APPENDIX 1

Following Schreiber et al (2003), we created the reliability statistic as follows.

Let $\{x_i\}$ and $\{y_i\}$ be the spike times from two trials with n and m spikes respectively. We can turn the $\{x_i\}$ into a continuous function by allowing each spike time to be a Dirac delta function about x_i. Thus

(1) $$\{x_i\} \rightarrow \sum_{i=1}^{n} \delta(t - x_i)$$

By convolving with a normalized Gaussian of width σ, we turn the discrete spike times into a well behaved, smooth curve.

$$x(t) = \int_{-\infty}^{\infty} \sum_{i=1}^{n} \delta(\tau - x_i) \frac{1}{\sqrt{2\Pi}\,\sigma} e^{\frac{-(t-\tau)^2}{2\sigma^2}} d\tau$$

(2) $$x(t) = \frac{1}{\sqrt{2\Pi}\,\sigma} \sum_{i=1}^{n} e^{\frac{-(t-x_i)^2}{2\sigma^2}}$$

Similarly

(3) $$y(t) = \frac{1}{\sqrt{2\Pi}\,\sigma} \sum_{j=1}^{m} e^{\frac{-(t-y_j)^2}{2\sigma^2}}$$

Schreiber et al (2003) defines the reliability between two spike trains as the normalized dot product of these functions.

(4) $$R_{xy} = \frac{x \cdot y}{|x|\,|y|}$$

However, whereas Schreiber et al (2003) advocates treating x and y as vectors, and calculating the dot product point by point over an arbitrary Δt, we chose to keep them as continuous functions.

(5) $$x \cdot y = \int_{-\infty}^{\infty} x(t)\, y(t)\, dt$$

$$= \int_{-\infty}^{\infty} \left(\sum_{i=1}^{n} \frac{1}{\sqrt{2\Pi}\,\sigma} e^{\frac{-(t-x_i)^2}{2\sigma^2}} \right) \left(\sum_{j=1}^{m} \frac{1}{\sqrt{2\Pi}\,\sigma} e^{\frac{-(t-y_j)^2}{2\sigma^2}} \right) dt$$

(6) $$x \cdot y = \frac{1}{2\Pi\sigma^2} \sum_{i=1}^{n} \sum_{j=1}^{m} \int_{-\infty}^{\infty} e^{\frac{-(t-x_i)^2 - (t-y_j)^2}{2\sigma^2}} dt$$

Manipulating the numerator of the exponent in (6) by expanding the terms and completing the square yields:

$$x \cdot y = \frac{1}{2\Pi\sigma^2} \sum_{i=1}^{n} \sum_{j=1}^{m} \int_{-\infty}^{\infty} e^{\frac{-(x_i - y_j)^2}{4\sigma^2}} e^{\frac{-\left(t - \frac{x_i + y_j}{2}\right)^2}{\sigma^2}} dt$$

(7) $$x \cdot y = \frac{1}{2\sqrt{\Pi}\,\sigma} \sum_{i=1}^{n} \sum_{j=1}^{m} e^{\frac{-(x_i - y_j)^2}{4\sigma^2}}$$

We can now write down the norms by defining $|x| = \sqrt{x \cdot x}$ and $|y| = \sqrt{y \cdot y}$.

(8) $$|\,x\,| = \sqrt{x \cdot x} = \sqrt{\left(\sum_{i=1}^{n} \sum_{j=1}^{n} e^{\frac{-(x_i - x_j)^2}{4\sigma^2}}\right)}$$

(9) $$|\,y\,| = \sqrt{y \cdot y} = \sqrt{\left(\sum_{i=1}^{m} \sum_{j=1}^{m} e^{\frac{-(y_i - y_j)^2}{4\sigma^2}}\right)}$$

Finally, we can insert (7-9) into (4) and explicitly write the reliability in terms of spike times and σ.

(10) $$R_{xy} = \frac{\sum_{i=1}^{n} \sum_{j=1}^{m} e^{\frac{-(x_i - y_j)^2}{4\sigma^2}}}{\sqrt{\left(\sum_{i=1}^{n} \sum_{j=1}^{n} e^{\frac{-(x_i - x_j)^2}{4\sigma^2}}\right)\left(\sum_{i=1}^{m} \sum_{j=1}^{m} e^{\frac{-(y_i - y_j)^2}{4\sigma^2}}\right)}}$$

Although the expression appears bulky, it significantly speeds computation time when compared with treating x and y as vectors. Reliability across N trials can be found by averaging the pairwise reliabilities:

(11) $$R = \frac{2}{N\,(N-1)} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} R_{ij}$$