The Chances of Excluding Paternity by the MNS Blood Group System[1]

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WIENER (1952) HAS CALCULATED the chances of exonerating a man of a false charge of paternity by means of the MNS blood groups. Wiener's formulas apply, however, to the case in which the bloods are tested with four sera, anti-M, anti-N, anti-S and anti-s. In actual practice the anti-s serum is usually not available, and the tests are done with three sera only. It would be desirable to be able to calculate the chances of exclusion in this case.

Formulas similar to those given by Wiener can be derived for the probabilities when the tests are done with three sera. If we represent, as does Wiener, the frequency of the genes MS, NS, Ms, and Ns by the letters a, b, c, and d respectively, and let $m = a + c$, $n = b + d$, $p = a + b$, and $q = c + d$, the formulas are:

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\begin{aligned}
P_{MS} &= n(1 - mn) \\
P_{M} &= n(1 - mn) + aq^2 + bcd \\
P_{MNS} &= 0 \\
P_{MN} &= pq^2 \\
P_{NS} &= m(1 - mn) \\
P_{N} &= m(1 - mn) + bq^2 + acd \\
P_{M,N,S} &= mn(1 - mn) + (ac^2 + bd^2)q^2 + 2cdpq^2 + cd(ad^2 + bc^2)
\end{aligned}
$$

where P_{MS} represents the probability of excluding the paternity of a man of group MS, P_{M} the probability for a man of group M, etc., and $P_{M,N,S}$ represents the chance of excluding paternity of an innocent man whose MNS blood group is not known.

For the English population used by Wiener to illustrate his calculations, which is sufficiently similar for our purposes to American populations, (data in Race and Sanger) we have $a = 0.2472$, $b = 0.0802$, $c = 0.2831$, $d = 0.3895$, and the probabilities are as follows:

Group of Man	Probabilities of Excluding Paternity Using Three Sera
MS	0.3527
M	0.4734
MNS	0
MN	0.1481
NS	0.3982
N	0.4618
Unknown	0.2390

For this population Wiener found that the chance of excluding paternity by the MNS system, when the group of the man is unknown, using four sera, is 0.315 (actually 0.3158). The chances using anti-M and anti-N alone are 0.1870, so it is apparent that the addition of the anti-S serum adds about 40% as much to the chances as does the addition of both anti-S and anti-s.

The value of the probability for the English population given by Race and Sanger (1954), and used by me in a previous paper (1954) is 0.2741, which is seen to be somewhat too high.

It was pointed out by Wiener (1952) that if the calculations for the MNS system, for the case in which four sera are used, are made using the simple but erroneous assumption that the MN and Ss systems are independent, the result is only a few per cent in error. In a previous paper (Boyd 1954) this method was described and used to estimate the probability for a population, and it was stated that the result was doubtless too high. It is remarkable how small the error actually is. For the English population referred to above the calculation making use of the erroneous assumption of independence gives 0.2415 instead of the exact 0.2390, an error of only 0.0025. For certain sets of gene frequencies the results of the two methods agree, and it can easily be shown that if the gene frequencies are connected by the relation $a/c = b/d$, the two methods will give identical results in the case of tests done with three sera. For certain other gene frequencies the approximate method may give a slightly smaller result than the exact method, as is found for the data from Lahore (in Boyd and Boyd 1954), where the approximate method gives 0.2428 and the exact method 0.2430. For the similar but hypothetical gene frequencies $a = 0.3, b = 0.1, c = 0.4, d = 0.2$, the difference in this direction is somewhat larger, viz. 0.0011. In most cases, however, the results of the approximate method are larger, but seldom more than 0.0200 greater than those of the exact method.

It is evident that the calculations done by the simpler method are close enough to the truth for all practical purposes, and under exceptional circumstances may be exact. It is not easy to see how this could have been ascertained, however, without first deriving the correct formulas.

SUMMARY

Formulas are given for calculating the chances of excluding paternity of a falsely accused man by means of the MNS blood group system, when the tests are done with anti-M, anti-N and anti-S sera. It is pointed out that the approximate calculations based on the erroneous assumption that the MN and Ss systems are independent produces but a very small error in the estimates in this case.

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