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Electronic appendices are refereed with the text. However, no attempt is made to impose a uniform editorial style on the electronic appendices.

Electronic Appendix A

We have defined the frequencies of $\mathbf{UA}_{\mathrm{H}}$, $\mathbf{TfT}_{\mathrm{H}}$, and $\mathbf{DE}_{\mathrm{H}}$ by x_1, x_2, x_3 and the frequencies of $\mathbf{UA}_{\mathrm{L}}$, $\mathbf{TfT}_{\mathrm{L}}$, and $\mathbf{DE}_{\mathrm{L}}$ by y_1, y_2, y_3. That is, the respective *strategy sets* of the high and the low quality players are given by

(A.1a)
$$\mathbf{X} = \{x_1\mathbf{e}_1 + x_2\mathbf{e}_2 + x_3\mathbf{e}_3 | x_1, x_2, x_3 \geq 0 \text{ and } x_1 + x_2 + x_3 = 1\}$$
$$\mathbf{Y} = \{y_1\mathbf{e}_1 + y_2\mathbf{e}_2 + y_3\mathbf{e}_3 | y_1, y_2, y_3 \geq 0 \text{ and } y_1 + y_2 + y_3 = 1\}.$$

where {$\mathbf{e}_1$, $\mathbf{e}_3$, $\mathbf{e}_3$} is the *standard basis* of $\mathbf{R}^3 = (-\infty,\infty)^3$, i.e.,

(A.1b)
$$\mathbf{e}_1 = \begin{pmatrix}1\\0\\0\end{pmatrix}, \mathbf{e}_2 = \begin{pmatrix}0\\1\\0\end{pmatrix}, \mathbf{e}_3 = \begin{pmatrix}0\\0\\1\end{pmatrix}.$$

Let us define a population strategy profile by $(\mathbf{x},\mathbf{y}) \in \mathbf{X}\times\mathbf{Y}$, then the payoff to a high quality player using strategy $\mathbf{a} \in \mathbf{X}$ is given by

(A.2a)
$$\mathrm{v}_1(\mathbf{a},(\mathbf{x},\mathbf{y})) = \mathbf{a}^{\mathrm{t}} \circ \mathrm{P}_{\mathrm{HH}}(\mathbf{x},\mathbf{y}) \circ \mathbf{x} + \mathbf{a}^{\mathrm{t}} \circ \mathrm{P}_{\mathrm{HL}}(\mathbf{x},\mathbf{y}) \circ \mathbf{y}.$$

Whereas the payoff to a low quality player using strategy $\mathbf{b} \in \mathbf{Y}$ is given by

(A.2b)
$$\mathrm{v}_2(\mathbf{b},(\mathbf{x},\mathbf{y})) = \mathbf{b}^{\mathrm{t}} \circ \mathrm{P}_{\mathrm{LH}}(\mathbf{x},\mathbf{y}) \circ \mathbf{x} + \mathbf{b}^{\mathrm{t}} \circ \mathrm{P}_{\mathrm{LL}}(\mathbf{x},\mathbf{y}) \circ \mathbf{y}.$$

The idea of evolutionary stability is based on the concept of an *invasion barrier*. In the current case, a strategy profile $(\mathbf{x}^*,\mathbf{y}^*) \in \mathbf{X}\times\mathbf{Y}$ is an ESS: if a population in which all high quality individuals play $\mathbf{x}^*$ and all low quality individuals play $\mathbf{y}^*$ cannot be invaded by mutants or emigrants, of either or both quality types, using any alternative strategies — provided the numbers of these invaders are sufficiently small relative to the invaded population.

To qualify the term "sufficiently small" we use the dynamical approach advanced by Cressman (1992, chapter - 3). Let a strategy profile $(\mathbf{x}^*,\mathbf{y}^*) \in \mathbf{X}\times\mathbf{Y}$ be given. We construct the dynamical form of the game prescribed by the system (6-9) relative to $(\mathbf{x}^*,\mathbf{y}^*)$ as follows.

