612.744.24

THE EFFICIENCY OF BICYCLE-PEDALLING, AS AFFECTED BY SPEED AND LOAD.

By SYLVIA DICKINSON[1].

(From the Department of Physiology and Biochemistry, University College, London.)

It has been shown both for the case of isolated frog's muscle and for certain human muscular movements (1, 3, 4) that the work done in a maximal contraction is diminished by increase of rate of shortening. The relation between the work done and the speed of shortening is found to be linear, so that the relation between the work W and the time t occupied by the contraction is given by the equation

$$W = W_0\left(1 - \frac{k}{t}\right) \qquad \text{......(1)},$$

where W_0 and k are constants. W_0 represents the theoretical maximum work and would be attained only if the contraction could take place infinitely slowly. k represents the theoretical minimum time and would be attained only if no external work were done. From experiments on isolated frog's muscle a relation has been found between the theoretical maximum work W_0, the energy H liberated in an isometric contraction excited by a maximal stimulus, and the time t during which the stimulus lasts. It is given by the equation

$$H = W_0 a\,(1 + bt) \qquad \text{......(2)},$$

where a and b are constants: a represents the energy required to set up a contraction capable of doing one unit of work under maximal conditions: the product ab represents the energy required per second of stimulus to maintain that contraction. From equations (1) and (2) it can be deduced (5) that the mechanical efficiency E of a muscular movement will be given by the equation

$$E = \frac{1 - (k/t)}{a\,(1 + bt)} \qquad \text{......(3)}.$$

[1] Working for the Industrial Fatigue Research Board of the Medical Research Council.

If equation (3) holds for any muscular movement then the efficiency would be independent of the force overcome, but would vary with change of the time occupied by the movement. The efficiency would tend towards zero for times given by $t = k$ and $t = \infty$ and would pass through a maximum value (1, 6) for a time of contraction given by

$$t = k\left(1 + \sqrt{1 + \frac{1}{bk}}\right).$$

The way in which efficiency varies with time of contraction has already been determined in the case of isolated frog's muscle (6), and for certain types of human muscular movements (2, 7, 8). In a previous paper (4) it was shown that the relation expressed by equation (1) holds for the movement of bicycle-pedalling, and it was predicted that the efficiency of bicycle-pedalling would vary with the speed and that there would be an optimum speed at which the efficiency is highest. In the present investigation the efficiency of bicycle-pedalling was determined at various speeds.

Experimental procedure. Determinations were made of the total amount of oxygen consumed by a subject during exercise and recovery[1], as the result of the performance of a known amount of work on a bicycle ergometer working against a given load at a given rate of pedalling. A Martin's friction ergometer was used. The arrangement adopted, the dimensions of the bicycle, the method of recording the number of turns and the rate of pedalling are described in a previous paper (4). For the measurement of some of the smaller loads a weight was hung at one end of the friction band directly over the bicycle wheel, and a more sensitive spring-balance was used at the other end.

A system of Douglas bags, and a Haldane gas-analysis apparatus were employed. The bags (9, p. 87) were supported on a table at the side of the bicycle. A tube from the expiratory side of the mouthpiece worn by the subject led to a three-way tap fixed on the handle-bars, and from this a tube led to a pipe connected with the system of bags.

Collections of expired air were made: (1) over a period of ten minutes immediately after the subject had remained at rest on the bicycle for half an hour; (2) over a period of half an hour, the first one to ten minutes of which (according to the speed of pedalling) were occupied by the exercise and (3) over a period of ten minutes immediately after the half-

[1] It was necessary to employ the method of measuring the oxygen "requirement" in this way, since a steady state cannot be attained at the higher rates of working, and therefore a measurement merely of the oxygen consumption during the exercise would give fallacious results.

hour period. From the first and third collections was determined the oxygen used per minute when sitting at rest on the bicycle immediately before the exercise and after recovery. From the second collection the total oxygen used during the period of exercise and recovery was found. The oxygen used for the exercise was taken as the difference between the total amount used during the thirty-minute period and the amount that would have been consumed during that period at the mean of the rates of oxygen consumption found for sitting on the bicycle before and after that period. It was assumed that the foodstuff utilized for the excess metabolism of the exercise was glycogen, in which case the energy value of 1 litre of oxygen would be 5·14 calories (10, p. 6), which is equivalent to 2190 kilogram-metres of energy expended. Hence the efficiency E is given by the formula

$$E = \frac{Rnl}{V \times 2190},$$

where R is the load at the rim of the wheel expressed in kilograms, l is the circumference of the wheel expressed in metres, n is the number of wheel revolutions occurring during the exercise, and V is the number of litres of dry oxygen at N.T.P. used for the performance of the exercise.

