Detecting natural changes of cone-excitation ratios in simple and complex coloured images

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SUMMARY

Ratios of excitations in each cone-photoreceptor class produced by light reflected from pairs of surfaces in a scene are almost invariant under natural illuminant changes. The stability of these spatially defined ratios may explain the remarkable ability of human observers to efficiently discriminate illuminant changes from changes in surface reflectances. Spatial cone-excitation ratios are not, however, exactly invariant. This study is concerned with observers' sensitivity to these invariance violations. Simulations of Mondrian paintings with either 49 or two natural surfaces under Planckian illuminants were presented as images on a computer-controlled display in a two-interval experimental design: in one interval, the surfaces underwent an illuminant change; in the other interval, the surfaces underwent the same change but the images were then corrected so that, for each cone class, ratios of excitations were preserved exactly. Although the intervals with corrected images corresponded individually to highly improbable natural events, observers systematically misidentified them as containing the illuminant changes, the probability of error increasing as the violation of invariance in the other interval increased. For the range of illuminants and surfaces tested, sensitivity to violations of invariance was found to depend on cone class: it was greatest for long-wavelength-sensitive cones and least for short-wavelength-sensitive cones. Spatial cone-excitation ratios, or some closely related quantities, seem to be the cues preferred by observers for making inferences about surface illuminant changes.

1. INTRODUCTION

An important aspect of human colour vision concerns the stability of the perceived colours of surfaces in a scene as the spectral composition of the light illuminating the scene changes; for example, when artificial tungsten light is substituted for natural daylight from an overcast sky. The impression of colour constancy is strong (Young 1807; von Helmholtz 1866; Land 1959*a*,*b*), but not always justified when subjected to experimental test (Arend & Reeves 1986; Tiplitz Blackwell & Buchsbaum 1988; Valberg & Lange-Malecki 1990; Cornelissen & Brenner 1995; Nascimento 1995; Lucassen & Walraven 1996): even under natural viewing conditions, the colours of surfaces may appear to alter as the illuminant is changed. Nevertheless, observers seem to have little difficulty in distinguishing between a change in illuminant and a change in materials comprising the scene (Craven & Foster 1992; Foster *et al.* 1992).

This ability to discriminate natural illuminant changes from changes in surface spectral reflectances might be explained by the constancy of perceived spatial colour relations under illuminant changes ('relational colour constancy') and the failures of this constancy under spectral-reflectance changes. A physical basis for relational colour constancy could lie in the stability under illuminant changes of the ratios of excitations in each class of cone photoreceptors responding to light reflected from pairs of surfaces in the scene. Notice that the ratios are computed within rather than between cone classes. The stability of these spatially defined cone-excitation ratios holds for a large class of pigmented surfaces and surfaces with randomized spectral reflectances, and for a large class of illuminants, including the phases of natural daylight and Planckian radiators with temperatures ranging from 2000 to 100 000 K (Foster & Nascimento 1994). It is an empirical property that extends beyond the predictions of some theories of colour constancy based on low-dimensional models of illuminant and reflectance spectra (see, for example, Brill & West 1981; West & Brill 1982; Worthey & Brill 1986; Maloney & Wandell 1986; Funt & Drew 1988; Maloney 1993; Finlayson *et al.* 1994; D'Zmura & Iverson 1994).

Spatial cone-excitation ratios obtained from natural scenes are not, however, exactly invariant under illuminant changes; the extent of the failure is a few per cent (Foster & Nascimento 1994). If observers do indeed use spatial cone-excitation ratios as a cue to illuminant changes, then violations of invariance under an illuminant change, if visually detectable in a given scene, should be misinterpreted as being due to something other than an illuminant change; moreover, the probability of misinterpretation should increase as the size of the violation of invariance increases.

