

# 'Optimal' vortex rings and aquatic propulsion mechanisms

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Fishes swim by flapping their tail and other fins. Other sea creatures, such as squid and salps, eject fluid intermittently as a jet. We discuss the fluid mechanics behind these propulsion mechanisms and show that these animals produce optimal vortex rings, which give the maximum thrust for a given energy input. We show that fishes optimize both their steady swimming efficiency and their ability to accelerate and turn by producing an individual optimal ring with each flap of the tail or fin. Salps produce vortex rings directly by ejecting a volume of fluid through a rear orifice, and these are also optimal. An important implication of this paper is that the repetition of vortex production is not necessary for an individual vortex to have the 'optimal' characteristics.

**Keywords:** swimming; fishes; squid; salps; vortices

## 1. INTRODUCTION

Observations of swimming fishes and other organisms such as salps reveal a series of vortex rings forming behind the animals, which play an important part in their mechanisms of propulsion. Fishes produce these vortices by an undulatory motion of the body and tail, controlling the periodic shedding of vorticity into the wake, and salps form them more directly, by ejecting fluid backwards through an orifice. In both cases the vortices roll up into three-dimensional (3D) ring-like structures. For fishes, the important question is what frequency and amplitude of the undulatory motion provide the most efficient propulsion? A similar question faces the oarsman of a gondola. For efficient motion, should the oar be moved slowly at large amplitude or quickly with a smaller amplitude? Here, we discuss this question, not in terms of a periodic motion, but by considering each flap of the tail or fin as a *single event*. This may be repeated periodically during steady swimming, but may be aperiodic during a turn or other manoeuvre. For other organisms using a jet for propulsion, the question is how long a jet provides the optimal efficiency.

Previous theoretical studies and observations have made good progress towards the understanding of both these mechanisms, and what constitutes optimal swimming behaviour. Two definitions of swimming efficiency are commonly used, both of which are appropriate for continuous swimming. The first is purely mechanical and relates the flow round and behind the body to the mean thrust this produces, while the second has a biological component and considers the energy supplied by the muscles and the resulting swimming speed. Here, we are concerned with the first definition, and discuss the efficiency in terms of the thrust achieved for a given amount of work.

The theory of vortex-ring production developed by Linden & Turner (2001)—this paper is hereafter referred to as LT—and summarized in § 2 was prompted by new measurements of vortices published by Gharib *et al.* (1998). They produced vortex rings by ejecting a length  $L$  of fluid from a circular pipe of diameter  $D$ , and observed that the maximum length of a plug of fluid ejected (using a piston) from the pipe, such that all the ejected fluid rolls up into a single vortex ring, is given by  $L/D = 4$ , and that this limit holds for various time histories of the piston motion. For larger  $L/D$ , secondary vortices formed in the wake behind the leading vortex. The LT theory matches four properties of the input plug to the corresponding properties of the vortex that forms, and predicts a maximum value of  $L/D = 3.5$ , close to the observed value  $L/D = 4$  and to that deduced from vortices shed behind aquatic animals. We show in § 2 that one crucial property of this critical case is that the vortex with  $L/D = 3.5$  has the maximum momentum (impulse) for a given energy input, i.e. it will produce the maximum thrust on the body generating it. It is in this sense that the vortex is 'optimal'; it is not necessary for similar vortices to be produced to measure the most efficient way of creating thrust.

## 2. THEORY OF VORTEX-RING PRODUCTION

The basic idea behind the theory in LT is that properties of the injected plug, in particular the circulation, impulse, kinetic energy and volume, are matched with a class of vortex rings. The class of rings chosen is that described by Norbury (1973), of which the classical thin-core vortex rings and Hill's spherical vortex are end members. These all have vorticity  $\omega$  proportional to the distance  $r$  from the axis of symmetry and confined to the vortex cores, which have a non-dimensional mean radius  $\varepsilon$  defined by  $\varepsilon^2 = (\text{area of core})/\pi X^2$ , where  $X$  is the distance from the axis to the centre of the core. The size of the core in relation to the spheroidal body of irrotational fluid carried along by the ring increases monotonically with  $\varepsilon$  until, at  $\varepsilon = 2^{1/2}$ , we reach Hill's vortex, for which

