This is an appendix to the paper by Randerson *et al.* 2000 Male killing can select for male mate choice: a novel solution to the paradox of the lek. *Proc. R. Soc. Lond.* B **267**, 867–874.

Electronic appendices are refereed with the paper. However, no attempt has been made to impose a uniform editorial style on the electronic appendices.

# Appendix B

## Details of the Simple Male Choice Model

The genotype frequencies are represented as follows (males: $y$, females: $x$)

|  | No MK | MK (females only) |
|---|---|---|
| No choice gene | $y_1, x_1$ | $x_4$ |
| Heterozygous for the choice gene | $y_2, x_2$ | $x_5$ |
| Homozygous for the choice gene | $y_3, x_3$ | $x_6$ |

The frequencies of matings in which there is no choice (i.e. those involving $y_1$) are simply the product of the respective male and female genotype frequencies. In our notation, $x_1 y_1 = m_{11}$, $x_2 y_1 = m_{21}$ etc. The frequencies of matings in which there is male choice (i.e. those involving $y_2$ and $y_3$) are as follows:

| Genotypes involved | Notation | Frequency |
|---|---|---|
| $x_1, y_2$ | $m_{12}$ | $y_2 (x_1(1-m) + ((1-m)(x_4 + x_5 + x_6) + m(x_1 + x_2 + x_3))(1-c)x_{12})$ |
| $x_2, y_2$ | $m_{22}$ | $y_2 (x_2(1-m) + ((1-m)(x_4 + x_5 + x_6) + m(x_1 + x_2 + x_3))(1-c)x_{22})$ |
| $x_3, y_2$ | $m_{32}$ | $y_2 (x_3(1-m) + ((1-m)(x_4 + x_5 + x_6) + m(x_1 + x_2 + x_3))(1-c)x_{32})$ |
| $x_4, y_2$ | $m_{42}$ | $y_2 (x_4 m + ((1-m)(x_4 + x_5 + x_6) + m(x_1 + x_2 + x_3))(1-c)x_{42})$ |
| $x_5, y_2$ | $m_{52}$ | $y_2 (x_5 m + ((1-m)(x_4 + x_5 + x_6) + m(x_1 + x_2 + x_3))(1-c)x_{52})$ |
| $x_6, y_2$ | $m_{62}$ | $y_2 (x_6 m + ((1-m)(x_4 + x_5 + x_6) + m(x_1 + x_2 + x_3))(1-c)x_{62})$ |
| $x_1, y_3$ | $m_{13}$ | $y_3 (x_1(1-m) + ((1-m)(x_4 + x_5 + x_6) + m(x_1 + x_2 + x_3))(1-c)x_{12})$ |
| $x_2, y_3$ | $m_{23}$ | $y_3 (x_2(1-m) + ((1-m)(x_4 + x_5 + x_6) + m(x_1 + x_2 + x_3))(1-c)x_{22})$ |
| $x_3, y_3$ | $m_{33}$ | $y_3 (x_3(1-m) + ((1-m)(x_4 + x_5 + x_6) + m(x_1 + x_2 + x_3))(1-c)x_{32})$ |
| $x_4, y_3$ | $m_{43}$ | $y_3 (x_4 m + ((1-m)(x_4 + x_5 + x_6) + m(x_1 + x_2 + x_3))(1-c)x_{42})$ |
| $x_5, y_3$ | $m_{53}$ | $y_3 (x_5 m + ((1-m)(x_4 + x_5 + x_6) + m(x_1 + x_2 + x_3))(1-c)x_{52})$ |
| $x_6, y_3$ | $m_{63}$ | $y_3 (x_6 m + ((1-m)(x_4 + x_5 + x_6) + m(x_1 + x_2 + x_3))(1-c)x_{62})$ |

$m$ is the probability of a mistake by a male and $c$ is the cost of mate searching (suffered if a male enters the second round of mate searching). $x_{12}$, $x_{22}$, $x_{32}$ etc. refer to the female genotype frequencies in the second round of mating. In the simple model in which population size is assumed to be infinite (so male choice in the first round has no effect on females genotype frequencies in the second round) these are the same as the original female genotype frequency. Recursion equations were obtained following

the assumptions in the section "Invasion of the choice gene" (so $x_1 = x_{12}$, $x_2 = x_{22}$ etc.). It was necessary to carry out recursions on adult individuals because fitness compensation affects broods as a whole.