This hypothesis was tested in psychophysical experiments in which simulations of Mondrian paintings with either 49 or two natural surfaces (but without the dark gridlines typical of those paintings) were presented as images on a computer-controlled display. These surfaces underwent illuminant changes and modified illuminant changes in which the images were corrected so that spatial cone-excitation ratios were preserved exactly. A computational simulation showed that these particular corrected images corresponded to highly improbable natural events. The results of these experiments suggested that even though spatial cone-excitation ratios are sometimes inaccurate indicators, observers prefer these kinds of cues for making inferences about surface illuminant changes.

2. METHODS

Images were generated by an RGB colour-graphics system with 10-bit resolution selected for each gun (Ramtek UK Ltd, Basingstoke, Hampshire, UK; 4660 series). The graphics system was under the control of a laboratory computer (Sun Microsystems Inc, Mountain View, CA, USA; type 3/160). The images were displayed on the screen of a 19 inch colour monitor (Sony Corp, Tokyo, Japan; Trinitron GDM-2036S) with a resolution of 1280×1024 pixels and a frame rate of 60 Hz.

The system was calibrated spectroradiometrically and photometrically with instruments (Bentham Instruments Ltd, Reading, Berks, UK; LMT GmbH, Berlin, Germany) whose calibrations could in turn be traced to the UK National Physical Laboratory.

Stimuli each consisted of 49 (7×7) abutting square coloured patches, the whole pattern subtending 6×6 deg visual angle at a viewing distance of 60 cm. Simpler stimuli, of the same overall size but consisting of two coloured patches, were displayed with 8-bit resolution on each gun. All patches were simulations of natural, Lambertian, pigmented surfaces illuminated by natural, spatially uniform, light sources; or they were modifications of these simulations to achieve the required levels of cone excitations, as explained later. (All subsequent references to surfaces and illuminants should be taken to refer to the corresponding simulations.) The mean luminance of the images was set to 4 cd m^{-2}, but a wide range of individual patch luminances around that value were presented. Images appeared in a dark field.

The pigmented surfaces were Munsell surfaces drawn at random from the *Munsell book of color* (Munsell Color 1976). Their spectral reflectances were generated from a set of eight characteristic vectors obtained from a principal components analysis by Parkkinen *et al.* (1989). A large set of natural spectral reflectances, such as those of flowers, flower clusters, leaves and berries, are included in the space spanned by this set of characteristic vectors (Jaaskelainen *et al.* 1990). The light sources were drawn at random from a continuum of Planckian radiators having temperatures in the range 2000–100 000 K and constant integrated spectral power. These sources were preferred to daylights since they offer a larger range of colour temperatures. In the CIE 1931 (x, y)-chromaticity diagram, the Planckian locus is close to and almost parallel with the locus of the daylight coordinates (Judd *et al.* 1964).

The CIE 1931 (x, y)-chromaticity coordinates and luminance of the Munsell surfaces illuminated by the Planckian light sources of variable temperature were computed by numerically integrating the product of the spectral power distribution from the surfaces with the colour-matching functions for the CIE 1931 Standard Colorimetric Observer (see Wyszecki & Stiles 1982). Integrations were performed over the range 400–700 nm, with step size 10 nm.

Cone excitations were calculated from a set of spectral sensitivities defined for light incident at the cornea and were based on transformations of D. B. Judd's modification of the colour-matching functions for the CIE 1931 Standard Colorimetric Observer (Smith & Pokorny 1972, 1975; see Wyszecki & Stiles 1982). As with the calculation of (x, y)-chromaticity coordinates, integrations were performed numerically over the range 400–700 nm with step size 10 nm. (For further details, see Foster & Nascimento 1994.) Notice that by the nature of the colorimetric transformations involved, cone excitations represent a physical specification of the stimulus; no assumption is entailed about the size of the sensory response (von Kries's 'modified effects'; see Foster *et al.* 1997).

A temporal forced-choice procedure was used in all experiments. Each experimental trial was divided into two 2 s intervals separated by 1 s. In each interval, two images, each lasting 1 s, were presented in succession, so as to represent a natural illuminant change or a similar change in which ratios of excitations were manipulated (as described later). The observer was instructed to choose the interval containing the change that appeared more like a natural illuminant change. Measurements were made in a darkened room and experimental sessions lasted about 1 h. Subjects were not given feedback on their performance. When several experimental conditions were involved, their order of presentation was randomized to minimize order and carry-over effects.