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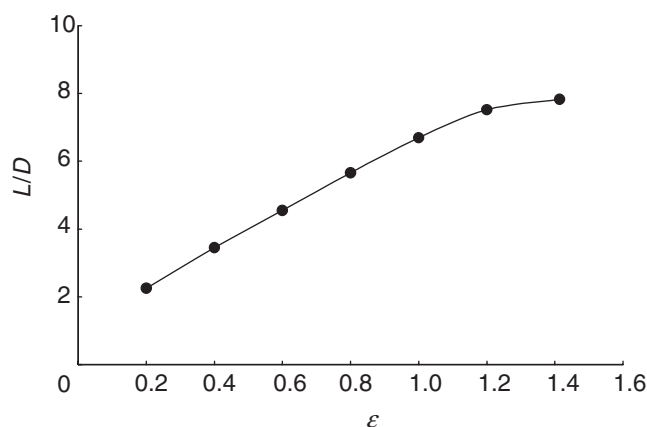


Figure 1. The aspect ratio of the impulsively produced plug, plotted against the core parameter  $\epsilon$  for the corresponding ring; reproduced from Linden & Turner (2001), fig. 3.

the whole sphere contains vorticity. Norbury (1973) calculated properties of these rings for discrete values of  $\epsilon$ , including (non-dimensional) values of the propagation velocity  $W_R$ , circulation  $K_R$ , fluid impulse  $P_R$  and kinetic energy  $T_R$ , and these have been used to match these properties to those of the injected plug.

The matching procedure is straightforward and the interested reader is referred to LT for the details. The main point is that if the length of the plug is too large, it is impossible to incorporate all the ejected fluid into the ring. This gives a maximum dimensionless plug length of  $L/D = 3.5$ , which, in turn, corresponds to a core-to-ring radius ratio of  $\epsilon = 0.42$ . The relationship between these two quantities is shown in figure 1. For the purposes of this discussion we consider the implications of matching the impulse (the momentum) and kinetic energy. These equations are

$$UX^3P_R = \frac{\pi}{4}U_P D^2 L, \quad (2.1a)$$

$$U^2 X^3 T_R = \frac{\pi}{8}U_P^2 D^2 L, \quad (2.1b)$$

where  $U_P$  is the velocity of the plug as it leaves the end of the pipe (taken here to be constant) and  $U$  is a reference velocity for the ring defined by  $U = \Omega X^2 \epsilon^2$ ;  $\Omega$  is the ‘vorticity constant’, defined by  $\omega = \Omega r$ , and is a measure of the magnitude of the vorticity in the core.

For fixed values of the plug (and ring) energy  $T_R$  and diameter  $D$ , equation (2.1b) implies that

$$LU_P^2 = \Lambda, \quad (2.2)$$

is a constant. Hence if  $L$  is increased while keeping the energy fixed, the plug velocity decreases,  $LU_P = \Lambda/U_P$  increases and, from equation (2.1a), it follows that the impulse increases, i.e. the member of the family with maximum  $L/D$  also has the maximum impulse. This is the optimal vortex ring and it is formed when the plug aspect ratio is  $L/D = 3.5$  and  $\epsilon = 0.42$ .

### 3. PROPULSION BY THE EJECTION OF FLUID FROM AN ORIFICE

The mode of swimming adopted by aquatic creatures that is most directly related to the flow of fluid from a pipe

is that which produces a backward-directed jet or pulse, commonly called jet propulsion. Madin (1990) has compared the cost of locomotion, defined as the energy required per unit mass to propel a body a unit distance, for various swimming animals. For squid, this quantity is in the range of  $7.6\text{--}12.6 \text{ J kg}^{-1} \text{ m}^{-1}$ , for solitary salps it is  $0.6\text{--}2.1 \text{ J kg}^{-1} \text{ m}^{-1}$  and for salmon it is  $1.9 \text{ J kg}^{-1} \text{ m}^{-1}$ . These figures imply that the swimming of squid is inefficient and that salps do well even when compared with fishes. We begin with a discussion of squid, and the reasons why they are not efficient swimmers according to this definition.