Results.

The relation between efficiency and speed of movement. Table I shows a series of measurements of efficiency with different rates of pedalling. The load in all these measurements lay between 2·8 kg. and 3·4 kg. at the rim of the wheel, except in the case of the two measurements given at the end of the table in which the load was 5·2 kg. The force opposing the movement of the pedal is roughly 3·7 times the load at the rim of the wheel. The results are shown graphically in Fig. 1, in which the abscissa represents the time in seconds of one foot movement (half a pedal revolution) and the ordinate represents the efficiency expressed as a percentage. The continuous curve shown in the figure is obtained from the experimental points. It is seen from the graph that the efficiency is low for both high and low rates of pedalling, and that it passes through a maximum value of 21·5 p.c. at a time for one foot movement of about 0·9 second.

Two theoretical points may be considered in relation to this curve. (A) From experiments described in a previous paper (4) it was found for the subject of these experiments that the theoretical minimum time of one foot movement when there is no external load is 0·16 sec. A theoretical point can therefore be added to the curve corresponding to a time of movement of 0·16 sec. and zero efficiency. (B) Since the energy

TABLE I.

Resting oxygen consump-tion: c.c. per minute	Oxygen used during exercise and re-covery: litres	Oxygen used for exercise: litres	Time of one foot move-ment: seconds	Load at rim of wheel: kg.	No. of turns of wheel	Work done: kg.-metres	Efficiency: p.c.
254 266	12·17	4·37	0.27	3·0	227	1090	11·4
279 278	13·33	4·98	0.46	3·1	400	1984	18·2
276 275	11·93	3·67	0.50	3·2	294	1505	18·7
277 279	12·75	4·41	0.63	3·1	400	1984	20·5
239 242	11·71	4·50	0.79	2·85	455	2074	21·0
262 276	12·68	4·61	0.78	3·2	396	2030	20·1
245 252	12·24 (31 min.)	4·55	0.90	3·4	400	2176	21·8
261 268	13·32	5·42	0.90	3·0	532	2554	21·5
242 242	11·20	3·94	1.11	2·75	400	1760	20·4
247 251	11·88 (31 min. 7 sec.)	4·38	1.20	3·05	400	1920	20·0
251 243	11·04	3·63	1.81	3·05	294	1411	17·7
273 267	12·89 (32 min.)	4·25	2.71	3·3	266	1405	15·1
267 ?	13·74 (33·5 min.)	4·80	3.62	3·2	271	1387	13·4
276 259	10·34	2·33	0.37	5·2	95	790	15·5
259 255	10·92	3·21	0.50	5·2	146	1214	17·3

Note. The time of exercise and recovery was 30 minutes except where otherwise stated.

required to maintain a muscular contraction increases with the time during which it is maintained, an infinitely slow contraction would require an infinite expenditure of energy. We may therefore suppose that the efficiency tends to zero as the time of movement becomes very great. That the efficiency should be zero at times $t = 0{\cdot}16$ and $t = \infty$ agrees reasonably well with the experimental points. The broken curve in Fig. 1 is a theoretical curve derived from equation (3) by giving to the constants the values $k = 0{\cdot}16$ sec., $a = 2{\cdot}8$ and $b = 0{\cdot}435$. It is seen from the figure that the experimental curve has the same general form as the theoretical curve.