In all, seven observers participated in the experiments: four in trials with complex (49-patch) images and three in trials with simple (two-patch) images. Each observer had normal colour vision, as assessed with the Farnsworth–Munsell 100-Hue test and Ishihara plates, and normal Snellen acuity. Each was unaware of the purpose of the experiments.

3. NATURAL ILLUMINANT CHANGES VERSUS INVARIANT CONE-EXCITATION RATIOS

This experiment determined the detectability of violations of invariance in spatial cone-excitation ratios during illuminant changes. The two intervals making up each experimental trial were constructed as follows: in one interval, the surfaces chosen at random from the Munsell set were subjected to an illuminant change chosen at random from a Planckian source; in the other interval, the surfaces were subjected to the same change but the images were then corrected (see Appendix 1) for any violations of invariance in the ratios of the excitations they produced in the short-, medium- and long-wavelength-sensitive cones. It is emphasized that these violations produced in the simulations were a natural consequence of the selection of illuminant and reflectance spectra; they were numerically small and occurred randomly (Foster & Nascimento 1994).

Trials were classified according to the degree of violation of invariance in spatial cone-excitation ratios for the pair of images in the illuminant-change interval. The degree of violation was calculated as follows. For any two patches in one of the images, three ratios of cone excitations can be defined: a ratio r_1 for short-wavelength-sensitive cones, a ratio r_2 for medium-wavelength-sensitive cones and a ratio r_3 for long-wavelength-sensitive cones. Let $\boldsymbol{r} = (r_1, r_2, r_3)$ be the three-dimensional vector consisting of these ratios r_1, r_2, r_3 in, say, the first image. Let $\boldsymbol{r}' = (r_1', r_2', r_3')$ be the corresponding vector of ratios in the second image. The difference between $\boldsymbol{r}$ and $\boldsymbol{r}'$ is small if the violations of invariance in the ratios are small, and large if the violations are large. A convenient summary measure of this difference is given by the quotient $|\boldsymbol{r} - \boldsymbol{r}'|/\min\{|\boldsymbol{r}|, |\boldsymbol{r}'|\}$, where the vertical bars signify the length of the vector; for example, $|\boldsymbol{r}| = (r_1^2 + r_2^2 + r_3^2)^{1/2}$. For brevity, the term *relative deviation* is used to refer both to the values of this quotient for simple (two-patch) images and to the average of these values taken over all adjacent pairs of patches for complex (49-patch) images. For complex images, relative deviations ranged from 0 to 0.1 and were quantized into bins of width 0.0125. For simple images, relative deviations ranged from 0 to 0.25 and were quantized into bins of width 0.05.

Figure 1 shows the results. The percentage of misidentifications (an ‘illuminant change’ response to an interval with images corrected for spatial cone-excitation ratios) is plotted against relative deviation in spatial cone-excitation ratios. Data for complex images are indicated by filled symbols and represent the means for the four observers, who performed 50 trials at each level of relative deviation. Data for simple images are indicated by open symbols and represent the means for the three observers, who performed 400 trials at each level of relative deviation. Vertical bars represent ±1 s.e.m. The continuous and broken curves each show the best-fitting inverse cumulative Gaussian transform of a quadratic function of the relative deviation.

Observers were clearly capable of detecting violations of invariance in spatial cone-excitation ratios, but, consistent with the assumed role of these ratios, they reported as illuminant-change intervals those intervals containing images corrected for spatial cone-excitation ratios, not the intervals containing natural illuminant changes: the larger the violation of invariance in the natural illuminant-change interval, the more likely it was to be misinterpreted.

When asked to comment on the appearance of the stimuli during the two kinds of changes, observers reported that for intervals containing images corrected for spatial cone-excitation ratios the change in appearance was ‘like a wash of colour’ over the scene; for intervals containing natural illuminant changes in which there were large violations of invariance in spatial cone-excitation ratios, they reported that the change in appearance was spatially uneven, with one or more patches being brought suddenly into prominence.

