Appendix B. Derivation of Endothelial Surface Layer (ESL) Compression Due to Red Cell Arrest

1. Governing Equations

The draining of fluid from the ESL due to red cell arrest is modeled in the following figure.

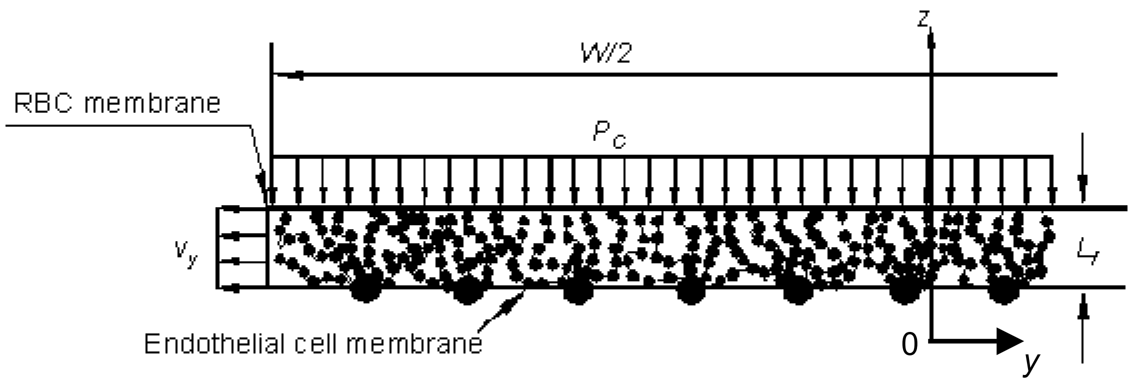

Fig. B1. Model for compaction of ESL beneath planar RBC membrane

In the above figure $v_y(y)$ is the local velocity of fluid in the ESL; W is the width of red cell; P_c is compression pressure of the red cell membrane on the ESL; and L_f is the height of the ESL. For a flexible membrane, L_f is a function of both y and t [see Wu and Weinbaum (1)]. However, for present purposes, where we are primarily interested in the characteristic time for the fluid drainage, we treat L_f as a rigid planar surface. In the absence of membrane curvature, P_c is equal to the internal cell pressure. As described in the text, the process is governed by Darcy's law,

$$\frac{\partial p}{\partial y} = -\frac{\mu}{K_p} v_y \ , \qquad \textbf{[B1]}$$

and continuity,

$$y\frac{dL_f}{dt}=-v_y L_f\,. \quad \textbf{[B2]}$$

Combining the above two equations, one obtains the pressure distribution beneath the red cell membrane,

$$p-p_0=\frac{\mu}{8K_p}\left(-\frac{dL_f}{dt}\right)\left(\frac{W^2-4y^2}{L_f}\right), \quad \textbf{[B3]}$$

where p_o is the ambient pressure at the edge of the compression zone. The average compression pressure on the ESL must equal the cell pressure. Thus,

$$P_c=\frac{2\int_0^{\frac{W}{2}}(p-p_0)dy}{W}=\frac{\mu W^2}{12K_p L_f}\left(-\frac{dL_f}{dt}\right)\,. \quad \textbf{[B4]}$$

Assuming P_c and W are constant, one obtains

$$t=\frac{\mu W^2}{12p_c}\int_{L_{f0}}^{L_f}\frac{-dL_f}{K_p L_f}\,, \quad \textbf{[B5]}$$

where L_{f0} is the initial ESL thickness. If K_p is constant, then

$$L_f/L_{f0}=\exp(-t/\tau)\ , \quad \textbf{[B6a]}$$

with

$$\tau=\frac{\mu W^2}{12P_c K_P}\,. \quad \textbf{[B6b]}$$

However, for the large compressions of the ESL considered herein, K_p is a function of the instantaneous solid fraction c.

2. Estimation of K_p

In ref. 2, the expression for the drag force on a single spherical scattering center along the core protein is

$$F = 6\pi K \mu r v_y, \qquad \textbf{[B7]}$$

where r is the radius of the scattering center and K is the drag force coefficient. Sangani and Acrivos (2) showed that for a face-centered cubic array of spheres,

$$K = \sum_{s=0}^{30} q_s \left[(c / c_{\max})^{1/3} \right]^s, \qquad \textbf{[B8]}$$

where $c_{\max} = 0.74$ is the maximum solid fraction for the face-centered array, and the q_s are coefficients given in ref. 2. From Darcy's law, one can show that K_p is related to K by

$$K_p = \frac{\varepsilon K^{-1}}{6\pi r}, \qquad \textbf{[B9]}$$

where $\varepsilon = \dfrac{4\pi r^3}{3c}$.

3. Estimation of other parameters

To solve the above equations, we have to estimate two other parameters: the solid fraction c and the width W of the red cell compression along the ESL. Mass conservation within the solid phase requires that

$$c = \frac{L_{f0} c_0}{L_f}, \qquad \textbf{[B10]}$$

where $c_0 = 0.13$ for the initial array of spherical scattering centers depicted in Fig.1. For a red cell volume of 90 μm^3, a capillary diameter of 5 μm and $L_f = 0.4$ μm, $W = 4.6$ μm initially and increases to 6.5 μm after maximum crushing of the ESL if the assumed shape is that of a circular cylindrical pellet. In our calculation, we let W be the mean of these two values.

4. Calculation

a. Constant K_p

Substituting t = 0.5 s into Eq. **B6a**,

$$\exp(-t/\tau) = \frac{L_{f\,\mathrm{min}}}{L_{f0}} = \frac{c_0}{c_{\mathrm{max}}} = 0.176.$$

Therefore,

$$\tau = 0.29 \text{ s}.$$

Substituting τ into Eq. **B6b** and rearranging, one obtains

P_c = 2,421 dyn/cm^2.

b. Variable K_p

Combining Eqs. **B7**, **B8,** and **B9**, one finds K_p is only a function of L_f. Thus, the integrand of the integral in Eq. **B5** is only a function of L_f. The integral in Eq. **B5** has been evaluated numerically. For P_c = 2,421 dyn/cm^2, the relationship between t and L_f is given in the following table and plotted in Fig. 6.

Table 1. Time-dependent compaction of ESL for variable K_P

***t*(*s*)**	**L_f/L_{f0}**	***t*(*s*)**	**L_f/L_{f0}**
0	1	0.28	0.55
0.015	0.95	0.36	0.50
0.033	0.90	0.47	0.45
0.053	0.85	0.62	0.40
0.076	0.80	0.86	0.35
0.10	0.75	1.29	0.30
0.14	0.70	2.29	0.25
0.17	0.65	7.03	0.20
0.22	0.60	25.5	0.176

Note that the time for maximum compaction has been extended by a factor of 50 from 0.5 s to 25 s.

1. Wu, R. & Weinbaum, S. (1982) *J. Fluid Mech.* **121,** 315-343.
2. Sangani, A. S. & Acrivos, A. (1982) *Int. J. Multiphase Flow* **8,** 343-360.