#### (a) Squid

Videler (1993) has expressed the above comparison in a different way, and states that ‘the squid *loligo opalescens* and the heaviest sockeye salmon share approximately the same mass, optimum swimming velocity and body length, but the squid, using jet propulsion, needs more than five times as much energy’ (p. 198). O’Dor & Webber (1986, p. 1592) also remarked that ‘The best performing squid uses more than twice the amount of energy to travel half as fast [mean speed] as the average fish’.

The reason is already to be found in the results presented by Johnson *et al.* (1972). Their measurements on squid showed that typically 200 ml of water is expelled in 0.2 s through an orifice of area  $1.5 \text{ cm}^2$ . This implies a velocity of  $600 \text{ cm s}^{-1}$  and a jet 120 cm long, so that  $L/D = 87$ . More recent measurements reported by Anderson & DeMont (2000), based on video recordings of time-varying squid profiles, give a more accurate measure of the jet velocity and length. Their estimate of a typical average jet velocity of  $0.54 \text{ m s}^{-1}$ , sustained for 0.5 s, implies a jet length  $L$  of 27 cm. The corresponding jet area was  $0.5 \text{ cm}^2$ , so that the mean diameter  $D$  was 0.8 cm and  $L/D = 34$ . This very long jet, ejected fast through a small opening, is far from the optimal condition we have identified in § 2 and, thus, is an inefficient way to produce thrust.

Note, however, the experimental results obtained by Bartol *et al.* (2001) who tracked particles in the surrounding flow for shallow-water brief squid of various sizes. They again found that large squid produced jets with large  $L/D$ , ranging from 10 to 40, but that some smaller squid (with lengths of less than 3.0 cm) kept  $L/D$  between four and seven throughout various jet cycles. They speculated that ‘smaller squid may be maximizing impulse per unit time and possibly for a given energy input’ (p. 3677), very much in accord with the arguments presented in this paper. They also reported measurements of ring radius and core radius for individual rings ejected by small squid, which give  $\epsilon = 2.6/4.8 = 0.54$ , close to the optimal value.

Of course, hydrodynamic efficiency, however defined, may not be the major factor that led to the evolution of this mechanism of propulsion in squid. Rapid acceleration to achieve the maximum speed during escape may be more important for their survival than the ‘efficiency’ during sustained swimming. Another factor that reduces the efficiency of the jet-propulsion mechanism used by squid is their need to refill the mantle cavity after each jet cycle by drawing in fluid in the opposite direction to the expelled jet.

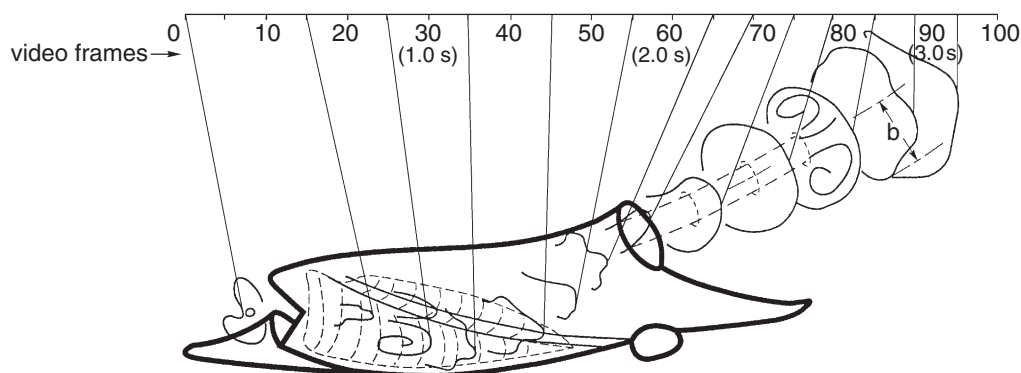


Figure 2. The water flow through a salp, *Salpa maxima*, after Madin (1990), with permission. The passage of a single pulse, made visible with uranine dye, was recorded on videotape, and the formation and progress of a vortex ring was clearly seen.