The effect of load on efficiency. Table II shows a series of measurements of efficiency with varying load, but with the speed of pedalling

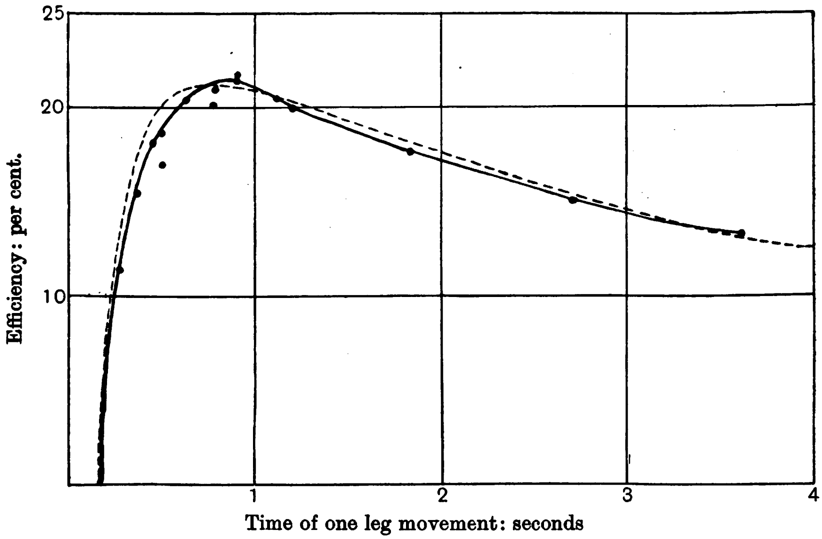

Fig. 1. The relation between E the efficiency and t the time of one leg movement. The plotted points represent the results of the experiments. The continuous curve is drawn through these points. The broken curve is a theoretical curve derived from the equation $E=\frac{1-(k/t)}{a\,(1+bt)}$ when the values of the constants are $k=0{\cdot}16$ sec., $a=2{\cdot}8$ and $b=0{\cdot}435$ per sec.

TABLE II.

Resting oxygen consumption: c.c. per minute	Oxygen used during exercise and recovery: litres	Oxygen used for exercise: litres	Time of one foot movement: seconds	Load at rim of wheel: kg.	No. of turns of wheel	Work done: kg.-metres	Efficiency: p.c.
229, 235	10·513	3·55	0.90	1·15	930	1711	22·0
235, 233	10·926	3·91	0.90	1·90	600	1824	21·2
261, 268	13·32	5·42	0.90	3·0	532	2554	21·5
245, 252	12·24 (31 min.)	4·55	0.90	3·4	400	2176	21·8
249, 256	12·37	4·80	0.90	5·0	267	2136	20·2*
252, 240	12·37	4·99	0.90	6·05	240	2323	21·4
256, 264	11·91	4·10	0.90	7·2	160	1843	20·6*

* The two low values were both obtained on the same occasion soon after the subject had had influenza. The low efficiency might be due to unskilled use of muscles.

constant at the value found to give the maximum efficiency in the previous series of experiments. The loads given are those at the rim of the wheel. Thus the range of force investigated was from 5·2 kg. to 26·6 kg. opposing the movement of the foot. The investigations were limited to this range because (*a*) with smaller loads the subject could not keep time with the metronome without "holding the legs back," and (*b*) with greater loads the subject could not move the pedals smoothly over the "dead points" and there was a tendency to "stand on the pedals" and so alter the type of movement. It is seen from the table that within the range used change of load has no appreciable effect on the efficiency if the speed of pedalling remains constant.

This result confirms the application of equation (3) to submaximal movements: strictly speaking it was deduced only for maximal ones, but on the assumption that the force exerted is graded merely by varying the number of muscle fibres exerted it should apply also to the more general case. To make the investigation complete would theoretically require the determination of an efficiency-time curve for each load. The present determinations, however, are sufficient to demonstrate the absence or at least the comparative smallness of the effect of load when the speed is constant at the chosen value.

The oxygen consumption for unloaded movement. Two experiments were made to determine the energy cost of unloaded pedalling at two different speeds. The results are shown in Table III. As would be ex-

TABLE III.

