bore of the tube should be quite small, otherwise the resistance of the gauge will be inconveniently low. The wall of the tube should be reasonably thick to allow the tube to be looped as indicated in Fig. 1 without causing collapse of the bore. The tube should also have a reasonably high extensibility (% extension per g tension) so that limb girth can increase up to 2 or 3 % without undue rise in gauge tension. Good quality rubber is essential to ensure reasonable life and negligible plastic deformation when stretched up to 5 % of its natural length. No. 1 Surgical Drainage Tube, with a bore and wall thickness of about 0·7 mm, can be used for multi-strand gauges, but the gauge resistance will be less than 1 Ω if this material is used for two-strand gauges. Also, a 1 % extension of this material is accompanied by a rise in tension of about 7 g. A much more satisfactory tubing is manufactured by Messrs Dunlop Special Products Ltd. This is a latex tube with a bore of approximately 0·5 mm, and a wall thickness of about 0·8 mm. A 1 % extension of this material is accompanied by a rise in tension of only 2·5 g. Two-strand gauges for use on human limbs have an electrical resistance of 2-3 Ω when manufactured with this material.

APPLICATIONS

The use of the gauges for human limb girth measurement

The manner in which the gauge can be mounted on the human limb is shown in Fig. 3. In order that limb contractions as well as limb expansions can be recorded it is necessary that the tubing of the gauge should always remain in tension during recording. In most circumstances this situation is achieved by initially mounting the gauge at a tension of 20–30 g, the tension in each strand of the gauge being, of course, this total tension divided by the number of strands in the gauge. This tension also serves to keep the gauge in position on the limb, no further location being necessary unless considerable movement is allowed to the limb. If, as may happen with continuous recordings over a long period of time, large changes in limb girth occur, the conditions of tension can be readily restored by manipulation of the adjusting nut of the gauge. This adjustment will ensure that the record of girth does not go off the photographic paper. It will be apparent that, if the amount made of such adjustment is recorded, the base-line after adjustment can be accurately related to the base-line before adjustment in terms of limb girth or limb volume.

With two-strand gauges there is little difficulty in mounting the gauge on the limb so that both strands are at approximately the same tension. With multi-strand gauges, mounted on a longer segment of the limb, the variations in limb girth at different levels of the segment are readily accommodated by pulling the gauge rubber through the semicircular grooves of the sliding block on the gauge mounting until all the strands appear to be at approximately the same tension. It does not seem to be important to achieve precise equivalence of tension in all the strands of the gauge, for direct physical measurement shows that the resistance of a gauge, for a particular total tension of mounting, is not appreciably altered if the distribution of this tension over the different strands is varied. It is, of course, necessary for all the strands to remain in tension during expansions and contractions of the limb.

If very free movement is to be allowed to the limb it becomes necessary to provide additional means for locating the position of the gauge on the limb. Narrow strips of adhesive plaster, bridging the gauge mounting and the strands at various places round the circumference of the limb and stuck to the skin on each side, appear to be adequate for this purpose (see Fig. 3). A gauge mounted in this manner has been left in position for hours without causing discomfort to the subject. The three-core cable (Fig. 1 (xi)) connecting the gauge with the bridge unit should be sufficiently long to allow the subject the requisite freedom of movement and it should be securely anchored to the subject at a point near the gauge—this is conveniently done, if there is a collection cuff mounted on the same limb as is the gauge, by including the cable in one of the cuff wrappings. If a three-pin connector plug and socket are included in the three-core cable, the subject can, on occasion, be given complete freedom of movement without dismounting the gauge.

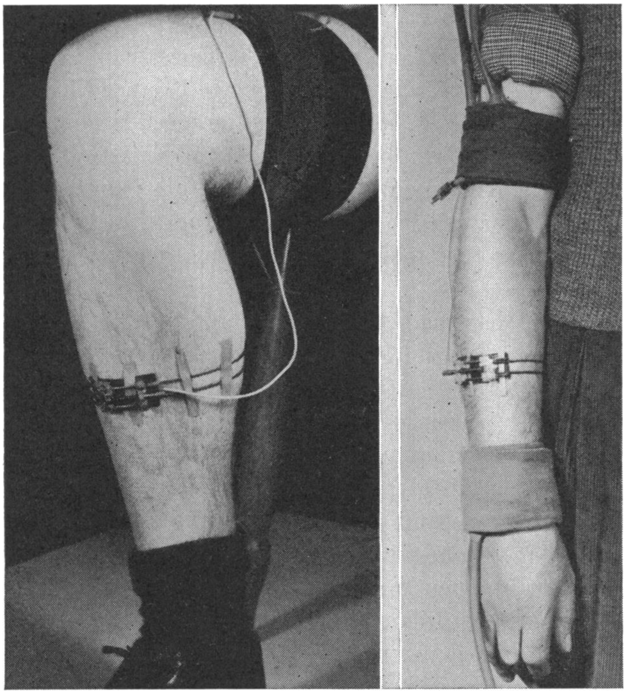

Fig. 3. Method of mounting gauge on forearm and calf. The small size of the gauge and the convenience with which it can be worn by the subject are illustrated. The simple method for preventing slipping of the gauge along the limb, by means of strips of adhesive plaster, is also shown.

The following procedure is adopted to ensure that the gauge is affixed to the limb at a known tension and to determine the girth of the limb at the position of the gauge. The gauge is opened out by moving the sliding block completely off its guides. The gauge mounting is then clamped in a stand so that the strands hang vertically from the mounting. The sliding block at the lower, looped end of the strands is then weighted to bring its total weight up to 25 g. With new gauge-rubber, the gauge is left hanging vertically in tension for 2 or 3 days to remove any irregularities in the strands. The gauge is then connected to its bridge circuit and, with the potentiometer R_7 set at the centre of its range, R_4 is adjusted for a null galvanometer reading. The dial knob position of R_7 is now identified for a gauge tension of 25 g, and the dial can, if it is so desired, be calibrated for tensions on either side of 25 g by varying the weights on the sliding block, the bridge being re-balanced for each particular tension by manipulation of R_7 only. The sliding block is now re-assembled on to the gauge mounting—it will be apparent from Fig. 1 that the sliding block need not be disturbed for mounting the gauge on a limb, the gauge being opened out for this purpose by moving aside the hinged cover to the semicircular groove cut into the block and so allowing the closed end of the rubber loop to be detached.