### (b) *Salps and jellyfish*

Salps pump water by muscular action through their tubular bodies (as illustrated in figure 2). They have orifices at both ends; they take in water at the front, and eject it at the back with the front opening closed, so that they can achieve more nearly steady swimming than other jet-producing creatures. Bone & Trueman (1983) have made a detailed study of the locomotion of salps, recording chamber pressures, thrust and muscle activity as well as the kinematics of freely swimming and tethered animals. The measurements were used to deduce thrust, drag and the energy input, and they concluded that 'jet propulsion' in salps is a more economical process than it is in other animals. From the data tabulated in the Bone & Trueman (1983) paper we can deduce that  $L/D$  for the ejected plugs lies in the range of 10–20 for various species of salp.

Other authors have also estimated  $L/D$  for salps, but the observations they have used are poorly constrained, because both the area of the exit opening and the pulse length are not known very accurately, and the values given vary over a wide range. Weihs (1977) quotes values of three to four, including one result based on Madin (1974), without giving any details of the calculation, and Madin (1990) himself suggested a value of  $L/D = 6.7$ , slightly longer than the optimum we have derived. At present we can recommend only that more detailed measurements should be made, using the particle-tracking techniques that have recently become available. However, one thing is clear from Madin's observations shown in figure 2. Dye placed in the body cavity of a salp, and followed as it was ejected, all ended up in a single vortex ring; the motion and expansion of this ring could be followed without any interference from a following vortex. Thus we can legitimately consider the propulsion mechanism in terms of the dynamics of single vortices.

Daniel (1983) has developed a theoretical model to describe the mechanics of medusan (jellyfish) jet propulsion. He has also quoted some rough experimental measurements, which we use here to estimate relevant parameters. The 'bell' of a typical jellyfish is roughly hemispherical, with a height of 1 cm and a diameter of 2 cm, with a ring of tissue around the opening restricting it to (approximately) half the bell area. Daniel states that vortex rings *ca.* 1 cm in diameter are produced during a contraction phase of 0.1 s, during which half the contained volume of water, *i.e.* *ca.* 2.1 ml, is expelled from the bell.

Equating this to the volume of the ejected plug of fluid, we find that  $L/D = 8.4/\pi D^3$ . This ratio is very sensitive to the diameter of the opening, which is poorly known. For example  $D = 1$  cm implies  $L/D = 2.67$ , while  $D = 0.85$  cm gives  $L/D = 4.35$ . A major conclusion of Daniel's (1983) analysis is that medusae are significantly less efficient than salps, because fluid is both drawn in and ejected through the same rear opening, and there is a large acceleration reaction that contributes to the drag as the velocity and mass change markedly over the cycle.

## 4. FISH PROPULSION BY UNDULATORY SWIMMING

Most marine organisms have only discrete mechanisms at their disposal, and cannot generate a continuous jet. Fishes swim by oscillating their pectoral fins or by passing a wave of curvature backwards along the body, shedding vorticity from the caudal fin into the wake, which rolls up into 3D ring-like structures. These two mechanisms are shown schematically in figure 3.

### (a) *Vortex wakes produced by pectoral fins*

The quantitative study of fish wakes has been transformed recently by the use of digital particle image velocimetry (DPIV) to visualize and measure the water flow pattern in the wake of the body and fins. Sheets of laser light orientated in various directions are used to illuminate small particles seeded into a recirculating flow tank, in which the fish is swimming at a fixed position relative to the light beams. Pairs of consecutive digital images are cross-correlated to produce a matrix of velocity vectors and other parameters, such as vorticity, to provide snapshots of the wake structure. From the measurements now available we have selected, and discuss in this and the following section, the papers that contain sufficient information to illustrate the principles and to test our predictions about the 'optimal' vortex ring.