Let $(\mathbf{x},\mathbf{y})$ be a arbitrary alternate strategy profile. Let N_{11} and N_{12} be the densities of the $\mathbf{x}^*$, and $\mathbf{x}$ players respectively. Similarly, let N_{21} and N_{22} be the densities of the $\mathbf{y}^*$, and $\mathbf{y}$ players $(N_{11} + N_{12} = N_1 : N_{21} + N_{22} = N_2)$. Let b_i be the background fitness of the individuals in the population (independent of the game in question), and let d_i be their death rate. Finally, let us define $p = N_{12} / N_1$, $s = N_{22} / N_2$. We can express the post-invasion population (average) strategies in terms of p and s as

(A.3a)
$$\mathbf{x}_{\mathrm{p}} = (1-p)\mathbf{x}^* + p\mathbf{x} = \mathbf{x}^* + p(\mathbf{x} - \mathbf{x}^*) \quad \text{and} \quad \mathbf{y}_{\mathrm{s}} = (1-s)\mathbf{y}^* + s\mathbf{y} = \mathbf{y}^* + s(\mathbf{y} - \mathbf{y}^*).$$

In these terms the dynamics of N_{11}, N_{12}, N_{21}, and N_{22} are given by

(A.3b)
$$\begin{aligned}
\frac{dN_{11}}{dt} &= N_{11}\left[b_1 + \mathrm{v}_1\left(\mathbf{x}^*, (\mathbf{x}_{\mathrm{p}}, \mathbf{y}_{\mathrm{s}})\right) - d_1\right] \\
\frac{dN_{12}}{dt} &= N_{12}\left[b_1 + \mathrm{v}_1\left(\mathbf{x}, (\mathbf{x}_{\mathrm{p}}, \mathbf{y}_{\mathrm{s}})\right) - d_1\right] \\
\frac{dN_{21}}{dt} &= N_{21}\left[b_2 + \mathrm{v}_2\left(\mathbf{y}^*, (\mathbf{x}_{\mathrm{p}}, \mathbf{y}_{\mathrm{s}})\right) - d_2\right] \\
\frac{dN_{22}}{dt} &= N_{22}\left[b_2 + \mathrm{v}_2\left(\mathbf{y}, (\mathbf{x}_{\mathrm{p}}, \mathbf{y}_{\mathrm{s}})\right) - d_2\right]
\end{aligned}.$$

And, consequently, the dynamics of p and s are given by

(A.3c)
$$p' = \frac{d}{dt}\left(\frac{N_{12}}{N_1}\right) = -p(1-p)\mathrm{v}_1\left(\mathbf{x}^* - \mathbf{x}, (\mathbf{x}_{\mathrm{p}}, \mathbf{y}_{\mathrm{s}})\right) \quad \text{and} \quad s' = \frac{d}{dt}\left(\frac{N_{22}}{N_2}\right) = -s(1-s)\mathrm{v}_2\left(\mathbf{y}^* - \mathbf{y}, (\mathbf{x}_{\mathrm{p}}, \mathbf{y}_{\mathrm{s}})\right) \;:\; p(0) = p_0 \text{ and } s(0) = s_0.$$

We see that an invasion fails *if, and only if*, (*iff*) $(p(t),s(t))$ converges to (0,0). Thus, we say that a strategy profile $(\mathbf{x}^*,\mathbf{y}^*)$ is *locally asymptotically stable* (las) with respect to the strategy profile $(\mathbf{x},\mathbf{y})$ *iff* exist $\delta_{\mathbf{xy}}$, $\varepsilon_{\mathbf{xy}} > 0$ such that whenever $0 < p_0 < \delta_{\mathbf{xy}}$ and $0 < s_0 < \varepsilon_{\mathbf{xy}}$, $(p(t),s(t))$ converges to (0,0). Clearly, $(\mathbf{x}^*,\mathbf{y}^*) \in \mathbf{X}\times\mathbf{Y}$ is an ESS *iff* $(\mathbf{x}^*,\mathbf{y}^*)$ is las with respect to any strategy profile $(\mathbf{x},\mathbf{y}) \neq (\mathbf{x}^*,\mathbf{y}^*)$.