Resting oxygen consumption: c.c. per minute	Oxygen used during movement and recovery: (15 min.) litres	Oxygen used for movement: litres	Time of one foot movement: seconds	Load	No. of turns	Work	Oxygen used per 100 turns: litres
258, 250, 258	4·35	0·54	0.82	—	266	—	0·203
	4·50	0·70	0.48	—	262	—	0·268

pected the cost is greater for the greater speed, but in both cases the value is higher than would be expected. It was found that when there was no load to work against it was very difficult to keep time with the metronome without using muscles to hold the legs back. It is probable, therefore, that the oxygen consumption found in an unloaded experiment is higher than that part of the oxygen consumption that is required for the work done in moving the legs themselves during a loaded experiment.

Discussion.

Previous work on the efficiency of human muscular movement. That the speed of limb movement has a marked effect on the efficiency of doing work against an external force, and that for any given type of movement there is an optimum speed at which the efficiency is a maximum, may be taken as satisfactorily established. Lupton for stair climbing (2) and Furusawa for arm movements working a lever (7) have found efficiency-time curves of the same general form as the one described in this paper for bicycle-pedalling. Atzler, Herbst, Lehmann and Müller (11) for the case of turning a crank with the arms have found a similar variation of efficiency with speed. Cathcart, Richardson and Campbell (8) from experiments on a "variable ergometer" using arm muscles, came to the conclusion that there was an optimum rate of movement for doing that type of work. For bicycle-pedalling Benedict and Cathcart (12, p. 140) found the highest efficiencies with the lowest rates of pedalling, but their experiments did not include any with very slow rates. In all cases in which a definite efficiency-time curve has been worked out it has the same general form as the one described here for bicycle-pedalling. The curve has a broad optimum zone so that it is not always possible to give exactly the optimum time, but it can be found to lie between certain limits.

The conclusions concerning the effect of load on efficiency drawn from various sources are not so definite as those concerning the effect of speed. The experiments described in this paper show no appreciable effect of load on efficiency when the speed of pedalling is constant at the speed chosen for those experiments. Furusawa (7) concluded from his experiments that change of load does not affect the energy expenditure for amount of work done at constant speed of movement and that there is no optimum load. Results given by McDonald and Duffield (13) show no effect of load on efficiency. On the other hand Cathcart (14) has found an increase of efficiency with increase of load from small to moderate ones and then a falling off of efficiency with further increase to excessive loads. This would seem to indicate the existence of an optimum load, but the differences found were small, especially as compared with those found for varying speeds. Experiments described by Atzler, Herbst, Lehmann and Müller (11) show the same effect as that found by Cathcart. Campbell, Douglas and Hobson (15) describe results that show a fall of efficiency with increase of load at constant speed. It would appear that the effect of load on efficiency at constant speed is, if not negligible, at any rate small. Where an effect is found, and the experiments include a large enough range of loads starting from very small ones, there is first a rise of efficiency and then a fall with increasing load. This may be explained by the operation of two factors which have not yet been considered.

In the first place, no account has been taken of the energy expended in guiding and moving the limbs. Since, for a given speed, this becomes a smaller fraction of the whole energy when the load is made greater, this factor will cause an increase of efficiency with increasing load. A more accurate determination of the efficiency would be obtained if from the oxygen consumption found for the work there was subtracted that required for moving the machine without a load at the same speed. This cannot be done in practice for the reasons given above. Benedict and Cathcart in their work on bicycle-pedalling tried the experiment of taking as base-line the cost of unloaded movement and came to the same conclusion (12, p. 119). One way of overcoming the difficulty is to take as base-line the cost of movement when working at the required speed against a small load that is just sufficient to enable the movement to be performed without the tendency to "hold the legs back." Any two loads may be taken, and the efficiency calculated by dividing the excess work with the larger load by the excess energy. Since the experiments described here show no appreciable change of efficiency with change of load, therefore either the cost of movement alone is small and the efficiency is constant, or else the cost of movement is appreciable and there is a fall of efficiency with increase of load which balances its effect. The experiments described by Campbell, Douglas and Hobson (15) give an efficiency of the excess work with a larger load that is definitely lower than the net efficiency working with a smaller load. On the other hand, Benedict and Cathcart (12) found the efficiency of excess work taking a moderate load as base-line to be 30–33 p.c., some measurements giving a value as high as 40 p.c. It must be noted, however, that the larger load used by Campbell, Douglas and Hobson was greater than that used by Benedict and Cathcart, and that the speed of pedalling used by Campbell, Douglas and Hobson was smaller.