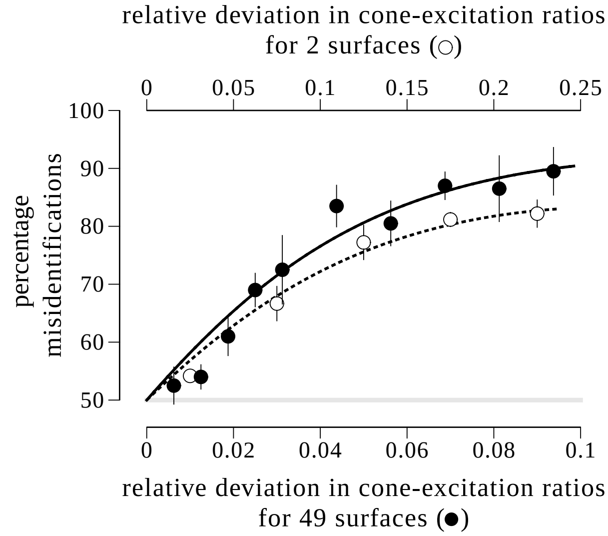

Figure 1. Discriminability of surface illuminant changes with natural images and images corrected for violations of invariance in spatial cone-excitation ratios. The percentage of misidentifications (‘illuminant change’ response to an interval with images corrected for spatial cone-excitation ratios) is plotted against relative deviation. Data for complex images are indicated by filled symbols and represent the means for four observers, who performed 50 trials at each level of relative deviation (bottom axis). Data for simple images are indicated by open symbols and represent the means for three observers, who performed 400 trials at each level of relative deviation (top axis). Vertical bars represent ±1 s.e.m. The continuous and broken curves each show the best-fitting inverse cumulative Gaussian transform of a quadratic function of the relative deviation.

4. RATIO-INVARIANCE VIOLATIONS FOR INDIVIDUAL CONE CLASSES

In the experiment described in the preceding section, observers discriminated between surfaces undergoing illuminant changes and the same surfaces undergoing similar changes, except that the images were corrected for any violations of invariance in the ratios of the excitations they produced in the short-, medium- and long-wavelength-sensitive cones. To determine whether sensitivity to violations of invariance was the same for each of the three cone classes, a modified version of the experiment was performed in which the images in the illuminant-change interval were also corrected for any invariance violations, but for only two of the three cone classes, chosen at random; any invariance violations in the third cone class were left intact. Only this cone class, therefore, could contribute to the discrimination task. Invariance violations in the other interval of the pair making up the trial were fully corrected as in the first experiment.

As before, trials were classified according to the relative deviation in spatial cone-excitation ratios in the illuminant-change interval, the deviation here defined only for the particular class of cones being tested. For complex images, relative deviations ranged from 0 to 0.0625 and were quantized into bins of width 0.0125. For simple images, relative devia-

