Chances of Proving Nonpaternity with a System Determined by Triple Allelic Codominant Genes

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In previous papers (Wiener *et al.*, 1930; Wiener, 1935), formulae were derived for the chances of proving nonpaternity with tests for a blood group system determined by two allelic codominant genes, such as the M-N system, and also for a system determined by three allelic genes, two codominant and the third an amorph, like the A-B-O blood group system (cf. MacCluer and Schull, 1963; Potthoff and Whittinghill, 1965; Wiener, 1966). Recently, other systems have been described, such as the acid phosphatase system of human red cells (Prokop and Uhlenbruck, 1966), which are determined by a series of three allelic genes, all of which are codominant. The purpose of this article is to derive the formulae for the chances of proving nonpaternity with tests of such a nature.

The procedure for deriving the formulae is first to calculate the chances of excluding paternity with the combinations in which tests on the mother's blood are not required, for example, putative father group AB and child group O; then to calculate the chances for the combinations in which the mother's type must also be determined, for example, putative father and mother both group O and the child group A; and then to sum the two probabilities to get the total chances.

If the three codominant genes are assigned the symbols G^R, G^S, and G^T, then there are six possible genotypes corresponding to the six phenotypes, namely, genotype G^RG^R—phenotype R; genotype G^SG^S—phenotype S; genotype G^TG^T—phenotype T; genotype G^RG^S—phenotype RS; genotype G^RG^T—phenotype RT; and genotype G^SG^T—phenotype ST. If the frequencies of the three allelic genes G^R, G^S, and G^T in the general population are r, s, and t, respectively, then in populations at genetic equilibrium, as will occur after a single generation of panmixia, the frequencies of the six phenotypes will be as follows: type R = r^2; type S = s^2; type T = t^2; type RS = $2rs$; type RT = $2rt$; and type ST = $2st$. Then the frequencies of the various mother-child combinations are readily derived, with the results shown in Table 1.

The formulae for the chances of proving nonpaternity in cases where the maternal phenotype is not known are readily derived, as shown in Table 2. For example, a falsely accused man of type R is excluded as a possible father when the child is of type S, or type T, or type ST, and the combined frequencies of these three combinations is

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$r^2(s + t)^2$. The frequencies of the other putative father-child combinations were derived similarly, and it can readily be seen that the total chance of exclusion for a falsely accused man is $2(r^2s^2 + r^2t^2 + s^2t^2) + 4rst$, in cases where the mother is not available to be tested. This has its maximum value when $r = s = t = \frac{1}{3}$, namely, $\frac{2}{9}$ or 22.2%. If we set $t = 0$, the situation reduces to a two-allelic system, as for the M-N blood types, and the chance of exclusion becomes $2r^2s^2$, which has its maximum value of $\frac{1}{8}$ when $r = s = \frac{1}{2}$.

The formula for the chances of proving nonpaternity in cases where the mother's phenotype must also be taken into account is derived with the aid of Table 1, as follows: When the putative father is type R, paternity is excluded for the mother-child combinations R → RT, or RS → RT, or RT → RS. Thus, the chances of exclusion for a falsely accused man of phenotype R is $r^2(1 - r) + 2rst$ (see Table 1), care being taken not to include the cases of exclusion given in Table 2. By multiplying by r^2, the frequency of phenotype R, one obtains the chances of occurrence in the general population of a falsely accused man of type R, with a mother-child constellation of phenotypes that excludes his paternity, namely, $r^4(1 - r) + 2r^3st$ (see Table 3). The

TABLE 1

FREQUENCIES OF MOTHER-CHILD COMBINATIONS IN A SYSTEM DETERMINED BY TRIPLE ALLELE GENES WITH CODOMINANCE

Type of Mother	Type of Child*					
	R	S	T	RS	RT	ST
R	r^3	0	0	r^2s	r^2t	0
S	0	s^3	0	rs^2	0	s^2t
T	0	0	t^3	0	rt^2	st^2
RS	r^2s	rs^2	0	$rs(r+s)$	rst	rst
RT	r^2t	0	rt^2	rst	$rt(r+t)$	rst
ST	0	s^2t	st^2	rst	rst	$st(s+t)$

* r, s, and t represent the frequency of the allelic genes G^R, G^S, and G^T, respectively.

