Mathematical relationship between waiting times and appointment interval for doctor and patients

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SUMMARY. Appointment systems try to achieve a balance between the time the doctor waits for patients to arrive and the time patients spend waiting to be seen. Mathematical analysis reveals that the time a patient can expect to wait increases exponentially as the appointment interval is reduced. An appointment interval that is less than the median consultation length can result in long waits for patients with no saving of time for the doctor. More frequent, shorter surgeries can save time for patients with no increase in the doctor's waiting time. Methods of improving the efficiency of use of surgery time are discussed.

Introduction

THE question of what constitutes an effective consultation has stimulated much research and discussion. The optimum length of a consultation has been considered from the point of view of the doctor,[1-3] the manager,[4,5] and the patient.[6,7]

Such considerations may influence our behaviour as clinicians, but we may be cautious about altering our appointment system for fear of having to wait for patients to arrive and thus wasting time. If the mean time spent in consultation is longer than the booked time for appointments, will patients have to wait much longer, or will the difference be hardly noticeable? This study aimed to investigate the effect of different appointment intervals on the efficient use of time by both doctor and patients.

Method

The length of consultation was determined for 248 consecutive patients attending a series of general practice surgeries in one urban practice. The practice had two full-time principals, one part-time principal, a trainee and 5200 patients. The surgeries were held by a single practitioner at various times of day on various days of the week. In order that the consultation times were not limited by the doctor's knowledge of the booked appointment length, each patient was asked when making the appointment whether he or she would like five, 10, 15 minutes or longer with the doctor. The earliest available appointment was offered. Their choice was unknown to the doctor and an electronic call system ensured he did not know how many patients were waiting to be seen. This sample reflects the distribution of various consultation lengths when the doctor does not feel obliged to keep to prearranged appointment times.

The next step was to calculate the expected mean waiting times for patients and doctor if these consultations had been booked using a fixed appointment interval and surgery duration. A specially written computer programme was used. The range of possible appointment intervals considered was five to 15 minutes in one minute steps, and surgeries lasting one and a half to three hours in half hour steps.

For example, consider an appointment system with five minute

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appointments and one and a half hour surgeries. The programme starts by randomly choosing one of the 248 patients. As this is the first patient and it is assumed that patients arrive at their appointment time, no time is spent waiting by either the doctor or the patient. Suppose this chosen patient had a recorded consultation time of only four minutes. The doctor has to wait one minute for the next patient, so after the first patient the doctor has waited one minute and the patient has waited zero minutes. A second patient is chosen at random from the remaining 247 and his consultation length may be seven minutes. This means that the consultation will overrun the next appointment time by two minutes and the third patient will have to wait two minutes. After the second patient the doctor has waited one minute and the patients have waited two minutes.

The process continues until the surgery is completed; in the example this would be when 18 patients had been seen. The programme then goes on to the second surgery and so on until all 248 patients have been seen. The total time waited by the doctor and by the patients is divided by 248 to give the mean waiting time for each consultation. For the sake of clarity whole minutes were used in the example, but the times used in the program are to the nearest second.

The result obtained from a single run depends on the order in which the patients are seen. If patients who had long consultations were randomly selected to be seen early in a surgery, this would increase the mean time waited by the patients, but if patients who had short consultations were selected to be seen first the mean time waited by the doctor would increase. The programme recalculates the results for a particular combination of appointments and surgeries until the mean of the results is constant.

The expected mean waiting times for a system of three five minute appointments followed by a five minute gap, a 6.7 minute appointment interval and for the times chosen by the patients were also calculated.

Results

Figure 1 is a frequency histogram of the actual consultation lengths of the 248 consultations. Most consultations lasted less than 15 minutes and the median consultation length was 8.5 minutes.

Figure 2 shows the computed relationship between various appointment intervals, surgery lengths and the expected mean waiting time for patients and doctor. A set of equations was found, empirically, to fit the results of the computation for mean patient waiting time, giving correlation coefficients greater than 0.99 for all curves. The equations are of the form:

$$t = e^{(k + c.i)} + 1$$

where t = mean time patient will wait in minutes, i = fixed time interval between appointments in minutes and k and c are constants which depend on the surgery length. For a one and a half hour surgery $k = 5.96$, $c = -0.48$; two hour surgery $k = 6.04$, $c = -0.45$; two and a half hour surgery $k = 6.31$, $c = -0.46$; three hour surgery $k = 6.54$, $c = -0.47$. The inclusion of $+1$ in these equations improves the correlation with the computed results in the five to 15 minute range, although it becomes inappropriate for longer appointment intervals.
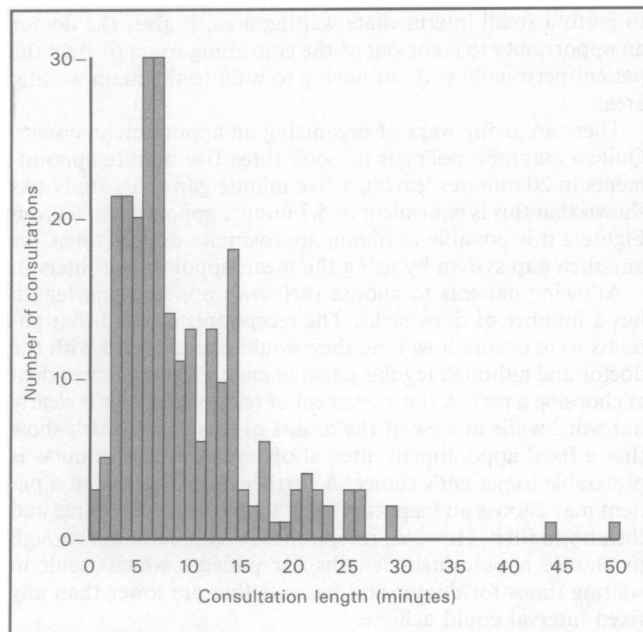
Figure 1. *Frequency distribution of the duration of the 248 consultations.*