We begin this discussion of the observations of fish wakes, and their interpretation in terms of 'optimal' vortices, with a description of the wake behind the pectoral fins of bluegill sunfish, as measured and described by Drucker & Lauder (1999). This is a simpler starting point than the vortices produced by whole-body oscillation and shedding at the caudal fin, because these authors have shown that discrete vortex rings are shed by the pectoral

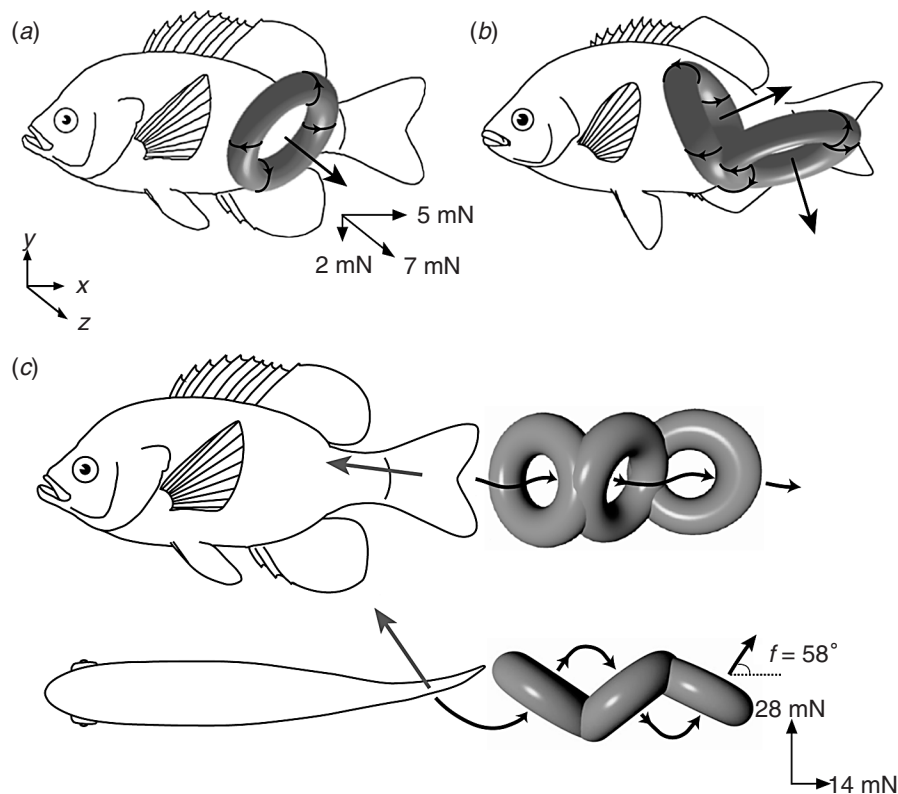


Figure 3. Vortex wakes of swimming fishes (reproduced from Lauder & Drucker (2002) fig. 2). (a) Bluegill sunfish and (b) black surfperch, swimming at 50% of their maximal pectoral-fin swimming speeds. These species shed wakes consisting of discrete vortex rings and linked vortex rings, respectively. (c) Lateral and dorsal views of sunfish swimming with the caudal fin, which generates a chain of linked vortex rings in the wake.

fins of these sunfish (see also Lauder & Drucker (2002)). Figure 4 shows the fluid vorticity components in three perpendicular wake planes behind the left pectoral fin of a bluegill sunfish swimming at 50% of its maximal pectoral-fin swimming speed. Note that the vorticity is concentrated into discrete cores; the arrows between them show the associated circulation, indicating the production of a single toroidal vortex.

There is no direct measure of a 'plug length' or  $L/D$  for pectoral-fin swimming, so we must use another property of the vortices produced to compare them with the proposed optimal condition. In several cases the radius  $R$  of the ring and the radius  $R_0$  of the vortex core have been measured. The ratio  $R_0/R$  is in fact close to the parameter  $\varepsilon$  and, because  $\varepsilon$  has a one-to-one correspondence with  $L/D$  (see figure 1), we can use this parameter as a measure of the optimal vortex ring. (Note that there is not necessarily an exact correlation between  $R_0/R$  and  $\varepsilon$  because the vorticity distribution in the core could be different from the Norbury vortices.) For the experimental measurements shown in figure 4, the vortex ring at the end of the stroke of the fin had  $R = 1.8$  cm and a mean  $R_0$  of 1.1 cm, so that  $\varepsilon = R_0/R = 0.61$ , a little larger than our theoretical optimal value. Drucker & Lauder (2001) have further considered the wake dynamics of turning manoeuvres in sunfish, and have shown that single rings with high momentum can be generated by the left and right pectoral fins in controlled asymmetrical directions to produce a rapid turn. This supports the view expressed in this paper, that the periodic production of rings is not essential for the effective generation of thrust.