Taking into consideration this factor of the work done in moving the limbs alone the results may be interpreted as showing the existence of a second factor, a fall of efficiency with excessive loads. The two factors together would cause a rise and then a fall of efficiency with increasing load. The fall of efficiency with excessive loads may be due to the use of different muscles and of different types of effort to overcome them.

This is obviously the case in such movements as lifting a weight, or walking along pulling or pushing against a force, or walking uphill: while in bicycle-pedalling excessive loads may require the use of the arms to keep the body in the saddle, and may lead to difficulties in passing over the "dead points."

From these considerations it would seem that the experimental results are not fundamentally inconsistent with the view that the efficiency of uncomplicated muscular work is unaffected by change of the force overcome.

The maximum efficiency observed in the present experiments is lower than that found by previous workers for bicycle-pedalling (12, 15). This may possibly be due to the fact that the bicycle used was of dimensions suitable for a bicyclist of larger build than the subject of these experiments. The length of pedal crank most commonly used for bicycles is $6\frac{1}{2}$ inches. This length is presumably adopted empirically as being suitable for a bicyclist of normal male build. The length of crank used in these experiments was 7 inches and the subject's height was only 5 ft. 3 in. The height of the saddle at its lowest point of adjustment was such that a considerable amount of extension of the ankle joint was required for the subject to reach the pedal at its lowest position. The experiments described, therefore, are not of value in supplying data as to the maximum efficiency attainable in bicycle-pedalling; their main object was to demonstrate the effect of speed and of load on efficiency.

The mechanical efficiency as measured in experiments on human limb movements is affected by the skill of the subject as well as by the mechanical conditions involved and by the efficiency with which a muscle does work. A skilled subject uses only those muscles required to perform the movement; an unskilled subject uses unnecessary muscles as well.

General theory. If the relation between factors involved in determining the efficiency of muscular work be adequately expressed by equation (3), then it follows that the efficiency at constant speed will be independent of the load and that an experimental efficiency-time curve will have the same general form as a theoretical curve derived from this equation. How far the experimental results are in accordance with this has been discussed in the previous section.

By an independent method (4) a value for the constant k was found for the subject of these experiments. The value found was 0·16 sec. Now it is seen from equation (3) that we may write $1 + bt = \frac{1}{a}\left(\frac{1-(k/t)}{E}\right)$, so that if the equation holds the relation between t and $\frac{1-(k/t)}{E}$ should be linear. By taking the experimental value for k a series of values for t and $\frac{1-(k/t)}{E}$ can be found from the results of the experiments. These are shown graphically in Fig. 2. It is seen that the points are roughly

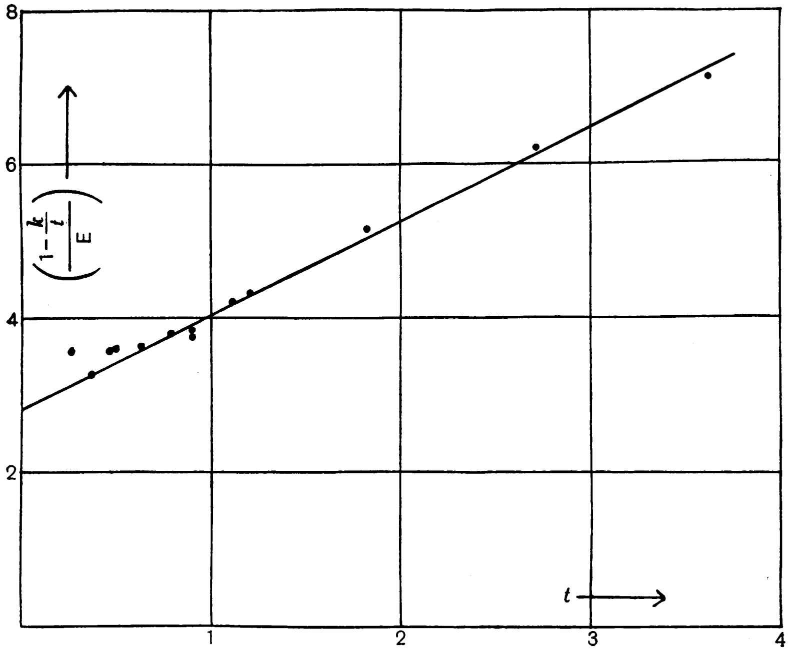

Fig. 2. The abscissa represents t the time of one leg movement. The ordinate represents $\frac{1-(k/t)}{E}$, where k is the "theoretical minimum time" and E is the efficiency, both experimentally observed. Note that the relation is linear, which demonstrates the validity of equation (3).