TABLE 2

COMBINATIONS EXCLUDING PATERNITY FOR WHICH TESTS ON THE BLOOD OF THE MOTHER ARE NOT NECESSARY

Type of Putative Father	Type of Child	Frequency of Exclusion
R	S, T, or ST	$r^2(s+t)^2$
S	R, T, or RT	$s^2(r+t)^2$
T	R, S, or RS	$t^2(r+s)^2$
RS	T	$2rst^2$
RT	S	$2rs^2t$
ST	R	$2r^2st$
Total	.....	$2(r^2s^2+r^2t^2+s^2t^2)+4rst$

chances for falsely accused men of the other phenotypes are readily derived as shown in Table 3. Thus, the combined frequency of the cases excluding paternity in which the phenotype of the mother must be taken into account are $(r^4 + s^4 + t^4) - (r^5 + s^5 + t^5) + 2rst + 4rst(r^2 + s^2 + t^2)$. The maximum chances of exclusions of this kind are obtained by taking $r = s = t = \frac{1}{3}$, namely, $\frac{4}{27}$ or 14.8%. The maximum chances for a two-allele system are obtained by taking $t = 0$ and $r = s = \frac{1}{2}$, namely, $\frac{1}{16}$ or 6.25%.

TABLE 3

COMBINATIONS EXCLUDING PATERNITY FOR WHICH TESTS ON THE BLOOD OF THE MOTHER ARE NECESSARY

Type of Putative Father	Types of Mother and Child	Frequency of Exclusion
R	R →RS or RT	$r^4(1-r)$
	RS→RT	$2r^3st$
	RT→RS	
S	S →RS or ST	$s^4(1-s)$
	RS→ST	$2rs^3t$
	ST→RS	
T	T →RT or ST	$t^4(1-t)$
	RT→ST	$2rst^3$
	ST→RT	
RS	R →RT	$2rst(r+s)^2$
	S →ST	
	RS→RT or ST	
RT	R →RS	$2rst(r+t)^2$
	T →ST	
	RT→RS or ST	
ST	S →RS	$2rst(s+t)^2$
	T →RT	
	ST→RS or RT	
Combined	.....	$(r^4+s^4+t^4)-(r^5+s^5+t^5)$ $+2rst+4rst(r^2+s^2+t^2)$

By combining the formulae of Tables 2 and 3, one obtains the total chances of excluding paternity for a falsely accused man, using a three-allele system without dominance, namely, $(r^2 + s^2 + t^2)^2 - (r^5 + s^5 + t^5) + 6rst + 4rst(r^2 + s^2 + t^2)$. The total maximum chances of excluding paternity with such a three-allele system, therefore, are $\frac{19}{27}$ or 37.0%, in contrast to only $\frac{3}{16}$ or 18.75% for a two-allele system.

REFERENCES

MACCLUER, J. W., and SCHULL, W. J. 1963. On the estimation of the frequency of nonpaternity. *Amer. J. Hum. Genet.* **15**:191–202.

POTTHOFF, R. F., and WHITTINGHILL, M. 1965. Maximum-likelihood estimation of the proportion of nonpaternity. *Amer. J. Hum. Genet.* **17**:480–494.

PROKOP, O., and UHLENBRUCK, G. 1966. *Lehrbuch der menschlichen Blut- und Serumgruppen.* 2d ed. Thieme, Leipzig, p. 342.

WIENER, A. S. 1935. Heredity of the agglutinogens M and N of Landsteiner and Levine. IV. Additional theoretico-statistical considerations. *Hum. Biol.* **7**:222–239.

WIENER, A. S. 1966. Estimation of nonpaternity. *Amer. J. Hum. Genet.* **18**:309–310.

WIENER, A. S., LEDERER, M., and POLAYES, S. H. 1930. Studies in isohemagglutination. IV. On the chances of proving non-paternity, with special reference to the blood groups. *J. Immun.* **19**:259–282.