Supplement – Reverse engineering gene networks — integrating genetic perturbations with dynamical modeling

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SUPPORTING TEXT.

Here we provide the equations for the single cell network, governed by 10 nonlinear equations describing the time evolution of both proteins (in capital) and genes. These equations represent the single cell in the network model of von Dassow *et al.* (1). We have therefore omitted diffusive coupling terms in the original model.

$$\frac{d\ en}{dt} = \frac{T_0}{H_{en}} \left[\frac{EWG\left(1-\frac{CN^{v_{CN,en}}}{\kappa_{CN,en}{}^{v_{CN,en}}+CN^{v_{CN,en}}}\right)^{v_{WG,en}}}{\kappa_{WG,en}{}^{v_{WG,en}}+EWG\left(1-\frac{CN^{v_{CN,en}}}{\kappa_{CN,en}{}^{v_{CN,en}}+CN^{v_{CN,en}}}\right)^{v_{WG,en}}} - en \right] \quad [1]$$

$$\frac{d\ EN}{dt} = \frac{T_0}{H_{EN}} \left[en - EN \right] \quad [2]$$

$$\frac{d\ wg}{dt} = \frac{T_0}{H_{wg}} \left[\frac{\alpha_{CI,wg}\left[\frac{CI\left(1-\frac{CN^{v_{CN,wg}}}{\kappa_{CN,wg}{}^{v_{CN,wg}}+CN^{v_{CN,wg}}}\right)^{v_{CI,wg}}}{\kappa_{CI,wg}{}^{v_{CI,wg}}+CI\left(1-\frac{CN^{v_{CN,wg}}}{\kappa_{CN,wg}{}^{v_{CN,wg}}+CN^{v_{CN,wg}}}\right)^{v_{CI,wg}}}\right]+\alpha_{WG,wg}\left[\frac{IWG^{v_{WG,wg}}}{\kappa_{WG,wg}{}^{v_{WG,wg}}+IWG^{v_{WG,wg}}}\right]}{1+\alpha_{CI,wg}\left[\frac{CI\left(1-\frac{CN^{v_{CN,wg}}}{\kappa_{CN,wg}{}^{v_{CN,wg}}+CN^{v_{CN,wg}}}\right)^{v_{CI,wg}}}{\kappa_{CI,wg}{}^{v_{CI,wg}}+CI\left(1-\frac{CN^{v_{CN,wg}}}{\kappa_{CN,wg}{}^{v_{CN,wg}}+CN^{v_{CN,wg}}}\right)^{v_{CI,wg}}}\right]+\alpha_{WG,wg}\left[\frac{IWG^{v_{WG,wg}}}{\kappa_{WG,wg}{}^{v_{WG,wg}}+IWG^{v_{WG,wg}}}\right]} \right] - \frac{wgT_0}{H_{wg}} \quad [3]$$

$$\frac{d\ IWG}{dt} = \frac{T_0}{H_{IWG}} \left[wg - IWG \right] \quad [4]$$

$$\frac{d\ ptc}{dt} = \frac{T_0}{H_{PTC}} \left[\frac{CI\left(1-\frac{CN^{v_{CN,ptc}}}{\kappa_{CN,ptc}{}^{v_{CN,ptc}}+CN^{v_{CN,ptc}}}\right)^{v_{CI,ptc}}}{\kappa_{CI,ptc}{}^{v_{CI,ptc}}+CI\left(1-\frac{CN^{v_{CN,ptc}}}{\kappa_{CN,ptc}{}^{v_{CN,ptc}}+CN^{v_{CN,ptc}}}\right)^{v_{CI,ptc}}} - ptc \right] \quad [5]$$

$$\frac{d\ PTC}{dt} = \frac{T_0}{H_{PTC}} \left[ptc/6 - PTC \right] \quad [6]$$

$$\frac{d\ ci}{dt} = \frac{T_0}{H_{ci}}\left[\frac{B(1-\frac{EN^{v_{EN,ci}}}{\kappa_{EN,ci}{}^{v_{EN,ci}}+EN^{v_{EN,ci}}})^{v_{B,ci}}}{\kappa_{B,ci}{}^{v_{B,ci}}+B(1-\frac{EN^{v_{EN,ci}}}{\kappa_{EN,ci}{}^{v_{EN,ci}}+EN^{v_{EN,ci}}})^{v_{B,ci}}} - ci\right] \qquad [7]$$

$$\frac{d\ CI}{dt} = \frac{T_0}{H_{CI}}(ci - CI) - T_0 C_{CI} CI\left[\frac{PTC^{v_{PTC,CI}}}{\kappa_{PTC,CI}{}^{v_{PTC,CI}}+PTC^{v_{PTC,CI}}}\right] \qquad [8]$$

$$\frac{d\ CN}{dt} = T_0 C_{CI} CI\left[\frac{PTC^{v_{PTC,CI}}}{\kappa_{PTC,CI}{}^{v_{PTC,CI}}+PTC^{v_{PTC,CI}}}\right] - \frac{T_0 CN}{H_{CI}} \qquad [9]$$

$$\frac{d\ hh}{dt} = \frac{T_0}{H_{hh}}\left[\frac{EN(1-\frac{CN^{v_{CN,hh}}}{\kappa_{CN,hh}{}^{v_{CN,hh}}+CN^{v_{CN,hh}}})^{v_{EN,hh}}}{\kappa_{EN,hh}{}^{v_{EN,hh}}+EN(1-\frac{CN^{v_{CN,hh}}}{\kappa_{CN,hh}{}^{v_{CN,hh}}+CN^{v_{CN,hh}}})^{v_{EN,hh}}} - hh\right] \qquad [10]$$

The half-maximal activation coefficient is governed by κ, saturability coefficient for enhancer α, half-life (inverse of degradation rate), and v is the Hill coefficient. For further details consult von Dassow *et al.* (1). Parameters are $\kappa = 0.1$, $H = 10$, $T_0 = 1$, $v = 1.1$, $EWG = 0.5$, $B = 1$ (dummy parameter for basal expression), and $\alpha_{CI,wg} = \alpha_{WG,wg} = 0.1$.

Reference

1. von Dassow, G., Eli, M., Munro, E. M. & Odell, G. M. (2000) *Nature* **406**, 188 - 192.