Supporting Text

In the spherical model of the deep-brain located AFR, the temperature distribution depends on a distance r from the center of the AFR, $T = T(r)$. Therefore the bioheat Eq. **2** reduces to

$$\frac{d^2T_j}{dr^2} + \frac{2}{r}\cdot\frac{dT_j}{dr} - \rho F_j \rho_b c_b \left(T_j - T_a\right) + q_j = 0\,, \qquad [\mathbf{9}]$$

where $j = i$ corresponds to the inner volume of the AFR ($r < R$) and $j = e$ corresponds to the surrounding tissue ($r > R$). The boundary conditions to Eq. **9** at the interface between the spherical AFR region and surrounding tissue (i.e., at $r = R$), corresponding to continuity of the temperature and of the heat flux across the surface $r = R$, are:

$$T_i(R) = T_e(R); \quad \left(\frac{\partial T_i}{\partial r}\right)_{r=R} = \left(\frac{\partial T_e}{\partial r}\right)_{r=R} \qquad [\mathbf{10}]$$

The solution of Eq. **9** with the boundary conditions in Eq. **10** can be written in the form:

$$T(r) = T_b + \Delta T_m \cdot \begin{cases} 1 - A_i(R)\cdot\sinh(\kappa_i r)/r, & r < R \\ A_e(R)\cdot\exp\left[-\kappa_e(r-R)\right]/r, & r > R \end{cases} \qquad [\mathbf{11}]$$

$$\begin{aligned} A_i(R) &= \frac{(\kappa_e R + 1)}{\left(\kappa_i\cosh(\kappa_i R) + \kappa_e\sinh(\kappa_i R)\right)} \\ A_e(R) &= \frac{\left[\kappa_i R\cosh(\kappa_i R) - \sinh(\kappa_i R)\right]}{\left[\kappa_i\cosh(\kappa_i R) + \kappa_e\sinh(\kappa_i R)\right]} \end{aligned} \qquad [\mathbf{12}]$$

where $\Delta T_m = T_{mi} - T_{me}$, $T_{mi} = \mathrm{OEF}_i \cdot T_m^*$ and $T_{me} = \mathrm{OEF}_e \cdot T_m^*$ are the "metabolic temperature shift" (MTS) within and outside the AFR, and parameters $\kappa_{i,e} = (\rho F_{i,e}\rho_b c_b / K)^{1/2}$ represent inverse characteristic lengths inside and outside the AFR: $\Delta_i = \kappa_i^{-1}$ and $\Delta_e = \kappa_e^{-1}$.

In the "flat" model of the AFR located in the superficial cortex, the temperature distribution depends only on a z coordinate normal to the brain surface, $T = T(z)$. Therefore the bioheat Eq. **2** reduces to

$$\frac{d^2T_j}{dz^2} - \rho F_j \rho_b c_b \left(T_j - T_a\right) + q_j = 0 \qquad [\mathbf{13}]$$

where $j = i$ corresponds to the inner volume of the GM AFR ($0 < z < d$) and $j = e$ corresponds to WM ($z > d$). The boundary conditions to the bioheat Eq. **13** at the GM/WM interface ($z = d$) and at the brain surface ($z = 0$) describing heat exchange with the environment are, respectively:

$$\begin{aligned} & T_i(d) = T_e(d); \quad \left(\frac{\partial T_i}{\partial d}\right)_{z=d} = \left(\frac{\partial T_e}{\partial z}\right)_{z=d}, \\ & K \cdot \left(\partial T_i / \partial z\right)_{z=0} = h \cdot \left(T_i(0) - T_{ext}\right) \end{aligned} \qquad [\mathbf{14}]$$

where T_{ext} is the ambient temperature and h is the "effective" heat transfer coefficient, accounting for an insulation of the brain from the environment by intermediate layers of cerebrospinal fluid (CSF), skull, scalp (25). To find the temperature distribution in the brain in the model with the intermediate layers, the bioheat equation should be solved not only in the brain itself but in all the additional layers, with corresponding boundary conditions at interfaces and at the external surface (scalp/air) with the environment similar to Eq. **14**. Resulting expressions for $T(z)$ are very cumbersome, however, the temperature distribution in the brain itself can be reduced to a simple exponential expression (25),

$$T_0(z) = T_b - A \cdot \exp(-\kappa z), \qquad A = \frac{h \cdot (T_b - T_{ext})}{(K\kappa + h)} \qquad [\mathbf{15}]$$

with an *effective* heat transfer coefficient h. We use this approach to describe heat exchange between the brain and environment for a brain model with the WM "core" and GM superficial cortex.

The solution of Eq. **13** with the boundary conditions in Eq. **14** can be written in the form:

$$\begin{aligned} T_i(z) &= T_b + \Delta T_m - \left[B_i^{(1)} \cosh \kappa_i z + B_i^{(2)} \sinh \kappa_i z \right] \\ T_e(z) &= T_b - B_e \cdot \exp\left[-\kappa_e (z - d)\right] \end{aligned} \qquad [\mathbf{16}]$$

$$B_i^{(1)} = \frac{1}{P} \cdot \left[K \cdot \Delta T_m \cdot k_i k_e + h\,\tilde{T} \cdot \left(k_i \cosh(\kappa_i d) + k_e \sinh(\kappa_i d) \right) \right]$$
$$B_i^{(2)} = \frac{h}{P} \cdot \left[\Delta T_m \cdot k_e - \tilde{T} \cdot \left(k_i \sinh(\kappa_i d) + k_e \cosh(\kappa_i d) \right) \right] \qquad [\mathbf{17}]$$
$$B_e = \frac{\kappa_i}{P} \cdot \left\{ h \cdot \tilde{T} - \Delta T_m \cdot \left[K\kappa_i \cdot \sinh(\kappa_i d) + h \cdot \cosh(\kappa_i d) \right] \right\}$$

where

$$\tilde{T} = T_b + \Delta T_m - T_{ext}$$
$$P = \kappa_i \cdot (h + K\kappa_e) \cdot \cosh(\kappa_i d) + (K\kappa_i^2 + h\kappa_e) \cdot \sinh(\kappa_i d) \qquad [\mathbf{18}]$$