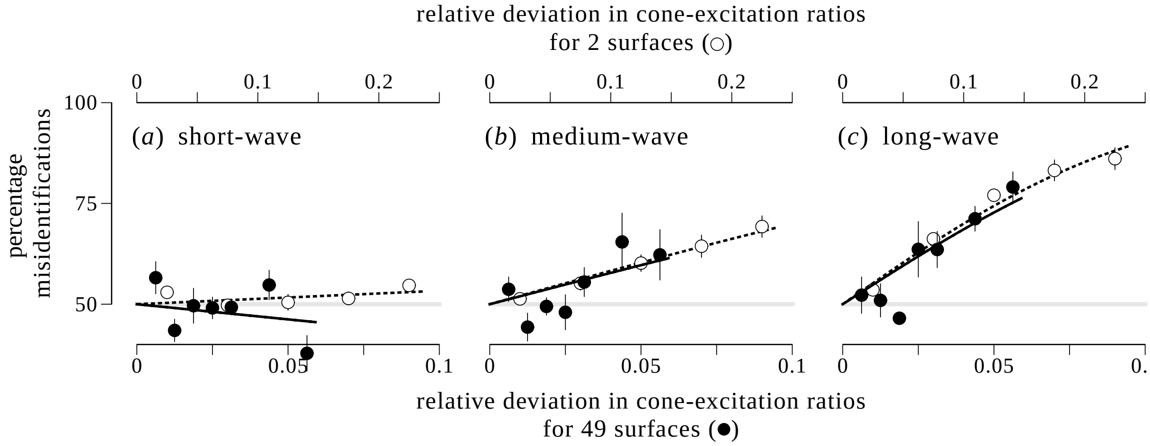

Figure 2. Discriminability of surface illuminant changes with partially corrected images and images fully corrected for violations of invariance in spatial cone-excitation ratios. The percentage of misidentifications ('illuminant change' response to an interval with images corrected for spatial cone-excitation ratios) is plotted against relative deviation for (*a*) short-, (*b*) medium- and (*c*) long-wavelength-sensitive cones. Data for complex images are indicated by filled symbols and represent the means for four observers, who performed at least 30 trials at each level of relative deviation (bottom axis). Data for simple images are indicated by open symbols and represent the means for three observers, who performed 300 trials at each level of relative deviation (top axis). Vertical bars represent ±1 s.e.m. The continuous and broken curves each show the best-fitting inverse cumulative Gaussian transform of a linear function of the relative deviation.

tions ranged from 0 to 0.25 and were quantized into bins of width 0.05.

Figure 2 shows the results. The percentage of misidentifications is plotted against relative deviation in spatial cone-excitation ratios for (*a*) short-, (*b*) medium- and (*c*) long-wavelength-sensitive cones. Data for complex images are indicated by filled symbols and represent the means for the four observers, who performed at least 30 trials at each level of relative deviation. Data for simple images are indicated by open symbols and represent the means for the three observers, who performed 300 trials at each level of relative deviation. Vertical bars represent ±1 s.e.m. The continuous and broken curves each show the best-fitting inverse cumulative Gaussian transform of a linear function of the relative deviation.

Sensitivity to violations of invariance in spatial cone-excitation ratios under changes in Planckian illuminant was greatest for long-wavelength-sensitive cones; intermediate for medium-wavelength-sensitive cones; and very small or absent for short-wavelength-sensitive cones.

5. EQUIVALENT NATURAL STIMULI WITH INVARIANT RATIOS

It might be argued that intervals containing pairs of images corrected for spatial cone-excitation ratios could still correspond to some naturally occurring combinations of surfaces and illuminant changes; in choosing these intervals, observers would therefore have been acting logically. But what is the probability of finding such naturally occurring combinations of surfaces and illuminant changes? The pool of potential combinations to be searched is not arbitrary: it has to be constrained to those combinations that could reasonably substitute for the image pairs that were corrected for spatial cone-excitation ratios. A reasonable requirement is that potential combinations should produce levels of changes in cone activities similar to those produced by the corrected image pairs. In general, however, the level of change in cone activities is correlated with the level of change in spatial cone-excitation ratios. This correlation was taken into account in the construction of table 1, which sets out the estimates of the required probabilities as a function of relative deviation in spatial cone excitations.

The table was constructed in stages.

(1) The range of relative deviations in spatial cone-excitation ratios was set at 0–0.1 and divided into ten equal bins of width 0.01. The centre values R of the bins are given in column 1.

