Appendix A. Solution of Eq. 4

Recoil of a bent fiber in a viscous fluid

For small deflections, the viscoelastic recoil of a fiber in a viscous fluid can be described by modified beam equation. In contrast to cilia which exert an active force, a bent fiber exerts a passive distributed drag force on the surrounding fluid due to the potential energy stored in its initial deformed shape. The modified beam equation describing this recoil is given by Eq. **4.**

$$EI\frac{\partial^4 y}{\partial x^4} = -\frac{\pi}{c}\frac{\mu r_f^{\,2}}{K_P}\frac{\partial y}{\partial t}, \qquad \textbf{[A1]}$$

where the coordinate system and local instantaneous velocity are shown in Fig. A1.

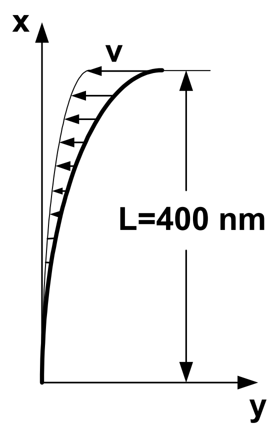

Fig. A1. Sketch of recoiling fiber

Eq. **A1** can be cast in dimensionless form

$$Y_{XXXX} = -Y_T\,, \qquad \textbf{[A2]}$$

by introducing the dimensionless coordinates $X = x/L$ and $Y = y/y(L,0)$, and dimensionless time $T = t/\tau$, where $\tau = kL^4/EI$. $k = (\pi/c)(\mu r_f^{\,2}/K_P)$, the coefficient of the velocity term in Eq. **A1,** is described in the main text.

Eq. **A2** satisfies the dimensionless boundary and initial conditions:

$$Y(0,T)=\frac{\partial Y(0,T)}{\partial X}=\frac{\partial^2 Y(1,T)}{\partial X^2}=\frac{\partial^3 Y(1,T)}{\partial X^3}=0\,, \qquad \textbf{[A3a, b, c, d]}$$

$$Y(X,0)=\frac{3}{2}X^2-\frac{1}{2}X^3\,, \qquad \textbf{[A3e]}$$

where Eq. **A3e** is the initial deflection due to a point load applied at X=1. The viscous loading term on the r.h.s. of Eq. **A2** is expressed in an infinite series of the form

$$-\frac{\partial Y}{\partial T}=a_2(T)\cdot X^2+a_3(T)\cdot X^3+a_4(T)\cdot X^4+a_5(T)\cdot X^5+a_6(T)\cdot X^6+a_7(T)\cdot X^7+\ldots\,, \qquad \textbf{[A4]}$$

where the $a_i(T)$, i = 2, 3, 4 … are unknown time dependent functions. Substituting Eq. **A4** into Eq. **A2**, integrating term by term and applying the first two boundary conditions (**A3a**, **b**), one finds that

$$Y(X,T)=f_2(T)\cdot\frac{X^2}{2}+f_3(T)\cdot\frac{X^3}{6}+a_2(T)\cdot\frac{X^6}{360}+a_3(T)\cdot\frac{X^7}{840}+a_4(T)\cdot\frac{X^8}{1680}+a_5(T)\cdot\frac{X^9}{3024}$$

$$+a_6(T)\cdot\frac{X^{10}}{5040}+a_7(T)\cdot\frac{X^{11}}{7920}+\ldots, \qquad \textbf{[A5]}$$

where $f_2(T)$ and $f_3(T)$ are also unknown functions of time. Differentiating Eq. **A5** with respect to T, provides another way to express Y_T, which is equivalent to Eq. **A4**. Comparing the coefficients of X^n in these two expressions for Y_T, one obtains the following relationship between the $a_i(T)$ and $f_2(T)$ and $f_3(T)$

$$a_2(T)=-\frac{f_2'(T)}{2}\,, \qquad \textbf{[A6a]}$$

$$a_3(T)=-\frac{f_3'(T)}{6}\,, \qquad \textbf{[A6b]}$$

$$a_4(T)=a_5(T)=0\,, \qquad \textbf{[A6c, d]}$$

$$a_6(T) = -\frac{a_2'(T)}{360} = \frac{f_2''(T)}{720}, \qquad \textbf{[A6e]}$$

$$a_7(T) = -\frac{a_3'(T)}{840} = \frac{f_3''(T)}{5040}, \text{ etc.} \qquad \textbf{[A6f]}$$

