Appendix

Existence and Uniqueness of Nash Equilibrium

Here we show that the vaccination game formulated in the main text always has a unique Nash equilibrium $p_{\text{ind}} \in [0, 1)$. We assumed in the main text that all delayers are either vaccinated or infected in the case of an outbreak, i.e., $\phi_{\text{v}}(p) = 1 - \phi_{\text{s}}(p)$, so we can rearrange Eq. **3** to obtain

$$E_{\text{del}}(p) = -r\left[\phi_{\text{s}}(p)(d_{\text{s}} - d_{\text{v}}) + d_{\text{v}}\right] . \qquad \textbf{[11]}$$

Therefore, because $\phi_{\text{s}}(p)$ strictly decreases with p and $d_{\text{v}} < d_{\text{s}}$, $E_{\text{del}}(p)$ strictly increases with p.

First consider the case where $E_{\text{vac}} > E_{\text{del}}(0)$. We identify a candidate Nash equilibrium as the mixed strategy p_{ind} corresponding to coverage levels where the payoff from playing vaccinator equals the payoff for playing delayer. Hence p_{ind} is given by the solution of the equation

$$E_{\text{vac}} = E_{\text{del}}(p_{\text{ind}}) \qquad \textbf{[12]}$$

$$i.e., \quad -d_{\text{v}} = -r\left[\phi_{\text{s}}(p_{\text{ind}})(d_{\text{s}} - d_{\text{v}}) + d_{\text{v}}\right] . \qquad \textbf{[13]}$$

Because $E_{\text{del}}(p)$ is a strictly increasing function and $E_{\text{vac}} > E_{\text{del}}(0)$, there is a unique value of p_{ind} that satisfies Eq. **12** (or, equivalently, Eq. **13**).

To show this is a Nash equilibrium, suppose a fraction $\epsilon (0 < \epsilon < 1)$ of the population adopts the alternative mixed strategy of playing vaccinator with probability $p_{\text{alt}} \in [0, 1)\backslash\{p_{\text{ind}}\}$. For p_{ind} to be a Nash equilibrium, the payoff $E(p_{\text{alt}})$ to individuals playing p_{alt} must be no greater than the payoff $E(p_{\text{ind}})$ to individuals playing p_{ind}, i.e., $E(p_{\text{alt}}) \leq E(p_{\text{ind}})$.

If a fraction ϵ of individuals play p_{alt}, then the overall proportion p_{tot} of the population choosing preemptive vaccination is

$$p_{\text{tot}} = (1 - \epsilon)p_{\text{ind}} + \epsilon p_{\text{alt}} . \qquad \textbf{[14]}$$

Under such conditions the payoff for playing p_{ind} is

$$E(p_{\text{ind}}) = p_{\text{ind}} E_{\text{vac}} + (1 - p_{\text{ind}}) E_{\text{del}}(p_{\text{tot}}) \qquad \textbf{[15]}$$

while the payoff for playing p_{alt} is

$$E(p_{\text{alt}}) = p_{\text{alt}} E_{\text{vac}} + (1 - p_{\text{alt}}) E_{\text{del}}(p_{\text{tot}})\,. \qquad \textbf{[16]}$$

Hence we must show that $E(p_{\text{alt}}) \leq E(p_{\text{ind}})$ for all $p_{\text{alt}} \neq p_{\text{ind}}$, i.e.,

$$\begin{aligned} & E(p_{\text{alt}}) \leq E(p_{\text{ind}}) \\ \iff \quad & p_{\text{alt}} E_{\text{vac}}(p_{\text{tot}}) + (1 - p_{\text{alt}}) E_{\text{del}}(p_{\text{tot}}) \leq p_{\text{ind}} E_{\text{vac}}(p_{\text{tot}}) + (1 - p_{\text{ind}}) E_{\text{del}}(p_{\text{tot}}) \\ \iff \quad & p_{\text{alt}}(E_{\text{vac}} - E_{\text{del}}(p_{\text{tot}})) \leq p_{\text{ind}}(E_{\text{vac}} - E_{\text{del}}(p_{\text{tot}})) \qquad \textbf{[17]} \end{aligned}$$

The following steps show that $E(p_{\text{alt}}) < E(p_{\text{ind}})$ when $p_{\text{alt}} < p_{\text{ind}}$:

$$\begin{aligned} p_{\text{alt}} < p_{\text{ind}} &\iff p_{\text{tot}} < p_{\text{ind}} && \text{Eq. } \textbf{14} \\ &\iff \phi_{\text{s}}(p_{\text{tot}}) > \phi_{\text{s}}(p_{\text{ind}}) && \text{because } \phi_{\text{s}}(p) \text{ is decreasing} \\ &\iff E_{\text{del}}(p_{\text{tot}}) < E_{\text{del}}(p_{\text{ind}}) = E_{\text{vac}} && \text{Eqs. } \textbf{11} \text{ and } \textbf{12} \\ &\iff E_{\text{vac}} - E_{\text{del}}(p_{\text{tot}}) > 0 && \\ &\iff p_{\text{alt}}(E_{\text{vac}} - E_{\text{del}}(p_{\text{tot}})) < p_{\text{ind}}(E_{\text{vac}} - E_{\text{del}}(p_{\text{tot}})) && \text{because } p_{\text{alt}} < p_{\text{ind}} \\ &\iff E(p_{\text{alt}}) < E(p_{\text{ind}}) && \text{Eq. } \textbf{17} \end{aligned}$$