To qualify this statement we follow Cressman in expanding $\mathrm{v}_1\left(\mathbf{x}^* - \mathbf{x}, (\mathbf{x}_{\mathrm{p}}, \mathbf{y}_{\mathrm{s}})\right)$ and $\mathrm{v}_2\left(\mathbf{y}^* - \mathbf{y}, (\mathbf{x}_{\mathrm{p}}, \mathbf{y}_{\mathrm{s}})\right)$ as power series in p and s to obtain

$$v_1\left(\mathbf{x}^* - \mathbf{x}, (\mathbf{x}_p, \mathbf{y}_s)\right) = (\mathbf{x}^* - \mathbf{x})^t \circ \left[P_{HH}(\mathbf{x}_p, \mathbf{y}_s) \circ \mathbf{x}_p + P_{HL}(\mathbf{x}_p, \mathbf{y}_s) \circ \mathbf{y}_s\right]$$

$$= (\mathbf{x}^* - \mathbf{x})^t \circ \left[P_{HH}(\mathbf{x}^*, \mathbf{y}^*) \circ \mathbf{x}^* + P_{HL}(\mathbf{x}^*, \mathbf{y}^*) \circ \mathbf{y}^*\right] +$$

(A.4a) $$p\left[(\mathbf{x}^* - \mathbf{x})^t \circ P_{HH}(\mathbf{x}^*, \mathbf{y}^*) \circ (\mathbf{x} - \mathbf{x}^*) + r(1-q)S(x_2 - x_2^*)(x_3 - x_3^*)\right]$$

$$+ s\left[(\mathbf{x}^* - \mathbf{x})^t \circ P_{HL}(\mathbf{x}^*, \mathbf{y}^*) \circ (\mathbf{y} - \mathbf{y}^*) + rqS(x_2 - x_2^*)(y_3 - y_3^*)\right] =$$

$$\alpha_{00}\left((\mathbf{x}^*, \mathbf{y}^*), (\mathbf{x}, \mathbf{y})\right) + p\alpha_{10}\left((\mathbf{x}^*, \mathbf{y}^*), (\mathbf{x}, \mathbf{y})\right) + s\alpha_{01}\left((\mathbf{x}^*, \mathbf{y}^*), (\mathbf{x}, \mathbf{y})\right)$$

and

$$v_2\left(\mathbf{y}^* - \mathbf{y}, (\mathbf{x}_p, \mathbf{y}_S)\right) = (\mathbf{y}^* - \mathbf{y})^t \circ \left[P_{LH}(\mathbf{x}^*, \mathbf{y}^*) \circ \mathbf{x}^* + P_{LL}(\mathbf{x}^*, \mathbf{y}^*) \circ \mathbf{y}^*\right] +$$

$$p\left[(\mathbf{y}^* - \mathbf{y})^t \circ P_{LH}(\mathbf{x}^*, \mathbf{y}^*) \circ (\mathbf{x} - \mathbf{x}^*) + r(1-q)S(y_2 - y_2^*)(x_3 - x_3^*)\right] +$$

(A.4b) $$s\left[(\mathbf{y}^* - \mathbf{y})^t \circ P_{LL}(\mathbf{x}^*, \mathbf{y}^*) \circ (\mathbf{y} - \mathbf{y}^*) + rqS(y_2 - y_2^*)(y_3 - y_3^*)\right] =$$.

$$\beta_{00}\left((\mathbf{x}^*, \mathbf{y}^*), (\mathbf{x}, \mathbf{y})\right) + p\beta_{10}\left((\mathbf{x}^*, \mathbf{y}^*), (\mathbf{x}, \mathbf{y})\right) + s\beta_{01}\left((\mathbf{x}^*, \mathbf{y}^*), (\mathbf{x}, \mathbf{y})\right)$$