The bridge circuit having been balanced and calibrated for tension, the gauge is now mounted on a rigid cylindrical former of suitable known circumference. With R_7 (Fig. 2) set at the required tension (say, 25 g) the gauge-adjusting nut (Fig. 1(ix)) is loosened or tightened for a null galvanometer reading. The gauge is now mounted on the former at a tension of 25 g, and the basic length of the gauge at this tension is obtained by subtracting the distance between the sliding block and the gauge-mounting terminal ((viii) and (ii) respectively, Fig. 1(*a*)) from the known circumference of the circular former. When the gauge is subsequently mounted on a limb and adjusted to give a null reading with 25 g set, the circumference of the limb at the level of the gauge is immediately determined by measuring the distance between the terminal and the sliding block and adding this distance to the basic length of the gauge as determined at 25 g tension on the rigid former.

The bridge-circuit settings and the basic length of the gauge, once determined, remain valid for several days of use. The gauge characteristics do, however, change slowly with time and usage and it is desirable to repeat the balancing procedure at regular intervals. In particular, the basic length appears to increase very gradually with use over an extended period. Corresponding adjustments in the bridge circuit should involve merely a re-setting of the variable resistor R_4.

Changes of limb volume are recorded by deflexions of the galvanometer about the null position which, of course, corresponds to the initial limb girth and volume. Usually a continuous record will be required, and photographic recording from a mirror galvanometer has been used for this purpose. Fig. 4 shows the type of record obtained when the gauge is used for blood-flow determination by the venous occlusion method. The sensitivity of the system is immediately variable by altering the voltage applied to the bridge. When recording high blood flows, a bridge voltage of only 0·1 V may be required, and

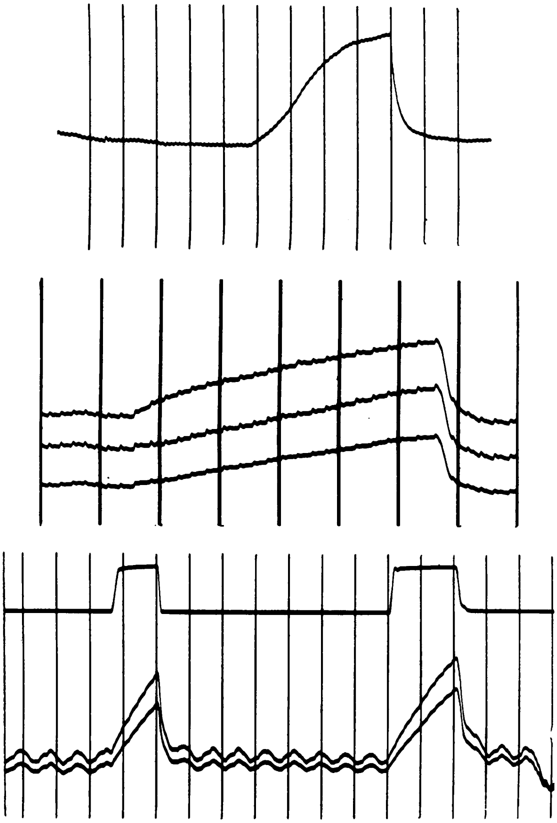

Fig. 4. Representative records obtained with gauges mounted on the forearm with venous occlusion, as for blood-flow determination. Time mark, 4 sec. The upper trace is a typical record from a single gauge when arterial occlusion is applied to the wrist followed by venous occlusion applied to the upper arm. The middle traces are for three separate gauges operating at higher sensitivity and show the pulse form obtained with the gauge. The lower traces are from two separate gauges with an accompanying trace for the venous occlusion pressure. The immediate response of the gauges is evident.

a simple circuit for dropping to this voltage from 2 V accumulator supplies is indicated in Fig. 2. During a record, the bridge voltage must be maintained constant, since the galvanometer output is directly proportional to this voltage.

In order to convert recorded galvanometer deflexions into equivalent limb volume changes, it is necessary to calibrate the apparatus at regular intervals during a continuous record. The need for such calibration was indicated when the theoretical aspects of limb girth measurement were considered. During an expansion, the increase of gauge length will be less than the increase of limb girth because, in the expanded condition, the increase in gauge tension will cause additional compression of the soft tissues. Similarly, during limb contraction, reduction in gauge length will be less than reduction in limb girth due to the fall in gauge tension. If, however, it can be assumed that the tissues are truly elastic over the range of compression to which they are subjected during measurements conducted with the gauge, then it can be shown (see Appendix II) that the change in gauge length (δL) will be proportional to the change in limb girth (δG), the ratio $\delta L/\delta G$ being dependent on the elasticity moduli of the gauge rubber and the soft tissues, on the actual girth of the limb, and on the proportion of bone (considered rigid) to soft tissue:

$$\frac{\delta G}{\delta L}=1+\frac{Ke}{GM},$$

e, modulus of gauge rubber (g/cm width); M, modulus of soft tissues (g/cm^2); G, girth of limb at gauge level (cm); K, constant, depending on proportion of bone to soft tissue at gauge level and equal approximately to 3 for 10 % of bone.

Frequent calibration of the gauge mounted on the limb is necessary to account for variation of the factors on which the value of $\delta G/\delta L$ depends. The principle on which calibration is carried out is as follows. Suppose that the gauge is mounted on a limb and that its tension has been adjusted to 25 g in the manner already described. The distance between the adjacent edges of the sliding block and the gauge-mounting terminal is then measured—this distance will be subsequently referred to as the 'gap'. The length of the gap added to the basic length of the gauge—determined as described above—gives the limb girth (G) at the level of the gauge and measured at a tension of 25 g. The gap is now increased or decreased by a definite amount—this can be done with adequate accuracy by turning the adjusting nut (Fig. 1(ix)) a definite number of complete turns, the pitch of the thread of the adjusting nut being known. If an alteration of the gap distance by 'p' mm causes a galvanometer deflexion of 'a' mm then the principle of gauge calibration assumes that this same deflexion of 'a' mm would be obtained if the limb girth altered by a percentage which, if measured under conditions of zero restraint, would be $100p/G$ %. In other words, the principle of calibration assumes that a galvanometer deflexion of 'a' mm is equivalent to an unrestrained alteration of limb girth of $100p/G$ %. The argument on which this assumption is based is given in Appendix II. Accepting this principle it follows that the percentage variation of limb girth (g_c) corresponding to a 1 mm galvanometer deflexion will be: $100p/Ga$ %, 'g_c' being the calibration factor required for the analysis of records taken with the gauge. If the suggested relationship between girth and volume change is accepted, it further follows that the percentage change of limb volume (V_c) corresponding to a 1 mm galvanometer deflexion will be equal to $2g_c$ or to $200p/Ga\,\%$.

