Supporting Text

Fitness increases with both x and z for the fitness function in Eq. **4**, provided that $b > 1$, $t < 0$, and $Y_0 > 0$, because $f_x = \frac{\partial f}{\partial x} = -(\log_e 10)\,(1-b)\,10^{(1-b)x}$, and $f_z = \frac{\partial f}{\partial z} = Y_0\,(1-b)\log_e(10) / zt$. The fitness contours for these parameter values are convex when viewed from below, as will now be shown. Implicit differentiation of Eq. **4** gives the slope of a fitness contour at (x, z) as

$$\frac{dz}{dx} = \frac{e^{x(1-b)\log_e 10}zt}{Y_0}$$, and its curvature as

$$\frac{d^2z}{dx^2} = \frac{1}{Y_0}\{(\log_e 10)(1-b)e^{x(1-b)\log_e 10}zt[1 + \frac{te^{x(1-b)\log_e 10}}{Y_0(1-b)\log_e 10}]\} .$$

For $b > 1$, $t < 0$ and $Y_0 > 0$, $\frac{d^2z}{dx^2}$ is necessarily positive, so the fitness contours are everywhere convex viewed from below.

Because we assume that the options sets are right triangles, it follows from the above that a male allocates its available resources each year to some combination of ornament and body, and the allocation lies on the hypotenuse of the options triangle. Mass balance implies that the allocation obeys the constraint $X + kY = c$, for some constants k and c. In the space with axes $x = \log_{10}X$ and $z = Y/X$, the options set is not exactly triangular, but the assumption of triangularity holds for small triangles and is a reasonable approximation for triangles of biologically realistic sizes.

Because the triangles are presumed to be small relative to the fitness contours, we can think of the fitness contours as wide parallel arcs through the triangles, as in Fig. 3*A*. There are three possibilities for the way the fitness contours intersect the triangles, with three corresponding strategies for optimal allocation of resources:

(*i*) if the fitness contours are steeper than the slope of the hypotenuse of the options triangle, allocate all resources to body growth;
(*ii*) if the fitness contours are shallower than the slope of the hypotenuse of the options triangle, allocate all resources to ornament;
(*iii*) if the hypotenuse of the options triangle is tangential to the fitness contours, allocate resources partly to body growth and partly to ornament. In particular, allocate to the point in the options triangle where the tangent touches the hypotenuse.

It is the last condition that results in positive ornament allometries. A fitness contour is, by definition, a curve with equation $f(x, z) = c$, for some constant c. The slope of the contour is obtained by implicit differentiation as $-f_x / f_z$, and setting this equal to the slope of the hypotenuse, t, gives Eq. **2**. The locus of the points defined by Eq. **2** represents the optimal allocation strategy.

Lastly, note that it is the relative positions of the fitness contours that are important, because they specify the relative intensities of selection on ornament size and body size. Because the optimal strategy is unaffected by monotonic increasing transformations of the fitness function f, the contours on which a species sits at any given time can be normalized and relabeled to give an average fitness of zero. Mathematically, the selection intensities f_z and f_x define a vector field that specifies, for each ornament and body size, the fitness advantages due to unit increases in z and x. This vector field determines the direction and rate of evolution.

Our explanation for why larger species have lower normalization constants depends on four assumptions: (*i*) the species differ in body size; (*ii*) the species' fitness surfaces have the same b and t parameters (so, for example, the fitness surfaces in Fig. 3 *C* and *D* have the same b parameter, and so do Fig. 3 *E* and *F*); (*iii*) $b > 1$; and (*iv*) the species mature at a reproductive size corresponding to a fixed value of z.

Because the optimal strategy is given by Eq. **1** and b is assumed the same for the different species, the only parameter that varies between species is Y_0. From assumption *iv* above, reproductive size, which we label X_r, corresponds to a fixed value of z, say 0.05, thus $z = 0.05 = Y_0 X_r^{b-1}$. Because b is assumed > 1, it follows that as X_r increases, Y_0 decreases. In other words, the optimal strategy of larger species involves lower normalization constants, Y_0. In the text, this argument is illustrated by comparison of Fig. 3 *E* and *F*.