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Electronic appendices are refereed with the text. However, no attempt has been made to impose a uniform editorial style on the electronic appendices.

Tabulated Results

Table 1 shows the predicted and observed resonant and non-resonant periods for all the incidence time series we have studied.

As in the examples in Figure 1, all time series were broken into sections of roughly constant recruitment rate. This sectioning of the time series is important because substantial differences in recruitment rates yield different predictions for the PSD peaks, i.e., dynamical transitions are induced by substantial changes in recruitment rates (Earn *et al.* 2000).

In Table 1, a superscript 'b' indicates that a major change in recruitment rate was caused by a change in the number of births, while a superscript 'v' indicates that the proportion vaccinated changed substantially (either mass vaccination was initiated or a vaccine scare led to a dramatic reduction in vaccine uptake).

A large boost in births occurred during the 'baby boom' following the Second World War. The effects of birth rate changes on transmission dynamics are delayed, approximately, by the mean age at infection, A. Therefore, before calculating recruitment rates, $\nu(1-p)$, we translated the birth time series forward by A (4 years for measles, 7 years for chicken pox, 11 years for rubella and 4 years for whooping cough; *cf.* caption to Figure 1). Note that A will, of course, decrease (increase) as larger (smaller) birth cohorts age; however, correcting for this would lead only to a small difference in the amount by which the birth time series is translated forward.

In contrast, the effects of vaccination are almost immediate because — at least during the periods that the data cover — vaccine was administered primarily to children who were already experiencing high contact rates. In the case of whooping cough in London, England, whole cell vaccination was introduced in 1957, but there was a sharp decrease

in uptake in 1974 due to a vaccine scare. Vaccination against rubella was introduced in Canada in 1969, and vaccination against measles in England and Wales was started in 1967.

For vaccine era measles in London and Liverpool, England, and for whooping cough in London from 1978 onwards, the non-resonant period increased as vaccine uptake gradually increased; this is the reason for the ranges given in the table. To obtain the PSD peak range from these time series, we used a wavelet analysis (Torrence & Compo 1998, Grenfell *et al.* 2001). The midpoint period of the range is plotted in Figure 3 in these cases.

Parameter Estimates

Tables 2, 3 and 4 show the estimated parameter values that we used to obtain predictions for the resonant and non-resonant PSD peaks in all the incidence time series. Below, when we refer to published parameter estimates, they were taken from the monograph of Anderson & May (1991) and from standard vital statistics publications.

The mean latent period ($1/\sigma$) and mean infectious period ($1/\gamma$) for each disease are given in Table 2. Table 3 shows estimates of α, and Table 4 shows estimates of R_0 and $\langle\beta\rangle$.

Empirical estimates of the basic reproductive ratio R_0 yield $\langle\beta\rangle$ according to the relation $\langle\beta\rangle \simeq \gamma R_0$ (Anderson & May 1991). When available, we used published estimates of R_0 based on age-structured serological and incidence data, relevant either to the same place and time under analysis, or for a similar place and time. For other places and times it was necessary to estimate an effective R_0; we did this by starting with a known estimate of R_0 for another place or time, and then taking into account differences in birth rates and/or vaccine coverage (*cf.* Eq. 2 and Earn *et al.* (2000)). Whenever Eq. 2 was used in estimating the *effective* basic reproductive ratio, $R_{0,\mathrm{eff}}$, we flagged the corresponding entry in Table 4 with a superscript '*'.

For measles in London, England (1950–1967), New York City (1951–1963) and Baltimore (1951–1959) we used a published estimate of $R_0 \approx 17$ based on age-structured data for England and Wales (1950–1958). Then, inserting published birth records into Eq. 2, we estimated $R_{0,\mathrm{eff}}$ for the years before the baby boom in these locations. For

measles in Ontario (1904–1948) we again used a published estimate of $R_0 \approx 10.5$, based on age-structured data for Ontario (1912–1913).

For chicken pox in the Canadian provinces (1942–1955) and New York City (1928–1955), we used the estimate $R_0 \approx 10.5$ for chicken pox in Baltimore in 1943.