#### (b) Wakes shed by the oscillation of fish bodies and the caudal fin

Direct measurements of the flow around and behind various species of fish using DPIV have been reported by Müller *et al.* (1997), Videler *et al.* (1999), Wolfgang *et al.* (1999), Lauder & Drucker (2002) and Nauen & Lauder (2002). They show that, as the wave of curvature travels down the body of a fish, semicircular flows (called proto-vortices) form on each side, centred on the inflection points. These proto-vortices are shed when the inflection points reach the tail. The vortices produced by the tail itself are shed when the tail changes direction. The fish can vary the phase of the shedding, and hence the shape of the wake and the contribution to thrust of the body vortices. For mullet, for example, less than half the thrust is caused by the body; the tail produces most of the thrust as well as controlling the vorticity shed from the body. A chain of rings is produced in the wake, with the momentum directed away from the mean path of motion rather than directly backwards, as it is in a jet expelled from a body cavity (see figure 2). However, for a given angle of the ring axis to the swimming direction, the forward thrust is still maximized by the optimal vortex.

The regular shedding of vortices can be described by a Strouhal number,  $St$ , defined by

$$St = \frac{fA}{V}, \quad (4.1)$$

where  $f$  (hertz) is the frequency of vortex formation,  $A$  is a length scale related to the width of the jet, usually identified by the amplitude of the tail flap and  $V$  is the

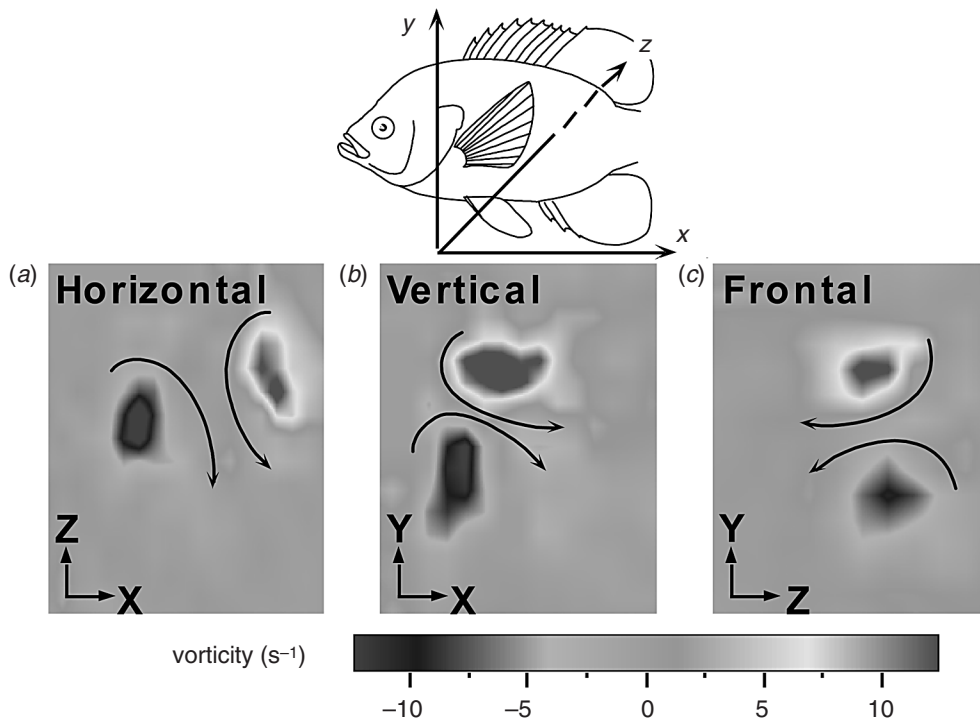


Figure 4. Fluid vorticity components in three perpendicular planes in the wake behind the fin of a sunfish swimming at  $0.5 \text{ L s}^{-1}$  (from Drucker & Lauder (1999) fig. 4): (a) horizontal; (b) vertical; (c) frontal. Note the confined vortex cores and the arrows showing the direction of motion in the associated fluid jets.