That is, the expansions of $v_1\left(\mathbf{x}^* - \mathbf{x}, (\mathbf{x}_p, \mathbf{y}_s)\right)$ and $v_2\left(\mathbf{y}^* - \mathbf{y}, (\mathbf{x}_p, \mathbf{y}_s)\right)$ as power series in *p* and *s* is analogous to the corresponding expansions for two-type games with constant payoff matrices. Thus (Cressman, 1992, ch - 3), $(\mathbf{x}^*,\mathbf{y}^*) \in \mathbf{X}\times\mathbf{Y}$ is an ESS of system (6-9) *if* for every strategy profile $(\mathbf{x},\mathbf{y}) \neq (\mathbf{x}^*,\mathbf{y}^*)$

(a) $\alpha_{00}\left((\mathbf{x}^*, \mathbf{y}^*), (\mathbf{x}, \mathbf{y})\right) \geq 0$ and $\alpha_{00}\left((\mathbf{x}^*, \mathbf{y}^*), (\mathbf{x}, \mathbf{y})\right) = 0 \Rightarrow \alpha_{10}\left((\mathbf{x}^*, \mathbf{y}^*), (\mathbf{x}, \mathbf{y})\right) > 0$

(b) $\beta_{00}\left((\mathbf{x}^*, \mathbf{y}^*), (\mathbf{x}, \mathbf{y})\right) \geq 0$ and $\beta_{00}\left((\mathbf{x}^*, \mathbf{y}^*), (\mathbf{x}, \mathbf{y})\right) = 0 \Rightarrow \beta_{01}\left((\mathbf{x}^*, \mathbf{y}^*), (\mathbf{x}, \mathbf{y})\right) > 0$

(A.5) If $\alpha_{00}\left((\mathbf{x}^*, \mathbf{y}^*), (\mathbf{x}, \mathbf{y})\right) = \beta_{00}\left((\mathbf{x}^*, \mathbf{y}^*), (\mathbf{x}, \mathbf{y})\right) = 0$ then

either

(c) $\alpha_{01}\left((\mathbf{x}^*, \mathbf{y}^*), (\mathbf{x}, \mathbf{y})\right) \geq 0$ or $\beta_{10}\left((\mathbf{x}^*, \mathbf{y}^*), (\mathbf{x}, \mathbf{y})\right) \geq 0$

or

(d) $\alpha_{10}\left((\mathbf{x}^*, \mathbf{y}^*), (\mathbf{x}, \mathbf{y})\right)\beta_{01}\left((\mathbf{x}^*, \mathbf{y}^*), (\mathbf{x}, \mathbf{y})\right) > \alpha_{01}\left((\mathbf{x}^*, \mathbf{y}^*), (\mathbf{x}, \mathbf{y})\right)\beta_{10}\left((\mathbf{x}^*, \mathbf{y}^*), (\mathbf{x}, \mathbf{y})\right)$

Remark. ESS points are not the only evolutionary stable solutions of evolutionary games (Cressman, 1992; ch - 6). There is also the possibility of a set of solutions exhibiting neutral stability among themselves, while being ESS-like in comparison with the strategies not in that set—*evolutionarily stable sets* (ES sets).

Electronic Appendix B

We find the evolutionary stable solutions of system (6-9) by a two step process.
In step – I we use the fact (cf. Weibull, 1996) that a strategy profile (**x***,**y***) is an evolutionary stable solution of system (6-9) *only if*

(B.0a) $$\mathrm{v}_1\left(\mathbf{x}^* - \mathbf{e}_{\mathrm{j}}, (\mathbf{x}^*,\mathbf{y}^*)\right)x_j = \mathrm{v}_2\left(\mathbf{y}^* - \mathbf{e}_{\mathrm{j}}, (\mathbf{x}^*,\mathbf{y}^*)\right)y_j = 0 \text{ for } j = 1,2 .$$

In step – 2 we apply the ESS criterion (A.5) to these potential ESS solutions.
In this specific case, system (6-9) has thirteen potential ESS solutions and one potential ES set solution.