For rubella, the published estimate is $R_0 \approx 6.5$ for England and Wales (1960–1970). It would not be unreasonable to adopt this value for R_0 in Ontario during the same time period. Unfortunately, we are unable to use Eq. 2 to estimate the $R_{0,\mathrm{eff}}$ after mass vaccination was initiated, because we have no data on rubella vaccine uptake levels in Ontario. Instead, we have adopted a method for estimating $R_{0,\mathrm{eff}}$ for rubella in Ontario that can be carried out for each of the three sections in the time series available to us. We used the approximation $R_0 \simeq G/A$, where G is the inverse of the per capita birth rate and A is the mean age at infection, which is derived in Anderson & May (1991). We estimated G from Ontario birth and population data for each of the three sections of the rubella time series. Since we do not have age-structured rubella data, we used published values for the mean age at infection for the relevant time periods in the USA (See Table 5.4 in Anderson & May (1991)). The mean age at infection by rubella in the USA (1978–1980) was $A_{\mathrm{USA}} \approx$ 13–16 years, so we took $A = 14.5$ years for Ontario (1970–1989) ($G = 69.9$). $A_{\mathrm{USA}} \approx 9.5$ for the USA (1966–1968), so we took $A = 9.5$ for Ontario (1957–1969) ($G = 55.4$). $A_{\mathrm{USA}} \approx 10.5$ in 1943, so we took $A = 0.5$ for Ontario (1929–1956) ($G = 48.3$).

Finally, for whooping cough, the published England and Wales (1944–1978) estimate of $R_0 \approx 17$ was used as the R_0 for London, England (1958–1974). The value for London (1978–1991) was computed using Eq. 2 and taking changes in birth rates and vaccine coverage into account. For whooping cough in London, vaccination was initiated in 1957 but we were able to obtain yearly vaccine uptake data only from 1966 (Miller & Gay 1997); following Rohani *et al.* (1999), we interpolated vaccine uptake from 60% in 1958 to 75% in 1966 with vaccine efficacy of 80%. The published estimate $R_0 \approx$ 10–11 for Ontario (1912–1913) was used for Ontario (1904–1913), and Eq. 2 supplied us with an estimate for Ontario (1925–1943).

References

Anderson, R. M. & May, R. M. 1991, *Infectious Diseases of Humans*, Oxford Univ. Press, Oxford.

Earn, D. J. D., Rohani, P., Bolker, B. M. & Grenfell, B. T. 2000, A simple model for complex dynamical transitions in epidemics, *Science* **287**, 667–670.

Grenfell, B. T., Bjornstad, O. N. & Kappey, J. 2001, Travelling waves and spatial hierarchies in measles epidemics, *Nature* **414**, 716–723.

Miller, E. & Gay, N. J. 1997, Epidemiological determinants of pertussis, *Dev. Biol. Stand.* **89**, 15–23.

Rohani, P., Earn, D. J. D. & Grenfell, B. T. 1999, Opposite patterns of synchrony in sympatric disease metapopulations, *Science* **286**, 968–971.

Torrence, C. & Compo, G. P. 1998, A practical guide to wavelet analysis, *Bull. Amer. Meteor. Soc.* **79**, 61–78.

Disease	Place & Time	Predicted Spectral Peak (yrs)		Observed Spectral Peak (yrs)	
		Nonres.	Res.	Nonres.	Res.
Measles	London, England 1950–1967^{v}	–	2	–	2
	London, England 1968^{v}–1988	2.0–3.5	1	2.0–3	1
	Liverpool, England 1944–1967^{v}	1.8	1	1.8	1
	Liverpool, England 1968^{v}–1988	1.8–3.5	1	1.8–3	1
	Ontario, Canada 1904–1948	2.5	1	2.7	1
	NYC, USA 1928–1950^{b}	2.4	1	2.5	1
	NYC, USA 1951^{b}–1963	–	2	–	2
	Baltimore, USA 1928–1950^{b}	2.8	1	2.6	1
	Baltimore, USA 1951^{b}–1959	–	2	–	2
Chicken Pox	Ontario, Canada 1942–1955	2.9	1	2.7	1
	Manitoba, Canada 1942–1955	2.9	1	3.5	1
	BC, Canada 1942–1955	2.9	1	2.4	1
	Sasketchewan, Canada 1942–1955	2.9	1	2.6	1
	NYC, USA 1928–1955	2.9	1	–	1
Rubella	Ontario, Canada 1929–1956^{b}	5.1	1	4.8	1
	Ontario, Canada 1957^{b}–1969^{v}	4.1	1	–	1
	Ontario, Canada 1970^{v}–1989	5.0	1	5.2	1
Whooping Cough	London, England 1948–1957^{v}	2.7	1	2.5	–
	London, England 1958^{v}–1974^{v}	4.1	1	3.8	1
	London, England 1978^{v}–1991	3.7–4.4	1	3.8–4	1
	Ontario, Canada 1904–1913	3.6	1	4.8	–
	Ontario, Canada 1925–1943	2.7	1	2.5	–