It has been mentioned above that the alteration of gap distances during calibration is effected by turning the adjusting nut of the gauge mounting. It should be added here that one turn of the adjusting nut does not alter the gap distance by an amount exactly equal to the pitch dimension of the screw on which the nut runs. The discrepancy arises partly from the fact that the sliding block moves on curved rails, but chiefly it is due to the fact that these rails need to be raised, for obvious constructional reasons, above the surface of the limb and because the bearings of the adjusting screw (Fig. 1 (vi)) are raised even further above the surface of the limb. If the precise dimensions are known the discrepancy can be calculated on simple geometrical principles. A No. 7 B.A. thread (pitch 0·48 mm) has been used for the adjusting screw, and a radius of curvature of 4 cm for the rails has been found suitable for all positions on the upper arm, forearm, thigh and calf of man. On the gauge mounting the rails are located 0·5 cm, and the adjusting screw bearings 1·0 cm, above the concave surface of the celluloid base (Fig. 1 (v)) of the gauge mounting. With these dimensions one complete turn of the adjusting nut alters the gap distance by 0·40 mm, and this value, which can be calculated from the geometry of the mounting, has been confirmed by measurements of the output of a gauge mounted on an expansible metal former. The effective pitch of the adjusting screw does not alter by more than 1 % for different positions of the sliding block along the curved rails or for variations in limb girth between 150–350 mm.

The reliability of limb volume variations determined with the gauge

The reliability of the gauge method for recording changes in limb volume depends, first, on the accuracy with which girth changes can be recorded; and, secondly, on the validity of the assumed relationship between girth and volume. In connexion with the first question a series of purely physical measurements on the gauge has been carried out, the results of which may be summarized as follows: (*a*) If the strands of the gauge are disposed horizontally so that they can be stretched or relaxed by precise amounts, the gauge output is entirely in accord with theory, it being possible to deduce changes in length from the gauge output and the known physical characteristics of the gauge and its circuit with a high degree of accuracy. (*b*) If the gauge is mounted on a rigid cylindrical former, the circumference of which can be altered by known amounts, the gauge output is again in agreement with prediction both when the output is obtained by altering the circumference of the former and when it is obtained by altering the gap distance by manipulation of the adjusting nut of the gauge. (*c*) If the gauge is mounted on a former, of circular or approximately circular cross-section, but this time covered with a layer of sorbo rubber, and if calibration of the gauge is carried out in the usual way, it is possible to deduce changes in the circumference of the former from the observed gauge output. The discrepancy between actual and deduced changes was never greater than 2 % of the actual change. The agreement obtained between actual and deduced changes is considered to be very good, in view of the difficulty of producing known changes of circumference in the deformable sorbo rubber surface.

To these physical tests may be added observations with the gauge mounted on an actual limb; gauge output being obtained by varying, with the adjustment nut, the gap distance at the gauge mounting as in normal calibration. Over a reasonable range of adjustment the gauge output is proportional to change in gap distance. From this it may be inferred that the living tissues behave as an elastic solid over the range of tensions likely to be employed in making measurements of limb girth.

The physical tests described above indicate that the gauge, if suitably calibrated, is capable of measuring circumference changes in elastically deformable objects of cylindrical or subcylindrical shape. The observations made with the gauge mounted on a limb indicate that, within a reasonable range of compression, the limb is subject to truly elastic deformation. The gauge output should, therefore, be a valid indication of changes of limb girth, the accuracy of this indication being largely determined by the errors of calibration. These errors have been investigated by repeatedly screwing up and unscrewing the adjustment nut of the gauge by the same amount when the gauge is mounted on a limb. The variation of the calibration outputs for the same alteration of gap distance on the gauge mounting shows that the error of basing gauge calibration on the measurement of only three successive calibration outputs is less than 3 %.

The validity of the suggested relationship between girth and volume changes in a limb could be tested most effectively by mounting a gauge on a limb, the volume of which could be varied precisely by a perfusion technique. Since this method has not been found practicable for testing the gauges on human limbs, the reliability of the gauges in recording limb volume changes had to be assessed by comparing the performance of the gauge with that of the standard water-filled plethysmograph. Such a comparison of the new gauge method with the established plethysmographic method is, in any case, desirable in view of the considerable use that has been made of the plethysmograph in the past. This method of comparison, however, does not provide an entirely satisfactory test of the absolute volumetric accuracy of the gauge method, for it makes the assumption that the plethysmograph can provide an accurate estimate of limb volume change. The absolute volumetric accuracy of the plethysmographic method cannot be inferred from a theoretical consideration of the system—in fact it can be shown theoretically (see Appendix II) that the water or air plethysmograph should always record a lower value than the actual limb volume change. On the practical side Lansdowne & Katz (1942) have indicated that considerable discrepancies can arise in deducing volume changes from the plethysmographic record. So far as is known no absolute check on the accuracy of the plethysmograph in recording limb volume change has yet been made.

In checking the performance of the gauge against that of the water plethysmograph the practical difficulty arises of obtaining strictly comparable measurements by the two techniques. The nature of this difficulty will become apparent in a consideration of the following section.

Comparison of the gauge method with the water plethysmographic method for recording volume changes in the human forearm

Two methods of comparison of the two techniques have been employed. With both methods the volumetric techniques were used in forearm blood-flow determinations by the usual venous occlusion principle.

Method 1. Forearm blood-flow determinations using gauges only on the forearm. The conditions under which the determinations were made simulated, as nearly as possible, those described in Barcroft & Edholm (1943) for determinations using the water plethysmographic technique and may be summarized as follows: forearm and hand immersed completely in water maintained at 35° C, venous occlusion by a cuff applied to the upper arm at a pressure of 50 mm Hg, arterial occlusion by a cuff applied to the wrist at a pressure of 200 mm Hg 1 min prior to the venous occlusion in each blood-flow determination. In each determination three separate gauge records were obtained from three gauges mounted at 3 cm intervals along the middle segment of the forearm.