(B.1a)
$$(\mathbf{x}_1,\mathbf{y}_1) = (\mathbf{e}_1,\mathbf{e}_3)$$
now
$$\alpha_{00}\left((\mathbf{x}_1,\mathbf{y}_1),(\mathbf{x},\mathbf{y})\right) = (S - C)(qrx_2 + x_3)$$
and
$$\beta_{00}\left((\mathbf{x}_1,\mathbf{y}_1),(\mathbf{x},\mathbf{y})\right) = (D - S)\left[y_1 + (1 - qr)y_2\right]$$

Hence (**x**$_1$,**y**$_1$) ↔ (**UA**$_{\mathrm{H}}$,**DE**$_{\mathrm{L}}$) is an ESS whenever

(B.1b) $$C < S < D .$$

(B.2a)
$$(\mathbf{x}_2,\mathbf{y}_2) = (\mathbf{e}_2,\mathbf{e}_3)$$
now
$$\alpha_{00}\left((\mathbf{x}_2,\mathbf{y}_2),(\mathbf{x},\mathbf{y})\right) = qr(C - S)x_1 + r(B - C + S)(\theta_2 - q)x_3$$
and
$$\beta_{00}\left((\mathbf{x}_2,\mathbf{y}_2),(\mathbf{x},\mathbf{y})\right) = rB(q - 1 + \theta_0)y_1 + rB(1 - \rho)(q - \theta_1)y_2$$
where
$$\rho = \frac{D - S}{B}\,;\ \theta_1 = \frac{r - \rho}{r(1 - \rho)}\,;\ \theta_2 = \frac{rB - C + S}{r(B - C + S)}\,;\ \theta_3 = \frac{\rho}{r}$$

Since r > C/B (Fishman *et al*., 2001), (**x**$_2$,**y**$_2$) ↔ (**TfT**$_{\mathrm{H}}$,**DE**$_{\mathrm{L}}$) is an ESS whenever.

(B.2b)
$$C < D - B,\ S < C, \text{ and } q < \theta_2$$
.
$$C > D - B,\ D - B < S < C,\ \rho < r, \text{ and } \theta_1 < q < \theta_2$$

(B.3)
$$(\mathbf{x}_3,\mathbf{y}_3) = (\mathbf{e}_3,\mathbf{e}_1) \neq (\mathbf{e}_1,\mathbf{e}_3)$$
but
$$\alpha_{00}\left((\mathbf{x}_3,\mathbf{y}_3),(\mathbf{e}_1,\mathbf{e}_3)\right) = C - S$$
and
$$\beta_{00}\left((\mathbf{x}_3,\mathbf{y}_3),(\mathbf{e}_1,\mathbf{e}_3)\right) = S - D$$

Thus, since D > C, (**x**$_3$,**y**$_3$) is not an ESS.

$$(\mathbf{x}_4,\mathbf{y}_4)=(\mathbf{e}_3,\mathbf{e}_2)\neq(\mathbf{e}_2,\mathbf{e}_3)$$

but

(B.4) $$\alpha_{00}\big((\mathbf{x}_4,\mathbf{y}_4),(\mathbf{e}_2,\mathbf{e}_3)\big)=(1-r+qr)(C-S)-qrB$$

and

$$\beta_{00}\big((\mathbf{x}_4,\mathbf{y}_4),(\mathbf{e}_2,\mathbf{e}_3)\big)=qrB-(1-r+qr)(D-S)$$

Thus, since $D > C$, $(\mathbf{x}_4,\mathbf{y}_4)$ is not an ESS.

$$(\mathbf{x}_5,\mathbf{y}_5)=(\mathbf{e}_3,\mathbf{e}_3)$$

now

(B.5a) $$\alpha_{00}\big((\mathbf{x}_5,\mathbf{y}_5),(\mathbf{x},\mathbf{y})\big)=(C-S)\big[x_1+(1-r)x_2\big]$$

and

$$\beta_{00}\big((\mathbf{x}_5,\mathbf{y}_5),(\mathbf{x},\mathbf{y})\big)=(D-S)\big[y_1+(1-r)y_2\big]$$

Hence $(\mathbf{x}_5,\mathbf{y}_5)\leftrightarrow(\mathbf{DE}_\mathrm{H},\mathbf{DE}_\mathrm{L})$ is an ESS whenever

