

## Appendix

By using different random variables, we can avoid the assumption of GW that dropping out is independent of the prior history of false positives. Let  $R_{tx} = 1$  if a subject with covariates at level  $x$  had  $t$  screenings and 0 otherwise. Because the only missingness is dropouts,  $pr(R_{tx} = 0 | R_{(t-1)x} = 0) = 1$ . Let  $Y_{tx} = 1$  if a subject with covariates at level  $x$  would have had a false positive if the subject received the  $t^{th}$  screening and  $Y_{tx} = 0$  if the result would not have been a false positive. Importantly,  $Y_{tx}$  (with realization  $y_{tx}$ ) is a *potential outcome* that is present regardless of whether or not a subject is screened.

We assume the probability of receiving the  $t^{th}$  screening depends on whether or not there was a previous screening,  $r_{t-1}$ , and the FP history up to and including the outcome on screening number  $(t - 1)$ , namely,  $y_1, y_2, \dots, y_{t-1}$ . Without loss of generality we describe the model for three screenings. Extensions to more screenings, such as the four screenings in our application, are straightforward. Let  $\phi$  denote a vector of parameters modeling the distribution of  $Y_1, Y_2, \dots, Y_{t-1}, Y_t$ . Under the assumed dropout mechanism, the probability of receiving only one screening with outcome  $y_{1x}$  is

$$pr(R_{2x} = 0 | R_{1x} = 1, y_{1x}) pr(R_{1x} = 1) pr(y_{1x}; \phi),$$

the probability of receiving exactly two screenings with outcomes  $y_{1x}$  and  $y_{2x}$  is

$$pr(R_{3x} = 0 | R_{2x} = 1, y_{1x}, y_{2x}) pr(R_{2x} = 1 | R_{1x} = 1, y_{1x}) pr(R_{1x} = 1) pr(y_{1x}, y_{2x}; \phi),$$

and the probability of receiving exactly three screenings with outcomes  $y_{1x}, y_{2x}$  and  $y_{3x}$  is

$$pr(R_{4x} = 0 | R_{3x} = 1, y_{1x}, y_{2x}, y_{3x}) pr(R_{3x} = 1 | R_{2x} = 1, y_{1x}, y_{2x})$$

$$pr(R_{2x} = 1 | R_{1x} = 1, y_{1x}) pr(R_{1x} = 1) pr(y_{1x}, y_{2x}, y_{3x}; \phi).$$

We use the above probabilities to formulate a likelihood. The key observation which obviates the unrealistic dropout assumption in GW is that the likelihood for the parameters  $\phi$  modeling the outcomes can be factored from the part of the likelihood modeling the dropout mechanism..

Let  $n_1(y_{1x})$  denote the number of subjects screened exactly once with outcome  $y_{1x}$ . Let  $n_2(y_{1x}, y_{2x})$  denote the number of subjects screened exactly twice with outcomes  $y_{1x}$  and  $y_{2x}$ . Similarly, let  $n_3(y_{1x}, y_{2x}, y_{3x})$  denote the number of subjects screened exactly three times with outcomes  $y_{1x}$ ,  $y_{2x}$  and  $y_{3x}$ . The part of the likelihood involving parameters modeling the false positive rate is

$$L_1 = \prod_{y_{1x}=0}^1 \prod_{y_{2x}=0}^1 \prod_{y_{3x}=0}^1 pr(y_{1x}; \phi)^{n_1(y_{1x})} [pr(y_{2x}|y_{1x}; \phi) pr(y_{1x}; \phi)]^{n_2(y_{1x}, y_{2x})} [pr(y_{3x}|y_{1x}, y_{2x}; \phi) pr(y_{2x}|y_{1x}; \phi) pr(y_{1x}; \phi)]^{n_3(y_{1x}, y_{2x}, y_{3x})}. \quad (\text{A.1})$$

It is helpful to regroup the factors in (A.1) as

$$L_1 = \prod_{y_{1x}=0}^1 pr(y_{1x}; \phi)^{n_1(y_{1x}) + n_2(y_{1x}, +) + n_3(y_{1x}, +, +)} \prod_{y_{1x}=0}^1 \prod_{y_{2x}=0}^1 pr(y_{2x} | y_{1x}; \phi)^{n_2(y_{1x}, y_{2x}) + n_3(y_{1x}, y_{2x}, +)} \prod_{y_{1x}=0}^1 \prod_{y_{2x}=0}^1 \prod_{y_{3x}=0}^1 pr(y_{3x} | y_{1x}, y_{2x}; \phi)^{n_3(y_{1x}, y_{2x}, y_{3x})}. \quad (\text{A.2})$$

where "+" denotes summation over the indicated variable.

To incorporate covariates, we modify the likelihood in (A.2). Let  $i$  index age at screening and  $j$  index time since the last screening. There are too few data to adequately fit a model that conditions on various prior patterns of FP. Instead, we simply condition

on whether an FP occurred previously ( $k = 1$ ) not ( $k = 0$ ). Thus instead of  $x$  we have covariates indexed by  $i, j, k$ .

Because we condition on whether or not an FP occurred previously (so a first screening is fundamentally different from a subsequent screening in predicting an FP), it is convenient to change the notation and specify distinct parameters for first screening,  $\alpha$ , and subsequent screening,  $\beta$ . We write

$$\begin{aligned} pr(Y_{1x} = y; \phi) &= pr(Y_{1i} = y; \alpha) \\ pr(Y_{tx} = y | y_{1x}, \dots, y_{(t-1)x}; \phi) &= pr(Y_{tij} = y | \text{previous history}; \beta) \\ &= pr(Y_{tijk} = y; \beta), \end{aligned}$$

where the relevant previous history under the model is captured by  $k$ .

Let  $m_{1yi}$  denote the number of subjects on the first screening in age interval  $i$  who had FP outcome  $y$ . Let  $m_{tyijk}$  denote the number of subjects on screening  $t$  in age interval  $i$  with time interval  $j$  since the last screening and false positive category  $k$  who had FP outcome  $y$ . Under this formulation we write the likelihood in (A.2) with  $n$  screenings as

$$L_2 = \prod_i \prod_{y=0}^1 pr(Y_{1i} = y; \alpha)^{m_{1yi}} \prod_i \prod_j \prod_k \prod_{t=2}^n \prod_{y=0}^1 pr(Y_{tijk} = y; \beta)^{m_{tyijk}}. \quad (\text{A.3})$$

The logistic regressions in the text are based on (A.3).