Method 2. With this method simultaneous records of forearm volume changes with the two techniques were made. A conventional type of water plethysmograph, constructed of aluminium and Perspex, was used, the water seal at each end consisting of a thick rubber diaphragm and thin rubber sleeves stuck to the skin with rubber solution. Diaphragms and sleeves were carefully made to measure for each subject, and the usual precautions were taken in obtaining and analysing the plethysmographic record. The plethysmograph was adapted so that two or three separate two-strand gauges could be mounted, at roughly 3 cm intervals, on the segment of arm enclosed in the plethysmograph. The gauges could be adjusted and calibrated without appreciable disturbance of the plethysmographic equipment. The plethysmographic space was filled with water and the complete hand and forearm immersed in a water-bath maintained at a constant temperature during recordings. The latter were obtained for the typical venous occlusion blood-flow determination procedure, carried out at various bath temperatures and either with or without an arterial occlusion cuff applied to the wrist. Each set of simultaneous volume/time records consisted, therefore, of one plethysmographic record on kymograph paper and two or three gauge records on the photographic paper.

Results

The results obtained with Method 1, obtained by the analysis of forty-four sets of gauge records (each set comprising three separate and simultaneous gauge records) made on five subjects, are summarized in Table 1. The table includes values obtained with the water plethysmograph by Barcroft & Edholm (1943) under similar conditions to those employed with the present determinations.

Table 1. Human forearm blood flow, arm fully immersed in water at 35° C

Summary of values deduced from gauge records taken for forty-four determinations on five subjects. The corresponding values given for the water plethysmograph are taken from data in Barcroft & Edholm (1943), these data being from sixty-eight determinations on five subjects. All values are in ml./100 ml. tissue/min

	Mean flow	S.E. of mean	S.D. of flows	Actual range of flows
Proximal gauge	3·40	0·23	1·53	1·82–7·70
Middle gauge	3·32	0·19	1·25	1·69–7·01
Distal gauge	3·19	0·14	0·94	1·82–5·08
Mean all gauges	3·30	0·17	1·13	1·94–6·40
Water plethysmograph	4·0	—	—	1·5–7·0

Reference to Table 1 shows that there is fair agreement between the two techniques, having regard to the circumstance that the subjects were different for the two techniques, agreement being seen both in the mean flow deduced for all determinations and in the range of blood flow which was encountered in the separate determinations. It seems probable that if determinations with the water plethysmograph had been made on the five subjects employed in determinations with the gauges a closer agreement between the two techniques would have been obtained.

A second feature of the gauge results noticeable in Table 1 is the general similarity of the results obtained at the three different levels of the middle segment of the forearm. Although there is an indication of a gradual decrease in the recorded flow towards the more distal part of the segment, this is not statistically significant. The value for the combined gauge flow, i.e. the average of the means of each set of three flows, is almost identical with mean flow deduced from the middle gauge records. A single two-strand gauge mounted on the middle of the forearm should, therefore, give blood-flow records which are representative of a segment of the forearm several cm in length. It cannot, of course, be assumed that this degree of axial uniformity exists in other parts of the body, and the results discussed above indicate how effectively the narrow, two-strand gauge could be used to detect departures from axial uniformity.

The results obtained by Method 2 for the comparison of the new with the old volumetric technique are summarized in Fig. 5, in which the forearm blood flow deduced from each separate gauge record is plotted against the value deduced from the simultaneous plethysmographic record. A linear relationship between the values obtained with the two techniques is apparent—linear correlation coefficients, either between the separate gauge values and the plethysmographic values, or between the mean gauge values lie between 0·80 and 0·95, and are therefore highly significant statistically. The differences between the slopes of the linear regression lines for the three gauge positions are not, however, significant. The line fitted to the data in Fig. 5 is for the mean gauge value against the plethysmographic value, and this shows that blood flows (and, therefore, volume changes) recorded by the gauges are consistently about 50% higher than simultaneous values deduced from the plethysmographic record.

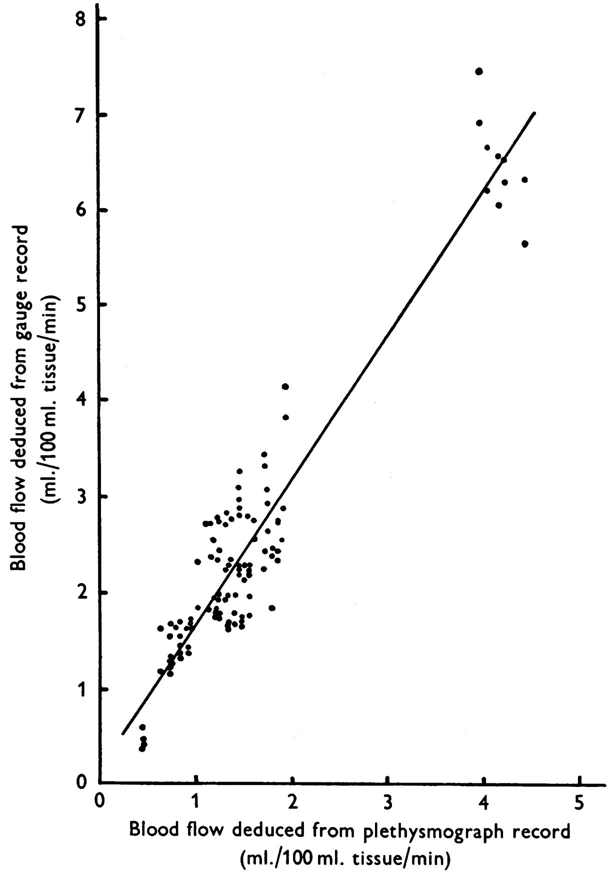

Fig. 5. Human forearm blood flow: values deduced from gauge records plotted against values deduced from simultaneous water plethysmograph records. The value for each separate gauge is plotted against its corresponding plethysmographic value, two or three gauge values being obtained for each plethysmographic value. Data are from fifty sets of determinations on two subjects.

The results obtained by Method 2 for comparing the two techniques are, therefore, at variance with those obtained by Method 1, in which tolerable agreement between the two techniques was indicated. In the simultaneous comparison of the two techniques, carried out in the manner described, the gauge values, though proportional to the plethysmographic values, are no longer equal to these values. Either, therefore, there is a moderately consistent discrepancy in the values obtained with one of the techniques—in which case it would be necessary to account for the agreement noted in the results of Method 1, or else Method 2 is not a valid means of comparing the two techniques. The following considerations suggest that Method 2 is not valid.