(B.5b) $$S<C.$$

$$(\mathbf{x}_6,\mathbf{y}_6)=\big(\mathbf{e}_1,\gamma\mathbf{e}_2+(1-\gamma)\mathbf{e}_3\big)\quad:\gamma=\frac{(1-qr)(D-S)}{qr(B-D+S)}$$

now

$$\alpha_{00}\big((\mathbf{x}_6,\mathbf{y}_6),(\mathbf{x},\mathbf{y})\big)=qr(S-C)(1-\gamma)x_2+\big[S-C+qrB\gamma\big]x_3=0\Leftrightarrow\mathbf{x}=\mathbf{e}_1$$

and

(B.6a) $$\beta_{00}\big((\mathbf{x}_6,\mathbf{y}_6),(\mathbf{x},\mathbf{y})\big)=qr(D-S)(1-\gamma)y_1$$

thus

$$\beta_{00}\big((\mathbf{x}_6,\mathbf{y}_6),(\mathbf{x},\mathbf{y})\big)=0\Leftrightarrow\mathbf{y}=\mathbf{y}_\lambda\equiv\lambda\mathbf{e}_2+(1-\lambda)\mathbf{e}_3\ \text{ for }\ 0\le\lambda\le1$$

but

$$\beta_{01}\big((\mathbf{x}_6,\mathbf{y}_6),(\mathbf{e}_1,\mathbf{y}_\lambda)\big)=qr(D-B+S)(\lambda-\gamma)^2$$

Hence $(\mathbf{x}_6,\mathbf{y}_6)\leftrightarrow(\mathbf{UA}_\mathrm{H},\mathbf{TfT}_\mathrm{L}\oplus\mathbf{DE}_\mathrm{L})$ is an ESS whenever

$$C<D-B<S<D,\ \rho<r,\ \text{and}\ \theta_3<q$$

(B.6b) or

$$D-B<C<S<D,\ \rho<r,\ \text{and}\ \theta_3<q$$

$$(\mathbf{x}_7, \mathbf{y}_7) = (\mathbf{e}_2, \mu\mathbf{e}_2 + (1-\mu)\mathbf{e}_3) \quad : \mu = (q - \theta_1)/q$$

now

$$\alpha_{00}((\mathbf{x}_7, \mathbf{y}_7), (\mathbf{x}, \mathbf{y})) = r(C-S)\theta_1 x_1 + (1 - r\theta_1)(D-C)x_3$$

thus

(B.7a)

$$\alpha_{00}((\mathbf{x}_7, \mathbf{y}_7), (\mathbf{x}, \mathbf{y})) = 0 \Leftrightarrow \mathbf{x} = \mathbf{e}_2$$

and

$$\beta_{00}((\mathbf{x}_7, \mathbf{y}_7), (\mathbf{x}, \mathbf{y})) = r(D-S)\theta_1 y_1$$

thus

$$\beta_{00}((\mathbf{x}_7, \mathbf{y}_7), (\mathbf{x}, \mathbf{y})) = 0 \Leftrightarrow \mathbf{y} = \mathbf{y}_\lambda \equiv \lambda\mathbf{e}_2 + (1-\lambda)\mathbf{e}_3 \text{ for } 0 \le \lambda \le 1$$

but

$$\beta_{01}((\mathbf{x}_7, \mathbf{y}_7), (\mathbf{e}_2, \mathbf{y}_\lambda)) = qrB(1-\rho)(\lambda - \mu)^2$$

Hence $(\mathbf{x}_7,\mathbf{y}_7) \leftrightarrow$ (**TfT**$_H$, **TfT**$_L \oplus$**DE**$_L$) is an ESS whenever

(B.7b) $$D - B < C,\ D - B < S < C,\ r > \rho \text{ and } q > \theta_1.$$

$$(\mathbf{x}_8, \mathbf{y}_8) = (\mathbf{e}_3, \chi\mathbf{e}_2 + (1-\chi)\mathbf{e}_3) \quad : \chi = \frac{(1-r)(D-S)}{qr(S+B-D)}$$

that is

$$0 < \chi \Leftrightarrow D - B < S < D$$

now

(B.8)

$$(\mathbf{x}_8, \mathbf{y}_8) \neq (\mathbf{e}_2, \mathbf{y}_8)$$

but

$$\alpha_{00}((\mathbf{x}_8, \mathbf{y}_8), (\mathbf{e}_2, \mathbf{y}_8)) = -\,qrB\frac{D-C}{D-S}\chi$$

and

$$\beta_{ij}((\mathbf{x}_8, \mathbf{y}_8), (\mathbf{e}_2, \mathbf{y}_8)) = 0 \text{ for } i, j = 0,1$$