Figure 2. *Predicted mean waiting time for a consultation for patients and doctor.*

The doctor's waiting time is virtually nil until the appointment interval is increased to a certain point, and then the waiting time increases linearly (Figure 2). This threshold point roughly corresponds to the median consultation length of 8.5 minutes. The patients' waiting time increases exponentially with decreasing appointment interval.

The four plots for different surgery duration are coincident for the doctor but not for the patients. To take an extreme example, if a change were made from a number of three hour surgeries to twice as many one and a half hour surgeries, the patients' mean waiting time per consultation would be shortened by up to 30 minutes with no change in the doctor's waiting time per consultation.

Figure 3 is a scatter plot of the appointment interval chosen by the patient and the actual consultation length. Most patients chose a five minute consultation and although the majority of these patients had a consultation lasting less than 10 minutes some consultations lasted 20–30 minutes.

Table 1 shows a comparison of the waiting times incurred with alternative appointment strategies. The choice of strategy can affect the patients' waiting time by up to 24 minutes, but the doctor's waiting time only varies by a matter of seconds. A system with three five minute appointments followed by a five minute gap gives similar results to a fixed interval of 6.7 minutes
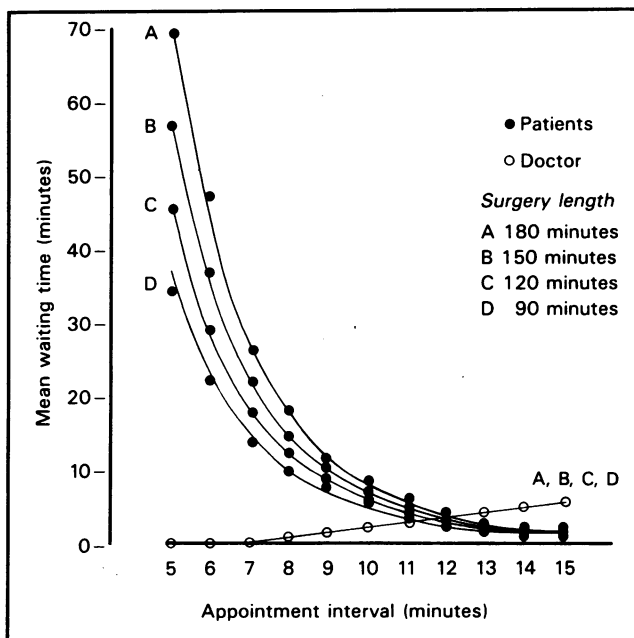
although the former saves the doctor a little time at the expense of patients waiting slightly longer. Either of the fixed intervals of seven or eight minutes is preferable to the patient's choice; a seven minute interval would save the doctor's time while an eight minute interval would save the patients' time.

It could be argued that the mean time patients wait may not be the best index of the dissatisfaction caused by waiting. For example, a patient who has a mean waiting time of 15 minutes may be satisfied with waiting 15 minutes on each visit, but not with a 45 minute wait on one visit out of three. To investigate this, the programme was adjusted to calculate the proportion of patients waiting longer than 10 or longer than 15 minutes. If these proportions are plotted against appointment intervals for each surgery length, the curves are the same exponential shape as Figure 2.

Discussion

The end of a consultation arises naturally and can be manipulated by the doctor or patient. In choosing a fixed appointment interval a balance must be struck: a very short appointment interval with long surgeries will result in virtually no waiting for the doctor but long waits for the patients, whereas long appointment intervals with short surgeries will result in

Table 1. Comparison of the mean waiting time for a consultation for alternative appointment strategies.