(*a*) It will be seen from Fig. 5 that there is considerable scatter about the fitted regression line—on some occasions the gauge value is equal to the plethysmographic value, on other occasions the former may be twice the latter. It is difficult to account for this variation except on the grounds that there was mutual interference between the two techniques, and that this interference was variable in extent. It is not possible to say which technique was chiefly affected by the interference, but it is difficult to visualize how the presence of the gauges could influence the plethysmographic record.

(*b*) There is some direct evidence that the presence of the plethysmograph on the arm interfered with the operation of the gauges. In discussing the results from Method 1 it was noted that the simultaneous gauge values at different levels of the middle segment of the forearm were almost equal, the values showing only a slight tendency to decrease from the proximal level to the distal level. With Method 2 bigger differences between the levels were obtained, and the gauge value obtained from the middle level was typically higher than the values recorded simultaneously from both the proximal and distal levels, the middle level values being, on the average, 7·7 % higher than the corresponding values recorded at the proximal level and 7·2 % higher than the distal level values. The statistical significance of these differences has been examined by using the analysis of variance to compare the regression coefficients (relating gauge values to plethysmographic values) for each pair of gauge levels. This analysis showed that, although the slopes of the three regression lines are not significantly different, the intercept of the middle gauge regression line on the ordinate (i.e. the 'Gauge Value') axis is significantly higher, at about the 1 % level of significance, than the intercepts of the proximal and distal gauge regression lines.

These observations suggest that the presence of the plethysmograph interferes with the normal pattern of volume change at different axial levels of the limb. It is reasonable to suppose that the chief source of interference is the sealing sleeves closing the ends of the plethysmograph, the sleeves exerting local restriction on volume changes resulting from the venous occlusion procedure used in the blood-flow determinations. The effect of this restriction on the accuracy of the plethysmographic record is discussed in Appendix II (see section headed: 'Errors of water or air plethysmograph records'). It has not been possible to estimate theoretically what effect such local restriction would have on the accuracy of the gauge record. It would be expected, however, that gauges mounted near to the sealing sleeves would record a lower local volume change, for a given volume change in the limb as a whole, than they would in the absence of the sleeves, and this effect would seem to account for the difference between the axial patterns of volume change in the forearm obtained with the gauges in the presence and absence of the plethysmograph. The effect of the sleeve restriction on the volumetric accuracy of the gauges could be extreme, for the theory on which the use of the gauges is based (see Appendix II) assumes that the only external restriction to volume changes in the limb is that exerted by the gauge itself, the effect of this restriction being accounted for in a calibration procedure which simulates the alterations, in the restriction exerted by the gauge, which would be produced by actual changes in limb volume. The additional restriction imposed by the plethysmograph sleeves therefore falsifies, to an unknown and probably variable extent, some of the assumptions on which the theory of the use of the gauge is based, and the alterations of this plethysmographic restriction with changes in limb volume are obviously not taken into account as a result of gauge calibration.

On the basis of the above argument it seems reasonable to conclude that the results of Method 2 are not indicative, either of the comparative accuracies of the gauge and plethysmographic techniques, or of the absolute volumetric accuracy of the gauge technique. The results of Method 1, in which the gauges were employed in a normal manner, do seem to indicate that results obtained under these circumstances should be of the same order as would be obtained with the plethysmographic technique. On the available experimental evidence it therefore seems reasonable to conclude that the gauge technique offers a sufficiently reliable method for recording changes in limb volume. This conclusion, which must be regarded as tentative until further use has been made of the gauge technique and until better means have been devised for assessing its absolute accuracy, is supported by the sound theoretical background on which the use and operation of the gauge can be based, and is further supported by the favourable results obtained when the gauge technique is tested in the measurement of inanimate objects under conditions very similar to those under which the living limb is measured.

The advantages of the gauge technique as compared with the plethysmographic technique

In view of the criticisms which have been made, in the opening paragraphs of this paper, of the established plethysmographic technique, it will be appropriate to summarize here the relative advantages of the gauge technique. Some of these advantages will have become evident during the preceding discussions of the construction and use of the gauges. In enumerating these advantages of the gauge technique the tentative conclusion will be assumed that the technique has a volumetric accuracy at least as good as that attainable with the current air or water plethysmograph. Application of the volumetric techniques for determining peripheral blood flow by the venous occlusion procedure and for investigating vasomotor changes are particularly stressed in the following list of suggested advantages:

(*a*) The gauge is small, light in weight, and does not cover a large area of the surface of the limb under examination. These features ensure that the gauge can be 'worn' by the subject for long periods without undue exertion or discomfort, that free movement on the part of the subject is not unduly restricted, and that normal relations of the surface of the limb with its surrounding environment are not appreciably influenced by the presence of the gauge on the limb (e.g. heat exchanges between the surface of the limb and its surroundings will not be seriously affected by mounting a gauge on the limb).

(*b*) The gauge can be rapidly mounted and dismounted from the limb, and the conditions of mounting—position on the limb and tension of the gauge—are readily controllable and accurately repeatable. Total mounting and adjusting time can be limited to less than 1 min in the case of the gauge, compared with a minimum of about 15 min which is required in the case of the simplest and most convenient plethysmograph. Moreover, owing to the simplicity of the electrical connexions which have to be made, the gauge can be mounted and adjusted on the subject at any time previous to the experimental period without unduly restricting the movements or occupation of the subject, connexion of the gauge with the recording apparatus being then possible within a few seconds of the experimental period. Since the conditions of mounting can be repeated with reliability the gauge could be employed as an accurate 'measuring tape' for recording changes in limb girth from day to day as, for example, in the investigation of oedematous conditions. If the subject is to be allowed much freedom when the gauge has been mounted it is advisable, as has been pointed out, to apply narrow adhesive strips to the skin, bridging the rubber strands at a number of points round the circumference. This procedure does not greatly increase the time required to mount the gauge.

(*c*) It is necessary, in the case of the plethysmograph, to avoid relative movement between the limb and the plethysmograph. This requirement typically leads to a considerable degree of immobilization of the limb. The light weight and small size of the gauge, and the flexibility of its electrical connexions ensures that the limb and gauge can move together without appreciable relative movement between the two.

(*d*) The electrical connexions between the gauge and the recording apparatus can be several yards long. This advantage confers considerable latitude in the placement of and in the movements allowed to the subject. It also means that all apparatus, apart from the gauge itself, need not be seen by the subject.