(2) For each such R, a set of 10 000 image pairs was obtained, each pair corresponding to a Mondrian pattern of 49 randomly selected Munsell surfaces illuminated in turn by two randomly selected Planckian sources, such that the relative deviation in spatial cone-excitation ratios was within $R \pm 0.005$. (The 10 000 image pairs with these values of relative deviation were a very small proportion of the total number of pairs that had to be generated in this simulation.) For each of these 10 000 image pairs, the relative deviation Q in cone excitations was calculated. This relative deviation in cone excitations was measured in the same way as for ratios of cone excitations; that is, $Q = |\boldsymbol{q} - \boldsymbol{q}'|/\min\{|\boldsymbol{q}|, |\boldsymbol{q}'|\}$, where $\boldsymbol{q} = (q_1, q_2, q_3)$ is the three-dimensional vector consisting of the excitations q_1, q_2, q_3 of short-, medium- and long-wavelength-sensitive cones under one illuminant, and $\boldsymbol{q}'$ is the corresponding vector of excitations under the other illuminant. (Q was actually the average of these quotients over all 49 patches.)

Table 1. *Estimated probability of almost no violation of invariance in spatial cone-excitation ratios for natural surfaces undergoing illuminant changes contingent on the level of change in cone excitations*

(The first column specifies the relative deviation R in spatial cone-excitation ratios. The second column gives for $R \pm 0.005$ the corresponding mean value $\bar{Q}$ of the relative deviation Q in cone excitations. The third column gives the estimated probability P of obtaining a relative deviation $\leqslant 0.01$ in spatial cone-excitation ratios at the level of relative deviation $\bar{Q} \pm 0.005$ in cone excitations given in the second column. Estimates at each level of relative deviation in cone excitations were based on 10 000 samples of images with 49 Munsell surfaces illuminated by Planckian sources.)

relative deviation R in excitation ratios	mean value of $\bar{Q}$ of relative deviation in excitations	estimated probability P of almost invariant excitation ratios
0.005	0.016	0.689
0.015	0.047	0.117
0.025	0.078	0.119
0.035	0.109	0.001
0.045	0.140	0.000
0.055	0.171	0.000
0.065	0.201	0.000
0.075	0.227	0.000
0.085	0.250	0.000
0.095	0.279	0.000

The mean value $\bar{Q}$ of Q over all image pairs in that bin is given in column 2.

(3) For each such $\bar{Q}$, a new set of 10 000 image pairs was obtained such that for each pair the relative deviation in cone excitations was within $\bar{Q} \pm 0.005$. The probability of obtaining almost no relative deviation in spatial cone-excitation ratios at that level was estimated by counting the number of samples in the set of 10 000 with relative deviations in spatial cone-excitation ratios within one bin width of zero; that is, less than or equal to 0.01. These estimated probabilities P are given in column 3.

As an example, the last line of the table would be interpreted as follows. For surfaces and illuminant changes violating the invariance of spatial cone-excitation ratios by about 0.095, the mean relative deviation in cone excitations is 0.279; for surfaces and illuminant changes producing about this level of change in cone excitations, the probability of there being almost no violation of invariance in spatial cone-excitation ratios is less than 10^{-4}. (As these probabilities are conditional on the level of relative deviation in cone excitations, they cannot easily be compared with previously published data in Foster & Nascimento (1994).)

From these results it is clear that, given even a modest level of relative deviation in spatial cone-excitation ratios, the probability of finding a combination of surfaces and illuminant change which gives the same relative deviation in cone excitations and which preserves spatial cone-excitation ratios is vanishingly small. It seems that in choosing image changes with corrected spatial cone-excitation ratios, observers were not choosing an approximation to a highly probable natural event.

6. DISCUSSION

Spatial cone-excitation ratios have long been of interest as a device for normalizing visual responses, especially in analyses of colour constancy; for example, in chromatic-adaptation models of the von Kries type, where they define a response-scaling mechanism (Ives 1912; West & Brill 1982; Worthey & Brill 1986; see also Brainard & Wandell 1992; Finlayson *et al*. 1994; Chichilnisky & Wandell 1995; Finlayson & Funt 1996; Whittle 1996); and in the Retinex models due to Land, where they appear as products relating each surface to the 'highest reflectance' surface of the scene (Land & McCann 1971) or to some average over a collection of surfaces (Land 1983; see Hurlbert 1986 and Brainard & Wandell 1986). It is, however, important to distinguish between assumptions about the role of spatial cone-excitation ratios in light adaptation, that is, von Kries' coefficient rule, and in representing the invariances in natural scenes under illuminant changes (see Foster *et al*. 1997).