Hence the payoff to strategy p_{ind} is higher than the payoff to any alternative strategy $p_{\text{alt}} < p_{\text{ind}}$. The same approach shows that $E(p_{\text{alt}}) < E(p_{\text{ind}})$ when $p_{\text{alt}} > p_{\text{ind}}$. Therefore the mixed strategy p_{ind} identified by Eq. **13** is, in fact, a strict Nash equilibrium. It also follows immediately that p_{ind} is a unique Nash equilibrium. Any alternative strategy $p_{\text{alt}} \in [0, 1)\backslash\{p_{\text{ind}}\}$ cannot be a Nash equilibrium, because the arguments discussed above show that $E(p_{\text{ind}}) > E(p_{\text{alt}})$ for all values of ϵ, i.e., any proportion of individuals playing p_{ind} get a higher payoff when the rest of the population plays any other strategy.

In the second case, where $E_{\text{vac}} \leq E_{\text{del}}(0)$, there is no mixed strategy that satisfies Eq. **13** due to the fact that $E_{\text{del}}(p)$ is strictly increasing. However, it can be shown in a similar way that the pure delayer strategy $p_{\text{ind}} = 0$ is the unique, strict Nash equilibrium.

Impact of Inclusion of Residual Immunity

We repeated our analysis with the inclusion of residual immunity. Approximately 15% of the current U.S. population was immunized during the preeradication era (1), but exactly how much immunity remains from this era is unknown. To determine the potential impact of residual immunity, we assumed that a proportion p_{init} of the population had such residual immunity. Consequently $p_{\text{gr}} > p_{\text{init}}$ and $p_{\text{ind}} > p_{\text{init}}$ (we assumed that individuals with residual immunity do not presently seek and are not given the option of immunization). We repeated our Monte Carlo sensitivity analysis for the same intervals as in Table 1, additionally using the interval $[0, 0.15]$ for possible values of p_{init}.

The inclusion of residual immunity has little effect on the difference Δp between the group optimum and individual equilibrium for most parameter values (Fig. 3). The same is true for the relative increase in mortality rate, $\Delta C/C$. The average value of the difference between the group optimum and individual equilibrium Δp is 0.16, where the average values of p_{ind} and p_{gr} are 0.49 and 0.65, respectively. All these values are similar to the results for $p_{\text{init}} = 0$.

Impact of Inclusion of Vaccine-Induced Morbidity in Cost Function

It is reasonable to suppose that the risk of vaccine-induced morbidity such as vaccinia may also sway choice of strategy. Incorporation of vaccine-induced morbidity into the analysis necessitates the use of a weighting factor of the cost of vaccine-induced morbidity relative to death. This allows both outcomes to be incorporated into the cost analysis for both the payoffs (for determining the individual equilibrium) and the cost function $C(p)$ (for determining the group optimum).

We included the risk of vaccine-induced morbidity in our model by replacing the term d_{v} everywhere by the term $d_{\mathrm{v}} + wc$, where $c = 0.00005$ is the risk of experiencing vaccine-induced morbidity (2) and w is the weighting factor (which was varied over the range $[0, 0.2]$ in the sensitivity analysis).

Although the group optimum and individual equilibrium are both lower when vaccine-induced morbidity is included, the average difference is essentially unchanged (Fig. 4). The average value of Δp is 0.19, where the average values of p_{ind} and p_{gr} are 0.17 and 0.36, respectively. As in the case where vaccine-induced morbidity is ignored, $\Delta C/C$ is typically substantial (but note that this quantity is now interpreted in terms of a generalized "health cost," not strictly in terms of expected mortality).

Sensitivity Analysis for Postattack Vaccination Rate v and Response Time t_{res}

We also carried out a sensitivity analysis with respect to the two parameters under most direct control of public health authorities, namely, the postattack vaccination rate v and the response time t_{res}. We explored the dependence of Δp and $\Delta C/C$ on these two parameters, where all other parameters were fixed at the values of Table 1 (third column). For most regions of this two-dimensional parameter space (Fig. 5), the values of Δp and $\Delta C/C$ are still substantial and often very high. Only for a very high postattack vaccination rate v and short response time t_{res} do the individual equilibrium and group optimum start to converge.

1. Henderson, D.A. (1998) *Emerg. Infect. Dis.* **4**, 488–492.

2. Bozzette, S.A., Boer, R., Bhatnagar, V., Brower, J.L., Keeler, E.B., Morton, S.C. & Stoto, M.A. (2003) *N. Engl. J. Med.* **348**, 416–425.