$W_x x_1' = W_y y_1' = m_{11} + m_{12}/2 + m_{21}/2 + m_{22}/4 + m_{41}(1-\alpha)\ \varphi + m_{42}(1-\alpha)\varphi/2 + m_{51}\ (1-\alpha)\varphi/2 + m_{52}(1-\alpha)\varphi/4$

$W_x x_2' = W_y y_2' = m_{12}/2 + m_{13} + m_{21}/2 + m_{22}/2 + m_{23}/2 + m_{31} + m_{32}/2 + m_{42}(1-\alpha)\varphi/2 + m_{43}(1-\alpha)\varphi + m_{51}\ (1-\alpha)\varphi/2 + m_{52}(1-\alpha)\varphi/2 + m_{53}(1-\alpha)\varphi/2 + m_{61}\ (1-\alpha)\varphi + m_{62}(1-\alpha)\varphi/2$

$W_x x_3' = W_y y_3' = m_{22}/4 + m_{23}/2 + m_{32}/2 + m_{33} + m_{52}(1-\alpha)\varphi/4 + m_{53}(1-\alpha)\varphi/2 + m_{62}(1-\alpha)\varphi/2 + m_{63}(1-\alpha)\varphi$

$W_x x_4' = (1-U)[m_{41}\ \alpha\varphi + m_{42}\ \alpha\varphi/2 + m_{51}\ \alpha\varphi/2 + m_{52}\ \alpha\varphi/4]$

$W_x x_5' = (1-U)[m_{42}\ \alpha\varphi/2 + m_{43}\ \alpha\varphi + m_{51}\ \alpha\varphi/2 + m_{52}\ \alpha\varphi/2 + m_{53}\ \alpha\varphi/2 + m_{61}\ \alpha\varphi + m_{62}\ \alpha\varphi/2]$

$W_x x_6' = (1-U)[m_{52}\ \alpha\varphi/4 + m_{53}\ \alpha\varphi/2 + m_{62}\ \alpha\varphi/2 + m_{63}\ \alpha\varphi]$

The invasion conditions for the choice gene in the presence of the MK were found by modifier analysis. Linearised recursion equations were obtained for the three genotypes in which the choice gene is heterozygous ($y_2$, $x_2$, $x_5$). The system was taken to be in equilibrium for the male killer. Hence $x_4 = p^*$ (equation 5), $x_1 = 1-p^*$, $y_1 = 1$ and $x_3 = y_3 = x_6 = 0$. In matrix form, the linearised recursions become:

$$\begin{bmatrix} y_2' \\ x_2' \\ x_5' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} y_2 \\ x_2 \\ x_5 \end{bmatrix} = \lambda \begin{bmatrix} y_2 \\ x_2 \\ x_5 \end{bmatrix}$$

Hence for invasion, the leading eigenvalue, $\lambda$ of the resultant 3x3 matrix must be greater than one. It therefore follows that:

$1 < a + bd + e - ae + cg - ceg + bfg + cdh + fh - afh + i - ai - bdi - ei + aei$

This revealed that for invasion:

$$c < \frac{(1 - 2m)(2\alpha\varphi - 1 - 2(\varphi - 1)(1 + \alpha(U - 1)\varphi)}{(1 + 2\alpha(U - 1)\varphi)(\alpha\varphi(1 + U(m - 1) - 2m) + (1 + \varphi)m - 1)}$$

*Finite Population Size*

If the population size is finite, the female genotype frequencies in the second round of mating (i.e. $x_{12}$, $x_{22}$ etc.) will not be the same as those in the first round due to unequal sampling by males. If $N_y$ and $N_x$ are the numbers of males and females in the population respectively, then the frequencies of female genotypes in the second round of mating are:

$$x_{12} = (x_1(N_x - N_y) + (y_2 + y_3)x_1\,m\,N_y)/((N_x - N_y) + (y_2 + y_3)N_y((x_4 + x_5 + x_6)(1-m) + m(x_1 + x_2 + x_3)))$$

$$x_{22} = (x_2(N_x - N_y) + (y_2 + y_3)x_2\,m\,N_y)/((N_x - N_y) + (y_2 + y_3)N_y((x_4 + x_5 + x_6)(1-m) + m(x_1 + x_2 + x_3)))$$

$$x_{32} = (x_3(N_x - N_y) + (y_2 + y_3)x_3\,m\,N_y)/((N_x - N_y) + (y_2 + y_3)N_y((x_4 + x_5 + x_6)(1-m) + m(x_1 + x_2 + x_3)))$$

$$x_{42} = (x_4(N_x - N_y) + x_4\,N_y(y_2 + y_3))/((N_x - N_y) + (y_2 + y_3)N_y((x_4 + x_5 + x_6)(1-m) + m(x_1 + x_2 + x_3)))$$

$$x_{52} = (x_5(N_x - N_y) + x_5\,N_y(y_2 + y_3))/((N_x - N_y) + (y_2 + y_3)N_y((x_4 + x_5 + x_6)(1-m) + m(x_1 + x_2 + x_3)))$$

$$x_{62} = (x_6(N_x - N_y) + x_6\,N_y(y_2 + y_3))/((N_x - N_y) + (y_2 + y_3)N_y((x_4 + x_5 + x_6)(1-m) + m(x_1 + x_2 + x_3)))$$