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## ELECTRONIC APPENDIX A.

List of symbols of model variables and parameters, and their definitions.

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<b>Symbol</b>	<b>Meaning</b>
$M$	Pollinator population density
$\gamma_1$	Rate of flower pollination by pollinators
$\gamma_2$	Rate of flower oviposition by pollinators
$\delta$	Pollinator mortality rate
$F_i$	Flower production of individual $i$
$P_i$	Fraction of flowers pollinated of individual $i$
$D_i$	Fraction of flowers parasitized of individual $i$
$R_{fl,i}$	Resources available for flower production by individual $i$
$R_{fr,i}$	Resources available for fruit production by individual $i$
$R_{t,i}$	Total resources available for reproduction by individual $i$
$\alpha$	Conversion efficiency of resources into flowers
$\beta$	Conversion efficiency of resources into mature fruit
$G_i$	Potential fruit production by individual $i$ , given available resources
$F_{s,i}$	Fruit set or initiation of individual $i$ , given pollination and resources
$W_{F,i}$	Female fitness (seed or fruit production) of individual $i$
$W_{M,i}$	Male fitness (pollen dispersal and donation) of individual $i$
$W_i$	Fitness of plant individual $i$

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## ELECTRONIC APPENDIX B

An analytic approximation of (6) for the ratio-dependent case is possible by expanding the exponentials, keeping up to second order terms, as

$$e^{-\gamma_2 M / F} \cong 1 - \gamma_2 M / F + (1/2)(\gamma_2 M / F)^2 \quad (\text{A1})$$

$$e^{-\gamma_1 M / F} \cong 1 - \gamma_1 M / F + (1/2)(\gamma_1 M / F)^2 \quad (\text{A2})$$

From equation (A1), when  $\beta R_t R_{fr} > PF$ , the right hand side of equation (7) can be approximated and solved to obtain the equilibrium value of  $M$ :

$$M = 2F(\gamma_2 - d) / \gamma_2^2 = 2\alpha R_{fl} R_t (\gamma_2 - d) / \gamma_2^2 \quad (\text{A3})$$

From equation (A2), when  $\beta R_t R_{fr} < PF$ , the right hand side of equation (8) is approximated and solved for equilibrium to obtain;

$$M = \beta R_t R_{fr} (\gamma_2 / \gamma_1) / d = \beta R_t (1 - R_{fl}) (\gamma_2 / \gamma_1) / d \quad (\text{A4})$$

(In this case only the first order terms of the expansion are used.) Now it can be seen that as  $R_{fl}$  is increased from zero to larger values, as long as  $\beta R_t R_{fr} > PF$ ,  $M$  increases linearly with  $R_{fl}$ .

However, when the inequality is reversed, so that  $\beta R_t R_{fr} < PF$ ,  $M$  decreases as a function of  $R_{fl}$ .

Thus,  $M$  is a unimodal function of  $R_{fl}$ . This is a reasonable quantitative approximation of the 'exact' equations. The above approximations for  $M$  can be used in equations (4) and (5). When  $\beta R_t R_{fr} > PF$  the female fitness function is  $W_F = \alpha R_t R_{fl} (e^{-2(\gamma_2 - d) / \gamma_2} - e^{-2\gamma_1 (\gamma_2 - d) / \gamma_2^2})$ . When  $\beta R_t R_{fr} < PF$ , the female fitness function is

$$W_F = \beta R_t (1 - R_{fl}) (e^{-\beta(1-R_{fl})[\gamma_2^2 / (d\gamma_1 \alpha R_{fl})]} - e^{-\beta(1-R_{fl})[\gamma_2 / (d\alpha R_{fl})]}) / (1 - e^{-\beta(1-R_{fl})[\gamma_2 / (d\alpha R_{fl})]}) .$$

### ELECTRONIC APPENDIX C

Values of flower production strategies,  $R_{fl}$ , for plant fitness for the ecological model and for evolutionarily stable strategies for a wide range of parameter values of  $\alpha$ ,  $\beta$ ,  $\delta$ ,  $\gamma_1$ , and  $\gamma_2$ , for standard male fitness. Results are for ratio-dependent functional responses.

$\alpha$	$\beta$	$\delta$	$\gamma_1$	$\gamma_2$	Flower Production Strategy ( $R_{fl}$ )	
					Ecological $R_{fl}$	ESS $R_{fl}$
1.0	1.0	0.5	6.0	3.0	0.83	0.73
2.0	1.0	0.5	6.0	3.0	0.72	0.66
3.0	1.0	0.5	6.0	3.0	0.64	0.62
4.0	1.0	0.5	6.0	3.0	0.60	0.59
8.0	1.0	0.5	6.0	3.0	0.46	0.55
1.0	2.0	0.5	6.0	3.0	0.90	0.79
1.0	4.0	0.5	6.0	3.0	0.94	0.84
2.0	1.0	1.0	6.0	3.0	0.59	0.60
2.0	1.0	0.5	5.0	3.0	0.72	0.66
2.0	1.0	0.5	4.0	3.0	0.72	0.66
2.0	1.0	0.5	3.0	2.0	0.64	0.62
2.0	1.0	0.5	3.0	1.0	0.54	0.57