(*e*) With the gauge technique it is possible to record limb volume changes over a considerable period without any practical limitations on the extent of the volume changes or on the temperature conditions to which the subject is exposed. This is made possible by the ease and comfort with which the gauge can be worn for long periods, by the trivial extent to which the presence of the gauge interferes with physical exchange (e.g. of heat) between the subject and the environment, and by the temperature compensation of the gauge. Positive adjustment of the gauge base line enables considerable volume changes to be continuously recorded without debasing the sensitivity of the system to keep the record on the recording paper.

(*f*) The gauge technique can be readily employed to record volume changes in very short limb segments, several simultaneous records from different parts of the same limb being obtained if necessary. In other words, greater discrimination can be achieved with the gauge than is possible with the plethysmograph. The gauge can also be mounted in positions (e.g. upper arm or thigh) where placement of a plethysmograph would be difficult, inconvenient or impossible.

(*g*) With the gauge technique it is not necessary to measure the volume of the limb segment under examination.

(*h*) The system of gauge calibration simulates, in all theoretical essentials, an actual volume change of the particular limb segment under examination. This feature, which has obvious advantages on the score of accuracy, is not possible in the case of the plethysmograph.

The chief criticism which can be made of the gauge technique is that, unlike the plethysmograph, it does not measure volume change directly. The evidence already given in this connexion suggests, however, that volume changes deduced from the gauge record of girth change are probably of acceptable accuracy.

Whilst it appears possible to employ the gauge technique, often with considerable advantage, under practically all the circumstances in which the plethysmograph has hitherto been used on the human subject, and whilst the technique can be used under some circumstances in which use of a plethysmograph would be inconvenient or impossible, certain limitations of the method should be mentioned here. For example, there is no obvious possibility of using the gauge to record changes in the whole hand or in the whole foot in the manner which is possible with the plethysmograph. It will also be obvious that the gauges cannot be used on parts of the body (e.g. in the neighbourhood of certain joints, such as the ankle or knee joints) where the encircling rubber strands cannot make contact with the limb surface at all parts of the circumference. It is possible that reliable records could be obtained with the gauge under these circumstances if the hollows in the surface are filled out with an elastic material (such as sorbo rubber), but this expedient has not, as yet, been investigated.

The use of the gauge for measuring tissue elasticity

Although it is not immediately relevant to the subject of limb volume measurement, one further application of the mercury-in-rubber strain gauge will be considered here—the measurement of tissue elasticity.

It has been pointed out above that the change in length of the gauge (δL) can be related to the change in girth of the limb (δG) by the following equation:

$$\delta G/\delta L = 1 + Ke/GM,$$

and it has been indicated that the same relationship holds between an alteration made in the length of the gauge gap with the adjustment nut and the change in gauge length which this adjustment produces. It will be obvious that, if the gauge is mounted on a rigid cylindrical former, the change in gauge length will be equal to the adjustment made. The following relationship can then be derived from the equation given:

$$\frac{\text{Galvanometer output per mm adjustment on rigid former}}{\text{Galvanometer output per mm adjustment on limb}} = 1 + Ke/GM.$$

In this equation, 'G' (girth of limb) is determinable by the method described above. The elasticity of the gauge rubber per unit width (e) can be found by direct physical measurement, and 'K' can be estimated from the proportion of bone to soft tissue for the segment of the limb under examination; it being sufficiently accurate to make use of standard anatomical diagrams for this purpose (e.g. Morton, Truex & Kellner, 1941). The value of 'M'—the Young's modulus for the soft tissues of the limb can, therefore, be deduced. The modulus so derived will, of course, be a combined value for the different soft tissues of the limb segment. In most cases, however, muscular tissue will predominate and the modulus obtained probably relates largely to this tissue. A few determinations of muscle modulus made in this way on two subjects indicate values for the modulus of the human forearm muscles between 600 and 900 g/cm^2. It has not been possible to compare these values with comparable measurements made on muscle by other workers because the relevant data provided with, for example, published load-extension curves for isolated muscle are usually inadequate for the computation of a modulus. It is interesting to note, however, that the modulus for the toad sartorius muscle at its natural resting length appears to be approximately 500 g/cm^2 (rough calculation from fig. 5 of Hill, 1949), and that the modulus deduced from the load-extension curve for the tibialis of the rabbit (Crawford, unpublished data) is approximately 700 g/cm^2.

SUMMARY

1. The subject of volume change measurement in human limbs is briefly reviewed and a new method, employing the mercury-in-rubber strain-gauge, is described. With the new method, percentage changes in limb girth are

measured directly and percentage changes in limb volume are deduced on the assumption that percentage change in limb volume is twice the percentage change in girth.

2. The following advantages are claimed for the new method, comparison being made in particular with the water or air plethysmograph:

(*a*) The apparatus mounted on the limb is much less cumbersome and normally consists only of a narrow, flat bracelet.

(*b*) The apparatus is very rapidly assembled on the limb and can remain attached to the limb during most movements and other working activities. Records of limb volume can be obtained immediately following cessation of such activities or, in many instances, actually during such activities.

(*c*) A system of temperature compensation is applied with the new method which allows accurate measurement of volume change without the imposition of restrictions on the temperature or on temperature changes in the limb. The gauge itself covers only a small area of the limb under examination and the gauge can be worn under loose clothing.

(*d*) The gauge itself is the only part of the apparatus which needs to be near the subject. Connexion of the gauge with the recording apparatus can be made by a single multicore cable several yards in length.

(*e*) The gauge records a percentage change directly. It is therefore unnecessary to measure separately the volume of any limb segment under examination. In addition, the percentage change as measured by the gauge is, on theoretical grounds, an accurate estimate of the unrestrained percentage change. The absolute limb volume change, as recorded by the water or air plethysmograph, must be dependent on the degree of restraint imposed on limb volume changes by the presence of the measuring apparatus.

(*f*) The system of calibration possible with the gauge simulates very closely a known limb volume change under the actual conditions of measurement. Such simulation is not possible with the water or air plethysmograph.

3. The theory on which the gauge method is based is fully examined and certain aspects of the theory have been checked by physical tests. No disagreements with the theory have been found.

4. The gauge method has also been checked by comparing its performance with that of the water plethysmograph in estimating human forearm blood flow. Ideal conditions for the comparison could not be achieved for practical reasons, but the results obtained with the two methods do not suggest that volume changes deduced from the gauge record are grossly different from those which would be recorded, under the same circumstances, by the usual type of water plethysmograph.