Appointment interval (minutes)	Mean doctor waiting time (minutes) for surgery durations of:				Mean patient waiting time (minutes) for surgery durations of:			
	90 minutes	120 minutes	150 minutes	180 minutes	90 minutes	120 minutes	150 minutes	180 minutes
5/5/5/gap[a]	0.20	0.15	0.14	0.12	18.1	23.4	28.7	33.6
6.7	0.29	0.21	0.16	0.15	16.2	22.1	26.5	32.8
7	0.37	0.28	0.25	0.22	13.6	19.0	22.8	26.6
8	0.74	0.64	0.57	0.48	9.6	12.4	14.3	17.1
Patient's choice	0.70	0.50	0.51	0.39	15.0	21.1	22.5	26.5

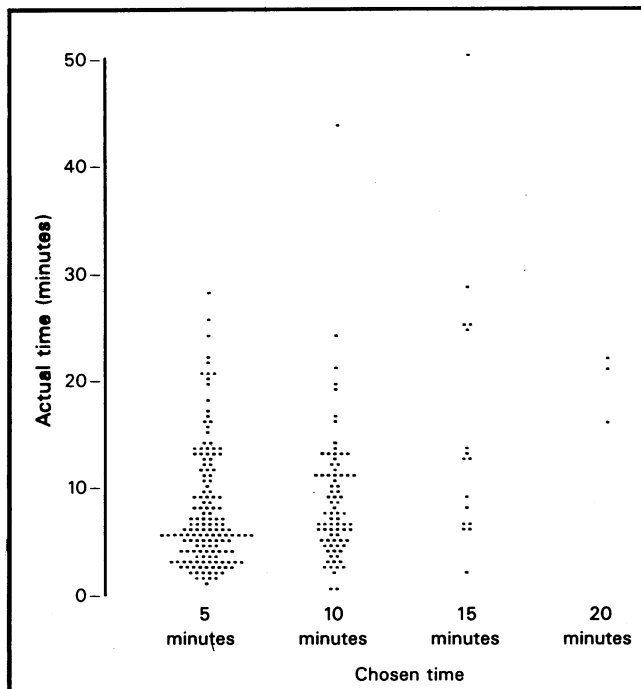[a] Three five minute appointments followed by a five minute gap.

Figure 3. *Scatter plot of consultation length for different appointment intervals chosen by the patient.*

relatively long waits between consultations for the doctor and relatively short waits for the patients. Between these two extremes there is a balance which allows the doctor to use his time efficiently without inconveniencing patients too much.

This study has shown that the doctor does not have to wait at all unless the appointment interval is less than the median appointment length. There is a straightforward explanation for this. If the majority of patients have consultations which last longer than the appointment interval, after the first few consultations there will be a queue of patients waiting which will grow as the surgery continues. Thus the doctor will have no waiting time between consultations. Decreasing the appointment interval will lengthen the queue but make no difference to the doctor's zero waiting time.

The patients' waiting time increases exponentially with decreasing appointment intervals. Decreasing the appointment interval much below the median consultation length can severely penalize patients with comparatively little benefit to the doctor. More frequent, shorter surgeries would result in less waiting for patients with no increase in the doctor's waiting time per consultation. However, this would not be practical if the doctor then had to make extra journeys to and from the clinic. If a long surgery could be conveniently split by an activity which did not have to be done at a fixed time, such as paperwork, then patients would benefit without detriment to the doctor.

Such a suggestion, which is based on the theoretical model described, should not mask other changes that can be made to improve the efficient use of surgery time. The distance patients have to walk between the waiting room and the consultation room may influence the time waited by both patient and doctor. A small intermediate waiting area may be helpful. Intercom call systems can also waste time if a patient has failed to attend and a personal call from a member of the reception staff is more efficient. If an intercom system has to be used, because of the design of the building for example, then it may be best routed via the reception area, so that the receptionist can ask the next available patient to enter. If the receptionist asks patients

to go to a small intermediate waiting area, it gives the doctor an opportunity to come out of the consulting room to greet the patient personally without having to walk to the main waiting area.

There are many ways of organizing an appointment system. Quite a common policy is to book three five minute appointments in 20 minutes leaving a five minute gap. This study has shown that this is equivalent to 6.7 minute appointments. From Figure 2 it is possible to obtain approximate waiting times for any such gap system by using the mean appointment interval.

Allowing patients to choose their own appointment length has a number of drawbacks. The receptionists found that patients were unsure how long they would like to spend with the doctor and although regular patients might become more adept at choosing a period, the investment of receptionist time is clearly not worthwhile in view of the results of this study which show that a fixed appointment interval of seven or eight minutes is preferable to patient's choice. A further drawback is that a patient may choose an inappropriately long consultation time and then try to fill it. However, receptionists might be skilful enough to choose appointment lengths for patients which result in waiting times for doctor and patients that are lower than any fixed interval could achieve.

The most radical alternative is to replace the appointment system with a queue. Assuming a normal list size, the doctor's waiting time is likely to be short; and although patients are likely to have correspondingly long waits they may not mind if the time can be spent usefully (for example, numbered tickets could allow them to leave the surgery). The main problem lies in the difficulty of planning ahead; there is no satisfactory means of informing patients of curtailed or cancelled surgeries, and the doctor may overrun his surgery if many patients arrive within the open period and are prepared to wait.

Waiting times will also be affected by emergency consultations, the flexibility of individual lists and any contingency plans to deal with emergency visits. Improving the conditions in which patients wait may make more difference to their satisfaction than simply reducing the waiting time.

In conclusion, there are many factors which affect the efficiency of an appointment system, but where a fixed interval is to be used, choosing an appropriate interval can considerably reduce the time patients spend waiting. Short, frequent surgeries are efficient, and may be used in conjunction with other time-saving devices.

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