distributed about a straight line. Since when $t = 0$, $a = \frac{1-(k/t)}{E}$ and when $\frac{1-(k/t)}{E} = 0$, $b = -(1/t)$, values for a and b can be found from the graph. These are found to be $a = 2{\cdot}8$, $b = 0{\cdot}435$. These are the values of the constants used in drawing the theoretical curve shown by a broken line in Fig. 1. By direct measurement on arm muscles Lupton (2) found the values $a = 2{\cdot}6$ and $a \times b = 0{\cdot}486$. His determination of $a \times b$ was made from experiments in which a maximal contraction was maintained statically, so that whatever factors are introduced by actual movement and doing work were not involved.

Although it is of interest to note the order of magnitude of the constants a and b so determined, the results obtained cannot be taken as indicating precisely the values of these constants for human muscles, because in applying equation (3) to interpret the experimental results certain assumptions have been made which are not necessarily correct. These are:

1. That the duration of the stimulus exciting the muscles is equal to the duration of the contraction.
2. That gradation of response is achieved by varying the number of fibres excited.
3. That the energy liberated by a muscle when it does work is the same as it is when the muscle responds isometrically to the same stimulus.
4. That only the muscles doing external work are involved in the limb movement.
5. That all fresh muscles or muscle fibres brought into play to increase the force work under the same mechanical conditions, and that the same muscles are used throughout the movement.
6. That the external mechanical conditions are such that the whole of the external force exerted by the muscles can be used in doing work.

1. The first of these points has been discussed in a paper by A. V. Hill(6).

2. About the second point there is not yet sufficient data to make a definite statement. Recent work by Adrian and Bronk(16) has shown that in some cases in skeletal muscle gradation of contraction is brought about not only by variation in the number of fibres in action but also by variation of the frequency of the discharge along the nerves supplying them. If this is so, then in a submaximal contraction doing work against a given force at a given rate of movement the number of fibres involved will be greater than if the fibres were excited to a maximal contraction and the efficiency would be less. To discover to what extent this factor will effect the efficiency it is necessary to know the conditions of energy liberation in an incomplete tetanus. This point is at present being investigated here by Dr Bronk. It would appear from the results he has so far obtained, taken in conjunction with the actual range of frequency of impulses found to occur in the mammal, that the effect of this factor on efficiency would not be large.

3. As regards the liberation of energy by a muscle doing work it was shown by Fenn(17) that a muscle excited by a maximal stimulus liberates more energy if it does external work than if it is kept under isometric

conditions. The matter has recently been further investigated by Hartree and Hill(18). Since the excess energy due to work is a complicated function of the work and of the duration of the contraction, a simple method of taking account of this factor in discussing the efficiency cannot be given. It is possible, however, to see qualitatively how this factor will affect the efficiency. The denominator of the ratio giving the efficiency must be greater than $\{a\,(1 + bt)\}$ by some quantity varying with $\{1 - (k/t)\}$. The efficiency will be less than it would be if there were no "Fenn effect." The general form of the efficiency-time curve will be the same in that the efficiency will be zero for times $t = k$ and $t = \infty$ and will pass through a maximum. The values of the constants a and b deduced from the simple equation (3) will be somewhat too high.

