## A plain instruction for ECF

To explain the ECF method intelligibly, it is applied to a plain network model where the EMCs are not uniquely determined. The flux of  $v_1$  is fixed to 100.



The flux distribution of  $\mathbf{v} = (v_1, v_2, ..., v_7)^t$  is decomposed into five elementary modes:

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{pmatrix} = \lambda_1 \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \lambda_2 \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \lambda_3 \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \lambda_4 \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} + \lambda_5 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

or

$$\begin{array}{c} 100\\ 60\\ 40\\ 30\\ 70\\ 30\\ 20 \end{array} \right| = \begin{pmatrix} 1 & 1 & 1 & 1 & 0\\ 1 & 0 & 0 & 1 & 0\\ 0 & 1 & 1 & 0 & 0\\ 0 & 0 & 1 & 0 & 1\\ 1 & 0 & 1 & 0 & 0\\ 0 & 1 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 & 1 \end{pmatrix} \quad \begin{pmatrix} \lambda_1\\ \lambda_2\\ \lambda_3\\ \lambda_4\\ \lambda_5 \end{pmatrix} = \mathbf{P} \cdot \mathbf{c} ,$$

where **P** is the elementary mode matrix and  $\mathbf{c} = (\lambda_1, \lambda_2, ..., \lambda_5)^t$  is the EMC vector of wild type. Ten EMC vectors for wild type  $(\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_{10})$  are calculated by maximizing or minimizing each  $\lambda_i$  (i = 1, 2, ..., 5) as follows:

	60	40	60	40	40	60	40	60	60	40)
$(\mathbf{r}_1, \mathbf{r}_2,, \mathbf{r}_{10}) =$	30	10	30	10	10	30	10	30	30	10
	10	30	10	30	30	10	30	10	10	30
	0	20	0	20	20	0	20	0	0	20
	20	0	20	0	0	20	0	20	20	0)

In a mutant, the activity of  $a_2$  is reduced to a half (0.5) and that of  $a_4$  is 2-fold enhanced. The relative enzyme activity of a mutant to wild type is given by:

$$(\boldsymbol{\theta}_p) = \begin{pmatrix} 1\\ 0.5\\ 1\\ 2\\ 1\\ 1\\ 1\\ 1 \end{pmatrix}$$

The matrix  $(\alpha_{pi})$  is provided by:

For the first and second columns for  $(\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_{10})$ , the intermediate EMC vectors for the mutant are given by:

$$\mathbf{inter}_{\mathbf{r}_{1}^{mutant}} = \begin{pmatrix} 30\\ 30\\ 20\\ 0\\ 0\\ 40 \end{pmatrix} = \begin{pmatrix} 60 \cdot 0.5^{1}\\ 30\\ 10 \cdot 2^{1}\\ 0\\ 20 \cdot 2^{1} \end{pmatrix} \text{ and } \mathbf{inter}_{\mathbf{r}_{2}^{mutant}} = \begin{pmatrix} 20\\ 10\\ 60\\ 10\\ 0 \end{pmatrix} = \begin{pmatrix} 40 \cdot 0.5^{1}\\ 10\\ 30 \cdot 2^{1}\\ 20 \cdot 0.5^{1}\\ 0 \end{pmatrix},$$

respectively, where  $\beta$  is set to 1. The others are calculated in the same manner. The resulting intermediate EMCs of {inter\_ $\mathbf{r}_{j}^{mutant} | j = 1, 2, ..., 10$ } are provided by:

(iı	nter_	$\mathbf{r}_{1}^{mutan}$	<sup>nt</sup> , int	er_r	mutant 2	, in	ter_	$\mathbf{r}_{10}^{mutant}$	<sup>t</sup> )	
	(30	20	30	20	20	30	20	30	30	20)
	30	10	30	10	10	30	10	30	30	10
=	20	60	20	60	60	20	60	20	20	60
	0	10	0	10	10	0	10	0	0	10
	40	0	40	0	0	40	0	40	40	0 )

For the vector of each column, the EMC vectors of the mutant  $\{\mathbf{r}_{j}^{mutant} | j = 1, 2, ..., 10\}$  are

calculated. Here, the first and second columns are used to calculate  $\mathbf{r}_1^{mutant}$  and  $\mathbf{r}_2^{mutant}$  as follows:

$$\mathbf{r}_{1}^{mutant} = \begin{pmatrix} 37.5\\ 37.5\\ 25\\ 0\\ 50 \end{pmatrix} = \frac{100}{(1,0,0,0,0,0,0,0)} \cdot \begin{pmatrix} 1 & 1 & 1 & 1 & 0\\ 1 & 0 & 0 & 1 & 0\\ 0 & 1 & 1 & 0 & 0\\ 0 & 0 & 1 & 0 & 1\\ 1 & 0 & 1 & 0 & 0\\ 0 & 1 & 0 & 1 & 0\\ 1 & 0 & 1 & 0 & 0\\ 0 & 1 & 0 & 1 & 0\\ 0 & 1 & 0 & 1 & 0\\ 0 & 1 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}} \cdot \begin{pmatrix} 30\\ 30\\ 20\\ 0\\ 40 \end{pmatrix}$$

and

$$\mathbf{r}_{2}^{mutant} = \begin{pmatrix} 20\\10\\60\\10\\0 \end{pmatrix} = \frac{100}{(1,0,0,0,0,0,0,0,0)} \cdot \begin{pmatrix} 1 & 1 & 1 & 1 & 0\\1 & 0 & 0 & 1 & 0\\0 & 1 & 1 & 0 & 0\\0 & 0 & 1 & 0 & 1\\1 & 0 & 1 & 0 & 0\\0 & 1 & 0 & 1 & 0\\0 & 1 & 0 & 1 & 0\\0 & 0 & 0 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 20\\10\\60\\10\\0\\0 \end{pmatrix},$$

respectively. The flux distributions of the mutant,  $\mathbf{v}_1^{mutant}$  and  $\mathbf{v}_2^{mutant}$ , are calculated as follows:

$$\mathbf{v}_{1}^{mutant} = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 37.5 \\ 37.5 \\ 25 \\ 0 \\ 50 \end{pmatrix} = \begin{pmatrix} 100 \\ 37.5 \\ 62.5 \\ 75 \\ 62.5 \\ 37.5 \\ 50 \end{pmatrix}.$$

and

$$\mathbf{v}_{2}^{mutant} = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 20 \\ 10 \\ 60 \\ 10 \\ 0 \end{pmatrix} = \begin{pmatrix} 100 \\ 30 \\ 70 \\ 60 \\ 80 \\ 20 \\ 10 \end{pmatrix},$$

respectively. The other flux distributions are calculated in the same manner. The mean flux for the mutant is provided by:

$$\mathbf{v}_{mean}^{mutant} = \frac{1}{10} \sum_{j=1}^{10} \mathbf{v}_{j}^{mutant}$$
$$= \begin{pmatrix} 100\\ 33.75\\ 66.25\\ 67.5\\ 71.25\\ 28.75\\ 30 \end{pmatrix}$$