5. The use of the gauge for measuring tissue elasticity of living human limbs is briefly described.

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APPENDIX I

The relationship between change of volume and change of circumference when the transverse section of the segment is not circular

If the length of the segment remains constant, the change in volume will be directly proportional to the change of cross-sectional area. Consider the division of the transverse section into a large number of narrow triangular elements by drawing radii from some point, O, lying within the circumference of the section (see Fig. 6). Let OAB be one such triangular element; AB being a small part of the circumference of the whole section. Consider an increase of the area of the whole section without change of shape. The element OAB changes, with such an increase, to $OA'B'$. If the shape of the section is to remain unaltered, the length of all radii drawn from O must increase in the same proportion, i.e.

$$\frac{AA'}{OA}=\frac{BB'}{OB}=k. \qquad (1)$$

It follows from simple geometrical principles that $A'B'$ will be parallel to AB. Let AB and $A'B'$ be produced to meet OCC' at right angles.

For the triangular element of the section:

$$\text{Geometrically: } \frac{A'B'}{AB}=\frac{OA'}{OA}=\frac{OA+AA'}{OA}=(1+k),$$

or
$$A'B'=AB(1+k), \qquad (2)$$

that is

$$\frac{\text{Increase in length of portion of circumference}}{\text{Initial length of portion of circumference}}=\frac{A'B'-AB}{AB}=k. \qquad (3)$$

Similarly
$$\frac{OC'}{OC}=\frac{OA'}{OA}=(1+k), \quad \text{or} \quad OC'=OC(1+k).$$

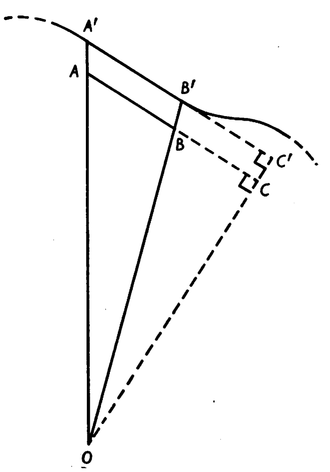

Fig. 6.

From (2) $$A'B' = AB(1+k).$$

Therefore $$\tfrac{1}{2}OC'.A'B' = \tfrac{1}{2}OC.AB(1+k)^2. \tag{4}$$

If the increase in size of the section is small, $(1+k)^2 \simeq 1+2k$. From (4)

$$\frac{\tfrac{1}{2}OC'.A'B' - \tfrac{1}{2}OC.AB}{\tfrac{1}{2}OC.AB} \simeq 2k,$$

or $$\frac{\text{Increase in area of triangular element}}{\text{Initial area of triangular element}} \simeq 2k. \tag{5}$$

Thus, for each triangular element of the section, it appears (from eqns (3) and (5)) that the proportionate increase in area of the element is, to a close approximation for small proportionate increases in area, equal to twice the proportionate increase of that part of the circumference of the whole section which is included in the element. Therefore, by summing over all the elements of the section, if an increase, δG, in the initial circumference, G, of the section accompanies an increase, δH, in the initial area, H, of the cross section, then

$$\frac{\delta H}{H} = 2\frac{\delta G}{G}$$

to a close approximation for small δH. Since, for uniform length of segment, $\delta H/H = \delta V/V$ if $\delta V/V$ is the proportionate increase in the volume of the segment, then

$$\frac{\delta V}{V} = 2\frac{\delta G}{G}$$

to a close approximation if δV is small compared with V.

APPENDIX II

The relationship between actual and measured limb girth changes

For theoretical purposes, the limb is considered to be a cylinder of radius R_0 (cm) with a rigid concentric core (the bone) of radius nR_0 covered by a layer of elastic material (the soft tissues) of modulus M (g/cm^2). The amount of elastic tissue in a given length of the cylinder can be increased or decreased; the whole of the increase or decrease being accommodated by a corresponding change, δR_0, in the radius of the cylinder, if the change is not accompanied by a change in pressure on the curved surface of the cylinder. If such a change in pressure occurs, so that the cylinder tends to change in length as well as in radius, then it is supposed that these changes in length of the elastic material are completely unopposed. It is considered that these conditions provide a good approximation to the circumstances prevailing for actual limb volume changes.

It can be shown, by the straightforward application of the mathematical theory of elasticity to such a system, that the effect of a pressure P (g/cm^2) exerted on the curved surface of the cylinder is to reduce the initial radius, R_0, of the cylinder by an amount aP where

$$a = \frac{R_0}{M}\frac{(1-n^2)(1-\sigma^2)}{(1+\sigma) - n^2(1-\sigma)}. \tag{1}$$

'σ' is the Poisson ratio for the elastic material and is equal to 0·5 if the volume of the material remains unaltered under stress. Such an assumption seems reasonable for living tissues subject to the small stresses which will be considered. The eqn (1) can therefore be simplified to

$$a = R_0\frac{A}{M} \quad \text{where } A = \frac{3(1-n^2)}{2(3-n^2)}. \tag{2}$$

The cylinder is now encircled by an elastic band (the gauge) of modulus e (g/cm width) at a tension T (g/cm width). Due to the pressure under the band, the radius of the cylinder is reduced from R_0 to R. Since the pressure exerted by the band on the cylinder is T/R (according to the law of Laplace) it follows that

$$R_0 = R + \frac{aT}{R}. \tag{3}$$

The elastic part of the cylinder is now subject to an expansion which, with the cylinder unrestrained externally, would result in an increase in radius of δR_0. With the restraint imposed by the band, the radius under the band increases by only δS and the tension in the band increases by $\delta T = e.2n.\delta S/2nR = e.\delta S/R$. Then obviously, from eqn (3):

$$(R_0+\delta R_0)=(R+\delta S)+\frac{a(T+e.\delta S/R)}{(R+\delta S)}. \tag{4}$$

Dividing (4) by (3) and simplifying:

$$\frac{\delta R_0}{R_0}=\frac{\delta S}{(R+\delta S)}\,\frac{R(R+\delta S)+a(e-T)}{(R^2+aT)}. \tag{5}$$