Table 1: Predicted and observed periods of non-resonant and resonant power spectral density (PSD) peaks. The model perfectly predicts all resonant PSD peaks. The correlation between predicted and observed non-resonant PSD peaks is shown in Figure 3. *'b'* denotes a transition induced by changes in birth rates and *'v'* denotes a transition induced by changes in vaccine uptake. Further details are given in the text.

Disease	$1/\sigma$ (days)	$1/\gamma$ (days)
Measles	8	5
Chicken Pox	10	5
Rubella	10	7
Whooping Cough	8	14

Table 2: Values of the mean latent period $1/\sigma$ and the mean infectious period $1/\gamma$ used in the analysis of the seasonally forced SEIR model. Source: Anderson & May (1991), Table 3.1.

Disease	Place & Time	α
Measles	London, England 1944–1988	0.20
	Liverpool, England 1944–1988	0.16
	Ontario, Canada 1939–1969	0.21
Chicken Pox	Ontario, Canada 1942–1955	0.22
	Manitoba, Canada 1942–1955	0.22
	BC, Canada 1942–1955	0.20
	Sasketchewan, Canada 1942–1955	0.22
Rubella	Ontario, Canada 1939–1989	0.21
Whooping Cough	London, England 1948–1991	0.10
	Ontario, Canada 1939–1969	0.14

Table 3: Estimates of the seasonal amplitude α, based on Eq. 9. For each disease in each location, the longest weekly incidence series available to us was used in Eq. 9. See Appendix B for a description of the estimation technique. Our estimates of α are consistent with more crude estimates of α reported previously (Earn *et al.* 2000, Rohani *et al.* 1999).

Disease	Place & Time	Data type	$R_{0,\mathrm{eff}}$	$\langle\beta\rangle$ (yr^{-1})
Measles	London, England 1950–1967	W	17	1241
	London, England 1968–1986	W	6.8*–17	496–1241
	Liverpool, England 1944–1967	W	25.6*	1869
	Liverpool, England 1968–1988	W	6.9*–25.6*	504–1869
	Ontario, Canada 1904–1948	M	11.5	840
	NYC, USA 1928–1950	M	12.2*	891
	NYC, USA 1951–1963	M	17	1241
	Baltimore, USA 1928–1950	M	9.9*	723
	Baltimore, USA 1951–1959	M	17	1241
Chicken Pox	Ontario, Canada 1942–1955	W	10.5	767
	Manitoba, Canada 1942–1955	W	10.5	767
	BC, Canada 1942–1955	W	10.5	767
	Sasketchewan, Canada 1942–1955	W	10.5	767
	NYC, USA 1928–1955	M	10.5	767
Rubella	Ontario, Canada 1929–1956	M	4.6	240
	Ontario, Canada 1957–1969	W	6.5	339
	Ontario, Canada 1970–1989	M	4.8	250
Whooping Cough	London, England 1948–1957	W	17	443
	London, England 1958–1974	W	8.0*	209
	London, England 1978–1991	W	7.2*–9.9*	188–258
	Ontario, Canada 1904–1913	M	10.5	274
	Ontario, Canada 1925–1943	M	17.3*	451

Table 4: Parameter values used in the analysis of the seasonally forced SEIR model. The 'Data type' indicates whether cases were reported weekly (W) or monthly (M). '*' indicates that an effective value of R_0 for a given place and time, $R_{0,\mathrm{eff}}$, was calculated based on an R_0 value from a different place or time, using Eq. 2 together with data on birth rates and vaccine uptake. Further details are given in the text.