Thus, $(\mathbf{x}_8,\mathbf{y}_8)$ is not an ESS.

$$(\mathbf{x}_9, \mathbf{y}_9) = (\delta_1\mathbf{e}_2 + (1-\delta_1)\mathbf{e}_3, \mathbf{e}_1) \quad : \delta_1 = \frac{(1-r+qr)(C-S)}{(1-q)r(B-C+S)}$$

that is

$$0 < \delta_1 \Leftrightarrow S < C < D$$

now

(B.9)

$$(\mathbf{x}_9, \mathbf{y}_9) \neq (\mathbf{x}_9, \mathbf{e}_2)$$

but

$$\alpha_{ij}((\mathbf{x}_9, \mathbf{y}_9), (\mathbf{x}_9, \mathbf{e}_2)) = 0 \text{ for } i, j = 0,1$$

and

$$\beta_{00}((\mathbf{x}_9, \mathbf{y}_9), (\mathbf{x}_9, \mathbf{e}_2)) = -\,r(1-q)(D-S)(1-\delta_1)$$

Thus, $(\mathbf{x}_9,\mathbf{y}_9)$ is not an ESS.

$$(\mathbf{x}_{10},\mathbf{y}_{10}) = \left(\delta_2\mathbf{e}_2 + (1-\delta_2)\mathbf{e}_3, \mathbf{e}_2\right)$$

$$\text{where } \delta_2 = \frac{(1-r)(C-S) - qr(B-C+S)}{(1-q)r(B-C+S)}$$

(B.10)

now

$$(\mathbf{x}_{10},\mathbf{y}_{10}) \neq (\mathbf{x}_{10},\mathbf{e}_3)$$

but

$$\alpha_{ij}\left((\mathbf{x}_{10},\mathbf{y}_{10}),(\mathbf{x}_{10},\mathbf{e}_3)\right) = 0 \text{ for } i,j = 0,1$$

and

$$\beta_{00}\left((\mathbf{x}_{10},\mathbf{y}_{10}),(\mathbf{x}_{10},\mathbf{e}_3)\right) = -(1-r)\frac{B(D-C)}{B-C+S}$$

Thus, $(\mathbf{x}_{10},\mathbf{y}_{10})$ is not an ESS.

$$(\mathbf{x}_{11},\mathbf{y}_{11}) = \left(\pi\mathbf{e}_2 + (1-\pi)\mathbf{e}_3, \mathbf{e}_3\right) \quad : \pi = \frac{(1-r)(C-S)}{(1-q)r(B-C+S)}$$

(B.11a)

now

$$\alpha_{00}\left((\mathbf{x}_{11},\mathbf{y}_{11}),(\mathbf{x},\mathbf{y})\right) = r(C-S)[1-(1-q)\pi]x_1 = gx_1$$

i.e.,

$$\alpha_{00}\left((\mathbf{x}_{11},\mathbf{y}_{11}),(\mathbf{x},\mathbf{y})\right)_1 = 0 \Leftrightarrow \mathbf{x} = \mathbf{x}_\lambda \equiv \lambda\mathbf{e}_2 + (1-\lambda)\mathbf{e}_3 \text{ for } 0 \le \lambda \le 1$$

and

$$\beta_{00}\left((\mathbf{x}_{11},\mathbf{y}_{11}),(\mathbf{x},\mathbf{y})\right) = (D-C+g)y_1 + (1-r)\frac{B(D-C)}{B-C+S}y_2$$

thus

$$\beta_{00}\left((\mathbf{x}_{11},\mathbf{y}_{11}),(\mathbf{x},\mathbf{y})\right) = 0 \Leftrightarrow \mathbf{y} = \mathbf{e}_3$$

but

$$\alpha_{10}\left((\mathbf{x}_{11},\mathbf{y}_{11}),(\mathbf{x}_\lambda,\mathbf{e}_3)\right) = r(1-q)(S-B+C)(\lambda-\pi)^2$$