swimming velocity. The relationship between  $St$  and the plug aspect ratio is  $L/D = St^{-1}$ , assuming that successive vortices are spaced as closely as possible without interfering. A wide variety of fishes swim with  $St$  in the range of  $0.25\text{--}0.35$  ( $L/D = 3\text{--}4$ ) (Triantafyllou *et al.* 1993), and it has been concluded that this value has evolved as the most efficient repetition rate for the production of thrust. But the experiments of Gharib *et al.* (1998) and the theoretical interpretation by LT summarized in § 2, show that a series of vortices can form by the breakdown of a jet ejected from an orifice, without any active control from a flapping foil.

Triantafyllou *et al.* (2000) have reviewed various engineering studies of the fish-swimming mechanism, which are actively being pursued because of the potential applications to the propulsion of vehicles. They discuss the production of thrust by a pitching foil, and provide a slightly different interpretation from that given in the previous paragraph. They considered the jet-like average flow behind the body, and found that such jets are convectively unstable, with a narrow range of frequencies where the amplification is at a maximum. This results in the formation of a staggered array of vortices with directions of rotation opposite to that in a Karman vortex street. They suggest that the jet and the large-scale vortex pattern can exist together only at the frequency of maximum amplification, and that this corresponds to the condition for optimal efficiency, i.e. where the maximum thrust is produced for a given input of energy. The corresponding non-dimensional frequency,  $St$ , is in the range of  $0.25$  to  $0.30$ . Triantafyllou *et al.* (2000) also review and tabulate data on the swimming characteristics of many fishes and dolphins, and conclude that a wide variety of fishes, swimming at near their maximum free-swimming speeds, have the same range of  $St$ .

#### (c) Detailed measurements on vortices shed by fish bodies

The process of vortex shedding into the wake has been compared in § 4b with the breakdown of a jet ejected from an orifice, to suggest that the vortices so formed will be optimal. There is, however, another direct method of making this comparison. We can consider the DPIV measurements on the individual shed vortices, comparable with those presented in § 4a for pectoral fins, for caudal-fin locomotion. As a particular example of the many studies now available, we will discuss the data presented by Nauen & Lauder (2002) referred to in § 4b. They have studied the swimming of chub mackerel (20–26 cm fork length (FL)) in a recirculating flow tank at two speeds, 1.2, and  $2.2 \text{ FL s}^{-1}$ .

DPIV in vertical and horizontal planes through the fish body was used to determine velocity vectors and other parameters in the flow behind the fish. They show that the wake consists of a series of strong counter-rotating elliptical vortices, linked together as vortex loops rather than separate (see fig. 8 of Nauen & Lauder (2002)). The ellipses are larger and more elongated at the higher speed. Using the vorticity data, the ring radius  $R$  to the centre of the core and the mean vortex-core radius  $R_0$  (along with other ring properties) were calculated and tabulated. The forces produced by the rings have been calculated: these are directed laterally at an angle to the wake, so that the effective forward thrust is about one-third of that calculated from the properties of the individual rings.

Using the mean radius for the elliptical vortices, the data in table 1 of Nauen & Lauder (2002) give  $R = 2.19 \text{ cm}$ ,  $R_0 = 0.8 \text{ cm}$  and  $R_0/R = 0.37$  for  $1.2 \text{ FL s}^{-1}$  and  $R = 2.58 \text{ cm}$ ,  $R_0 = 0.88$  and  $R_0/R = 0.34$  for  $2.2 \text{ FL s}^{-1}$ . These ratios, equivalent to  $\varepsilon$  defined in § 2, are slightly smaller than the value of  $\varepsilon = 0.42$  corresponding

to the optimal condition and  $L/D = 3.5$ , but are consistent with the efficient production of thrust. This paper also quotes the corresponding values of  $St$  defined by equation (4.1),  $St = 0.30$  and  $St = 0.26$  for the lower and higher speeds, respectively. Both of these are consistent with the core-radius measurements and the properties of the optimal vortex, and, quoting from the paper, 'are within the range of values indicating high performance, despite the production of high lateral forces' (p. 1721).