This study has been concerned with spatial cone-excitation ratios fulfilling a more direct role; namely, informing an observer about the constancy of spatial colour relations in scenes undergoing illuminant changes. It was hypothesized that if observers use the invariance of spatial cone-excitation ratios as a cue for discriminating illuminant changes from non-illuminant changes, then any natural violations of this invariance due to an illuminant change should be misinterpreted. Observers were able to detect violations of invariance in spatial cone-excitation ratios during illuminant changes; and, although corresponding to highly improbable natural events, the intervals they reported as illuminant-change intervals were indeed those containing images corrected for violations of invariance in spatial cone-excitation ratios, not the intervals containing natural illuminant changes. Moreover, the larger the violation of invariance during the illuminant change, the more likely the misinterpretation of that illuminant change. The same kind of performance was obtained with simple and complex images.

Restricting image changes so that only the ratios of excitations in one cone class varied showed that sensitivity to violations under changes in Planckian illuminants was greatest for long-wavelength-sensitive cones and least for short-wavelength-sensitive cones. The variation is superficially similar to that underlying the photopic luminance-sensitivity function: one formulation (Ingling & Tsou 1977) gives the proportions of the contributions of long-, medium- and short-wavelength-sensitive cones as 1.5:1:0; another (Vos *et al.* 1990) as 2:1:−0.06. From such data it might be inferred that observers' general ability to discriminate between illuminant and non-illuminant changes in a scene is a property of a luminance channel rather than of individual cone pathways. Despite the suggestive variation in the contributions of the three cone classes, it is unlikely that discrimination performance can be attributed to luminance signals alone: in a modified version of the experiment of § 3, violations of invariance in luminance ratios in the presence of random changes in surface colour were found to provide a much poorer cue. Furthermore, discrimination between illuminant and non-illuminant changes can be efficient with chromatic isoluminant images where all luminance information is removed (Nascimento 1995, § 5.2).

Although luminance ratios alone may be inadequate, there are other quantities related to spatial cone-excitation ratios that could provide biologically plausible cues. For example, ratios of opponent combinations of cone excitations are also stable under illuminant changes (Nascimento & Foster 1994; Zaidi *et al.* 1996). This stability was demonstrated in the following simulation. In each of 1000 iterations, a pair of Munsell surfaces was chosen at random from the Munsell set and a pair of Planckian illuminants was chosen at random with colour temperature in the range 2000–100 000 K. The excitations of each of the three cone classes were calculated as described earlier and used to form two linear combinations with varying weights, as follows. Let q_i, $i = 1, 2, 3$, be the excitations of short-, medium- and long-wavelength-sensitive cones, respectively, and let α be a weighting coefficient ranging from 0 to 1. A red–green difference signal 'r–g' was defined as $q_3 - \alpha q_2$ and a blue–yellow difference signal 'b–y' as $q_1 - \alpha(q_2 + q_3)$. For r–g signals, relative deviations in the ratios were lowest at intermediate values of α rather than at $\alpha = 0$ where they corresponded to excitations of long-wavelength-sensitive cones alone (compare Finlayson *et al.* 1994). For b–y signals, there was no reduction in relative deviations at non-zero values of α: the degree of invariance was highest with short-wavelength-sensitive cones alone.