Therefore, Eq. **A5** can be rewritten as

$$Y(X,T) = f_2 \cdot \frac{X^2}{2} + f_3 \cdot \frac{X^3}{6} - f_2' \cdot \frac{X^6}{720} - f_3' \cdot \frac{X^7}{5040} + f_2'' \cdot \frac{X^{10}}{5040 \times 720} + f_3'' \cdot \frac{X^{11}}{5040 \times 7920} + \ldots \qquad \textbf{[A7]}$$

Conditions **A3c**, **d** require that $f_2(T)$ and $f_3(T)$ satisfy,

$$f_2 + f_3 - \frac{f_2'}{24} - \frac{f_3'}{120} + \frac{f_2''}{5040 \times 8} + \frac{f_3''}{5040 \times 72} + \ldots = 0, \qquad \textbf{[A8a]}$$

$$f_3 - \frac{f_2'}{6} - \frac{f_3'}{24} + \frac{f_2''}{5040} + \frac{f_3''}{5040 \times 8} + \ldots = 0, \qquad \textbf{[A8b]}$$

where higher order terms are neglected since Eq. **A7** converges rapidly. Terms involving f_2'' and f_3'' are also neglected because they introduce inertia which is very small in a viscous dominated flow. The coupled equations for f_2 and f_3 thus simplify to

$$\frac{1}{2880} \cdot \frac{d^2 f_3}{dT^2} + \frac{1}{12}\frac{df_3}{dT} + f_3 = 0. \qquad \textbf{[A9]}$$

Assuming a solution for f_3 of the form

$$f_3(T) = A \cdot e^{-\frac{T}{\xi}}, \qquad \textbf{[A10]}$$

and substituting it into Eq. **A9**, one finds

$$1 - \frac{\xi}{12} + \frac{\xi^2}{2880} = 0, \qquad \textbf{[A11]}$$

whose roots are ξ_1=0.0044 and ξ_2=0.0789.

The solution for f_3 is

$$f_3(T) = A_1 e^{-\frac{T}{\xi_1}} + A_2 e^{-\frac{T}{\xi_2}}. \quad \textbf{[A12]}$$

Then, the simplified version of Eqs. **A8a** and **b** gives

$$f_2(T) = -\frac{3}{4} f_3 - \frac{1}{480} f_3{}' = \frac{1}{4} A_1 e^{-\frac{T}{\xi_1}} \left(\frac{1}{120\xi_1} - 3 \right) + \frac{1}{4} A_2 e^{-\frac{T}{\xi_2}} \left(\frac{1}{120\xi_2} - 3 \right). \quad \textbf{[A13]}$$

A_1 and A_2 are the integration constants which can be determined by applying the initial condition **A3e**.

$$A_1 = -\frac{3\xi_1(-1 + 120\xi_2)}{-\xi_2 + \xi_1}, \quad \textbf{[A14a]}$$

$$A_2 = \frac{3\xi_2(-1 + 120\xi_1)}{-\xi_2 + \xi_1}. \quad \textbf{[A14b]}$$

It should be pointed out that only first two terms in Eq. **A7** are required to satisfy initial condition **A3e**.

$\xi_1 \cdot \tau$ is the short dimensional time constant that accounts for the initial transient change in fiber shape after the point force P is released. $\xi_2 \cdot \tau$ is the long dimensional time constant that describes the long time recoil. We require $\xi_2 \cdot \tau$ be equal to the single exponential, β =0.38 s, obtained by curvefitting the experimental data in ref. 1. This matching leads to our theoretical prediction that EI = 700 pN·nm^2.

1. Vink, H., Duling, B. R. & Spaan, J. A. E. (1999) *FASEB J.* **13,** A11 (abstr.).