DPIV has also been used to measure the properties of vortices produced by starting and turning manoeuvres of fishes, and shows clearly that continuous periodic shedding is not essential for the efficient production of thrust. Müller *et al.* (2000) have described the mode of swimming used by zebra danios, which consists of a double flick of the tail followed by a coasting phase. A large-core vortex ring is produced with  $R_0 = 4.7$  mm,  $R = 7.9$  mm and hence  $R_0/R = 0.60$ , rather larger than that deduced above for continuous swimming of mackerel.

For a fish making a sharp turn the bending of the body is followed by a double stroke of the tail, which forms a vortex generated first by the body, with the tail fin adding to the shed momentum. The controlled release of vorticity into the wake provides thrust in the new swimming direction. Measurements by Wolfgang *et al.* (1999) show that the core radius of the ejected vortices is more than double that of those produced in straight swimming of the same fish, which is consistent with the very large thrust.

## 5. SUMMARY AND DISCUSSION

We have presented the results of a theory (Linden & Turner 2001) that describes the production of a vortex ring by forcing fluid out of a circular pipe, as in the experiments of Gharib *et al.* (1998). We deduced that the maximum length  $L$  of plug that can produce a single ring from a pipe of diameter  $D$ , while conserving circulation, impulse, kinetic energy and particularly volume, is given by  $L/D = 3.5$ ; the corresponding mean experimental value is about four. For these rings the ratio  $\varepsilon$  of the radius of the vorticity-containing core to the ring radius is 0.42. The 'optimal' vortex formed in this way has the maximum impulse for a given kinetic energy, and, if it is ejected behind a swimming body, it produces the maximum thrust. This has led us to propose a new definition of efficiency of propulsion, which is based on an individual optimal vortex. This new approach is fundamentally different from the traditional view that efficiency is linked to the separation of vortices in the wake, and is not related to the repeated production of vortices by a flapping tail or fins.

The wake of a steadily swimming fish contains a regular series of vortices with the  $St$  falling in the narrow range of 0.25–0.35, and this is interpreted as representing the most efficient rate of undulation for the production of thrust in continuous swimming. The theory of vortex-ring production summarized in § 2 provides an alternative definition of efficiency. We consider each vortex formed in the wake of a swimming body as an individual efficient thrust-producing vortex; the observed range of  $St$  implies that these have formed from lengths of the jet close to  $L = 3.5D$ .

Note that there is no minimum rate of production of vortices for the process to be efficient in the sense we are suggesting. In fact fishes use a double flap of their tails to change direction or to swim a short distance, and they can wait as long as they like before repeating the process and creating another 'optimal' vortex. The idea of an optimal vortex has also been supported by recent DPIV measurements of the wakes behind the fins and bodies of fishes, which show that the sizes of the vortex cores are close to the values predicted for the 'optimal' condition.

The 'jet propulsion' mechanism is much less well documented, although it seems clear that squid produce a long fast jet that gives them high speed and manoeuvrability at a high energy cost, while salps eject individual vortices with properties not far from optimal. Different biological requirements determine the behaviour of various species. The existing measurements are not as precise or convincing as they are for fishes, however, and this problem would benefit from further experimental work. Ocean engineers have made great progress in building and understanding devices that mimic the undulatory motions of fish bodies and tails, but so far there has been relatively little thought given to the potential of intermittent jet production for the propulsion of ocean vehicles. Our definition of efficiency suggests new approaches worth exploring, and more detailed measurements on salps could be particularly rewarding. When individual rings become the focus of attention, then the frequency with which they are produced can be chosen on other grounds. For example, a vehicle designed to travel near the surface could eject rings at a rate related to the wave frequency, or alternatively energy could be stored over a longer period until enough has been accumulated to produce a single optimal ring.

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