The gauge, at tension T and radius R, is calibrated on the cylinder by approximating its ends by a known amount and noting the galvanometer output due to the stretching of the gauge. To simplify the argument, it is supposed that the gauge ends are approximated by $2\pi.\delta R$, δR being chosen so that the gauge itself is stretched by $2\pi.\delta S$, the gauge tension rising during calibration by $\delta T = e.\delta S/R$. It will be obvious that a gauge length increase of $2\pi.\delta S$ during calibration corresponds to an increase of cylinder girth of $2\pi.\delta R$ under uniform tension T, for the rise in gauge tension attending such an increase in cylinder girth could be exactly eliminated by increasing the distance between the ends of the gauge by an amount $2\pi.\delta R$. Therefore, the galvanometer output on calibration (which is directly proportional to $2\pi.\delta S$, the amount by which the gauge is stretched) corresponds exactly to an increase of cylinder girth, $2\pi.\delta R$, measured at a constant tension T. $2\pi.\delta S$ is, by definition, the increase in cylinder girth, measured at tension $(T+\delta T)$, corresponding to an unrestrained increase in cylinder girth of $2\pi.\delta R_0$. The gauge calibration procedure therefore deduces a measured girth change of $2\pi.\delta R/2\pi.R = \delta R/R$ for an actual, unrestrained girth change of $2\pi.\delta R_0/2\pi.R_0 = \delta R_0/R_0$. Theoretical validation of the gauge method therefore depends on establishing that $\delta R/R$ is approximately equal to $\delta R_0/R_0$.

In the preceding paragraph, it was established that a cylinder girth of $2\pi(R+\delta R)$ at tension T becomes a girth of $2\pi(R+\delta S)$ at tension $(T+\delta T)$, where $\delta T = e.\delta S/R$. Obviously,

$$(R+\delta R)=(R+\delta S)+ae.\delta S/R^2, \tag{6}$$

or

$$\delta S=\frac{\delta R.R^2}{(R^2+ae)}.$$

Eqn (5) can now be rewritten. Substituting δS from (6), putting $a = RA/M$ and making the allowable approximation $(R+\delta S) = R$ we obtain

$$\frac{\delta R_0}{R_0}\simeq\frac{\delta R}{R}\,\frac{MR+A(e-T)}{MR+A(e+T)+A^2eT/MR}. \tag{7}$$

For practical values of M, R, A, e and T the term A^2eT/MR is quite insignificant compared with $[MR+A(e+T)]$ and can therefore be ignored. Also, e and T are not entirely independent in practice, for the gauge must be stretched sufficiently to allow decrease of limb girth to be followed. T is, therefore, replaced by et, where t has a lower limit of the order 0·02 (corresponding to an initial stretching of the gauge by 2 % to accommodate limb volume decreases up to 4 %). Substituting for T and making allowable approximations

$$\frac{\delta R_0}{R_0}\simeq\frac{\delta R}{R}\left(1-\frac{2t}{1+MR/Ae}\right). \tag{8}$$

Eqn (8) establishes that the measured change of limb girth ($\delta G/G = \delta R/R$) recorded at a tension $T = te$, is greater than the actual change in limb girth ($\delta G_0/G_0 = \delta R_0/R_0$). In practice, however, the difference is very small, e.g. putting in (8): $t = 0{\cdot}02$, $M = 800$ g/cm², $R = 4$ cm, $A = 0{\cdot}48$ (corresponding to $n = 0{\cdot}25$) and $e = 2000$ g/cm, it will be seen that $\delta G_0/G_0 = 0{\cdot}992\ \delta G/G$. Thus, for practical conditions, the measured limb girth change is within 1 % of the actual limb girth change. Limb volume changes deduced from the gauge record will also be within 1 % of the actual changes.

Errors of water or air plethysmograph records

The conditions operating when a limb expands in an air or water plethysmograph are somewhat obscure. It is not clear, for example, if the rigidity of the diaphragms closing the ends of the water or air-filled chamber can always be assumed. One point can, however, be examined on the basis of the theory outlined above—the effect of the sealing sleeves attached to the diaphragms. These are usually fabricated of thin rubber sheet, and many previous workers have emphasized the need for putting the sleeves on at the lowest possible tension. Employing the same nomenclature as previously, it is supposed that the unrestrained increase of limb radius is δR_0, and that this is the actual expansion of the limb mid-way between the two sealing sleeves. At the diaphragms the radius will increase by δS only, the relationship between δR_0 and δS being derived from eqn (5) above. After substituting for 'a' from eqn (2), we obtain

$$\delta R_0 = \delta S[1 + A(e - T)/MR]. \tag{9}$$

Appropriate values appear to be: $A = 0{\cdot}48$, $e = 1000$ g/cm, $T = 100$ g/cm, $M = 800$ g/cm^2 and $R = 4$ cm. For these values δS is 14% less than δR. Since there is no restraint mid-way between the diaphragms, the effect of the sleeves averaged over the length of the limb segment will be about one-half the restraint at the sleeves. Thus the increase in limb volume recorded by the plethysmograph will be about 7% less than the actual increase in limb volume. Since the limb volume is measured separately, this error will be perpetuated in any deductions (e.g. blood flows) from the record. In this respect, the greater accuracy of the gauge method compared with the plethysmograph method is seen to be due to the smaller effect which restraint has on relative compared with the absolute changes.

Effect of relative amounts of elastic and non-elastic tissue

The equation given in the main paper relating change in gauge length (δL) to change in limb girth under gauge tension (δG) is derived directly from substitution for 'a' $= RA/M$ in (6), and subsequent substitution of $G = 2\pi R$ and $K = 2\pi A$:

$$\frac{\delta G}{\delta L} = \frac{\delta R}{\delta S} = 1 + \frac{Ke}{GM}. \tag{10}$$

The value of 'K' in this equation varies with the ratio of the rigid core radius to the total cylinder radius. For practical purposes 'K' can be expressed in terms of the percentage of non-elastic material (bone) in the segment being measured:

% of bone ($=100\,n^2$)	5%	10%	15%	20%	25%
$K[=3\pi(1-n^2)/(3-n^2)]$	3·03	2·92	2·81	2·69	2·57

In practice, measurements of limb volume changes are confined to fleshy parts of the limb, where the percentage of bone is comparatively small. For example, measurements made on cross-sections of the human forearm at different levels show that the percentage of bone varies between 5 and 10% over the 20 cm of the forearm situated mid-way between wrist and elbow joints. When using eqn (10) to deduce tissue elasticity (M), therefore, it will usually be sufficiently accurate to assume a value of 3·00 for 'K'. In limb girth and limb volume change measurements carried out with the gauge, variations of 'K' are, of course, compensated for by the gauge calibration procedure.

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