4. When work is done by human limb movements, muscles have to be used to maintain the posture of the body as a whole and to fix in position parts of the limbs that would otherwise be moved by the contraction of the muscles that do work. For example, in bicycle-pedalling muscles must be used to prevent flexion at the ankle joint and with heavy loads to keep the body in the saddle. If it be assumed that the force that must be exerted by the statically contracting muscles is proportional to the external force exerted by the muscles doing work, then it is possible to find for the efficiency an equation that takes account of this factor. From this equation it can be shown: (1) that the efficiency, as in the simpler case, will be independent of the load; (2) that the general form of the efficiency-time curve will be the same; (3) that with slow speeds the falling off of efficiency will be greater than that predicted for the simpler case, and (4) that in fitting a theoretical curve to the experimental results the values of the constants a and b deduced will be less than those obtained by fitting a curve derived from the simpler equation.

5. The effect of bringing fresh muscles into play and changing the type of movement has already been considered in an earlier part of the discussion.

6. The external mechanical conditions under which a muscle works will affect efficiency. When dealing with isolated muscles a mechanical device is employed so that the whole of the external force exerted can be used in doing measured work. When a machine is worked by human limb movements this condition is not necessarily fulfilled. For example, in the case of bicycle-pedalling only the component perpendicular to the pedal crank of the force actually exerted on the pedal is used in

doing external work. In general, therefore, the efficiency found must be less than would be found under more advantageous mechanical conditions.

The way in which the measured efficiency of human muscular movements would differ, as a result of the operation of these factors, from a predicted efficiency based on equation (3) (supposing the values of the constants were known) may be summarised as follows. All the factors except the first will operate so as to make the measured efficiency lower than the predicted efficiency. The general form of the efficiency-time curve will be the same, but the detailed form of the curve will be altered in such a way that the apparent values of the constants a and b found on the assumption that the efficiency is given by equation (3) will be greater than their actual values.

SUMMARY.

1. The relation previously found between the maximum speed of pedalling a bicycle and the force overcome led to the prediction that the mechanical efficiency of bicycle-pedalling would vary with the speed, and that there would be an optimum speed at which the efficiency is highest.

2. Experiments were made to determine the efficiency of bicycle-pedalling over as wide a range of speeds as possible. The efficiency was found to vary with speed in accordance with the prediction. The optimum time of one foot-movement (half a pedal revolution) was found to be 0·9 sec.

3. At a constant speed of 33 complete pedal revolutions per minute it was found that the efficiency was not appreciably affected by change of load within a range of 5 to 26 kg. at the pedal.

4. The agreement of the experimental results with the theoretical efficiency-time curve shows that the chief factors involved in human muscular movement are taken account of in the equation given. It is pointed out, however, that various complicating factors exist which prohibit too full an acceptance of the exact values of the constants deduced.

My thanks are due to Professor A. V. Hill for his interest and help throughout the investigation, to the Medical Research Council for a grant to enable the work to be done, and to Mr J. L. Parkinson for assistance with the experiments.

REFERENCES.

1. Hill, A. V. This Journ. 56. p. 19. 1922.
2. Lupton. Ibid. 57. p. 337. 1923.
3. Best. Proc. Roy. Soc. B, 103. p. 218. 1928.
4. Dickinson. Ibid. 103. p. 225. 1928.
5. Hill, A. V. Ibid. 102. p. 30. 1927.
6. Hill, A. V. Ibid. 103. p. 234. 1928.
7. Furusawa. Ibid. 99. p. 155. 1926.
8. Cathcart, Richardson and Campbell. This Journ. 58. p. 355. 1924.
9. Hill, Long and Lupton. Proc. Roy. Soc. B, 97. p. 84. 1924.
10. Hill, A. V. "Muscular Work." McGraw-Hill Book Company, New York and London. 1927.
11. Atzler, Herbst, Lehmann and Müller. Archiv für Physiol. 208. p. 218. 1925.
12. Benedict and Cathcart. "Muscular Work." Publication No. 187, Carnegie Institute of Washington. 1913.
13. McDonald and Duffield. This Journ. 58. Proceedings, p. xiii. 1923.
14. Cathcart. Report of Brit. Ass. p. 164. 1922.
15. Campbell, Douglas and Hobson. Phil. Trans. Roy. Soc. B, 210. p. 21. 1920.
16. Adrian and Bronk. This Journ. 66. p. 81. 1928.
17. Fenn. Ibid. 58. p. 373. 1924.
18. Hartree and Hill. Proc. Roy. Soc. B, 104. p. 1. 1928.