Other functions of cone activity have been proposed as underlying psychophysically defined opponent-colour channels. Irrespective of the particular functional form, however, the effect of opponency is generally to sharpen and separate the spectral sensitivities of the medium- and long-wavelength-sensitive colour mechanisms, as revealed in traditional two-colour increment-threshold measurements for test sensitivity (Sperling & Harwerth 1971; King-Smith & Carden 1976; Foster & Snelgar 1983), and field sensitivity (Foster 1981), the latter being measurable over a greater wavelength and sensitivity range. As the width of the spectral sensitivities decreases, the ratios of the corresponding responses become less and less illuminant dependent (Barlow 1982; Worthey & Brill 1986; Buchsbaum & Gottschalk 1984; Maloney 1986). In theories of colour constancy based on low-dimensional models of illuminant and reflectance spectra, sharpening can be regarded as a transformation of a given set of sensor spectral sensitivities that makes possible a generalized von Kries approach to colour constancy (Finlayson *et al.* 1994; Finlayson & Funt 1996). Expressed as a matrix transformation, relational colour constancy can be considered as diagonal invariance (Wyszecki & Stiles 1982; Finlayson *et al.* 1994; see also Brainard & Wandell 1992), which is important in several computational approaches to colour and lightness constancy (Horn 1974; Brill 1978; Blake 1985; Hurlbert 1986; Funt & Finlayson 1995).

Why long-wavelength-sensitive cones should apparently give the greatest sensitivity to violations of invariance in spatial cone-excitation ratios under changes in Planckian illuminant and short-wavelength-sensitive cones the least is not clear. It seems unlikely that the difference is due to differences in the Fechner fraction for each cone class, for although the Fechner fraction for long-wavelength-sensitive cones is much smaller than that for short-wavelength-sensitive cones, it is about the same as that for medium-wavelength-sensitive cones; yet the contribution of medium-wavelength-sensitive cones to detecting invariance violations was smaller than that of long-wavelength-sensitive cones. Whatever the explanation of the differences in sensitivity between the three cone classes, spatial cone-excitation ratios—or some closely related quantities based on opponent-colour combinations—seem to be the preferred cues for making inferences about surface illuminant changes. Such quantities are preferred even when they sometimes correspond to highly unlikely natural events.

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APPENDIX 1.

The colorimetric specifications of the images corrected for violations of invariance in spatial cone-excitation ratios were calculated, in each trial, as follows. For each of the three cone classes, the levels of cone excitations produced by light from each Munsell surface under the first and the second Planckian illuminants were computed for all the 49 surfaces of the Mondrian pattern (as described in § 2). For each cone class, therefore, 49 pairs of excitations were obtained, each pair corresponding to a surface under the two illuminants. More precisely, fix the cone class and let $q_1(j)$ and $q_2(j)$ be the corresponding excitations produced by light from, respectively, the first and second illuminants on each surface j of the Mondrian pattern. (Notice that the subscripts refer here to the illuminant, not the cone class, as elsewhere.) When plotted in Cartesian coordinates, the points defined by the pairs $(q_1(j), q_2(j))$, with $j = 1, 2, \ldots, 49$, are distributed about a straight line passing through the origin. This straight line defines an ideal change in which, as is shown shortly, spatial cone-excitation ratios are perfectly preserved. The line does not, in general, have unit gradient: its slope depends strongly on the two illuminants, but it may be estimated from a least-squares linear fit to the data points. The individual levels of excitations $q_2(j)$ produced by light from the second illuminant on each surface j were adjusted so that the points fell precisely on the fitted line; that is, if k is the slope, then the adjusted excitations $\hat{q}_2(j)$, say, are given by $\hat{q}_2(j) = kq_1(j)$. To see that spatial cone-excitation ratios are now preserved exactly, consider two surfaces l, m under the first illuminant, producing, respectively, cone excitations $q_1(l)$, $q_1(m)$. Under the second illuminant, these cone excitations become $kq_1(l)$, $kq_1(m)$. The ratios are therefore unaltered: $q_1(l)/q_1(m) = kq_1(l)/(kq_1(m))$. The same correction procedure was applied for each cone class in turn.

The resulting corrected cone excitations $\hat{q}_2(j)$ were converted to CIE 1931 (x, y)-chromaticity coordinates with aid of the Vos (1978) conversion formula; luminance was determined under the assumption that Judd's modified photopic luminous efficiency function coincided with the non-modified function, which was acceptable as stimuli in the far blue were unavailable owing to limitations on the chromaticities of the monitor guns. Only those trials in which the chromaticities and luminances of the original and modified patches were such that all patches were displayable on the monitor were selected for the experiment.

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