Hence $(\mathbf{x}_{11},\mathbf{y}_{11}) \leftrightarrow (\mathbf{TfT}_H \oplus \mathbf{DE}_H, \mathbf{DE}_L)$ is an ESS whenever

(B.11b) $$B/2 < C,\ B-C < S < C, \text{ and } q < \theta_2.$$

$$(\mathbf{x}_{12},\mathbf{y}_{12}) = \left(\eta_1\mathbf{e}_1 + (1-\eta_1)\mathbf{e}_3, \kappa_1\mathbf{e}_2 + (1-\kappa_1)\mathbf{e}_3\right)$$

$$\eta_1 = \frac{(C-S)(S+B-D) - (1-r)B(D-S)}{(1-q)rB(D-S)}; \ \kappa_1 = \frac{C-S}{qrB}$$

here

$$\kappa_1 > 0 \Leftrightarrow S < C < D$$

(B.12)

now

$$(\mathbf{x}_{12},\mathbf{y}_{12}) \neq (\mathbf{e}_2,\mathbf{y}_{12})$$

but

$$\alpha_{00}\left((\mathbf{x}_{12},\mathbf{y}_{12}),(\mathbf{e}_2,\mathbf{y}_{12})\right) = -\frac{(C-S)(D-C)}{D-S} < 0$$

and

$$\beta_{ij}\left((\mathbf{x}_{12},\mathbf{y}_{12}),(\mathbf{e}_2,\mathbf{y}_{12})\right) = 0 \text{ for } i,j = 0,1$$

Thus, ($\mathbf{x}_{12}$,$\mathbf{y}_{12}$) is not an ESS.

$$(\mathbf{x}_{13},\mathbf{y}_{13}) = \left(\gamma_2\mathbf{e}_2 + (1-\gamma_2)\mathbf{e}_3, \kappa_2\mathbf{e}_1 + (1-\kappa_2)\mathbf{e}_3\right)$$

$$\gamma_2 = \frac{D-S}{(1-q)rB}\,;\ \kappa_2 = \frac{(D-S)(B-C+S)+(1-r)B(C-S)}{qrB(C-S)}$$

now

(B.13) $$(\mathbf{x}_{13},\mathbf{y}_{13}) \neq (\mathbf{e}_1,\mathbf{y}_{13})$$

but

$$\alpha_{00}\left((\mathbf{x}_{13},\mathbf{y}_{13}),(\mathbf{e}_1,\mathbf{y}_{13})\right) = -(D-C)$$

and

$$\beta_{ij}\left((\mathbf{x}_{13},\mathbf{y}_{13}),(\mathbf{e}_1,\mathbf{y}_{13})\right) = 0 \text{ for } i,j = 0,1$$

Thus, ($\mathbf{x}_{13}$,$\mathbf{y}_{13}$) is not an ESS.

$$(\mathbf{x}_\lambda,\mathbf{y}_\mu) \in \Lambda = \left\{\left(\lambda\mathbf{e}_1 + (1-\lambda)\mathbf{e}_2, \mu\mathbf{e}_1 + (1-\mu)\mathbf{e}_2\right) \middle|\ \lambda,\mu \in [0,1]\right\}$$

now

(B.14a) $$\alpha_{00}\left((\mathbf{x}_\lambda,\mathbf{y}_\mu),(\mathbf{x},\mathbf{y})\right) = \{S - C + rB[1-(1-q)\lambda - q\mu\}x_3 \quad .$$

and

$$\beta_{00}\left((\mathbf{x}_\lambda,\mathbf{y}_\mu),(\mathbf{x},\mathbf{y})\right) = \{S - D + rB[1-(1-q)\lambda - q\mu\}y_3$$

Hence Λ is an ES set whenever

(B.14b) $$S > D.$$

References for Appendices A and B.

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