

Supporting Text

A Model For Random Cell Turnover

Clonal Distribution as a Function of Turnover. A model was developed to calculate the effect of random turnover in a closed cell population on the distribution of single and multicopy clones. The model considers multiple cycles of two alternating steps: (i) killing of a single cell at random in the population, and (ii) division of one of the remaining cells, also at random. After the first step, a clone that contained n members will contain $n - 1$ members if one of its members is killed. In the second step, a clone that contained n members will contain $n + 1$ members if one of its members divides. The number of cells killed from a clone is 0 or 1. The expected number of such killed cells is therefore $1 \cdot p$ (a cell is killed from this clone), i.e. the probability that a cell is killed. Therefore, the number of clones containing n members after the first step will be decreased by the probability that a cell from one of these clones was killed, but increased by the probability that a cell from one of the clones containing $n + 1$ members was killed. Similarly, after a single cell division, the number of clones containing n members will be decreased by the probability that a cell from one of these clones divided, but increased by the probability that a cell from one of the clones containing $n - 1$ members divided. Let $f(n, k)$ be the number of clones containing n members after the killing step of turnover cycle k , and $F(n, k)$ be the number of clones containing n members after the division step of turnover cycle k . Then

After the first step $f(n, k) = F(n, k - 1) / c - n \cdot F(n, k - 1) / c + (n + 1) \cdot F(n + 1, k - 1) / c$

After the second step $F(n, k) = f(n, k) - n \cdot f(n, k) / (c - 1) + (n - 1) \cdot f(n - 1, k) / (c - 1)$

where c is the total number of cells in the population, and the probability of killing or division of a cell of a clone containing n members is equal to the total number of cells of

clones containing n members, divided by the total number of cells (c , before the first step, and $c - 1$ before the second step).

If we start with c cells, each of which is a member of a different clone, then with no cycles of killing and division

$$F(n = 1, 0) = c$$

$$F(n > 1, 0) = 0$$

And by definition:

$$f(0, k) \equiv 0$$

$F(n, k)$ may be calculated by recursive application of equations for the first and second steps for any value of k and $1 \geq n \geq c$.

Adjustment for Efficiency of Detection. The efficiency with which a cell is detected will influence the observed distribution of single and multicopy clones in a population. If a clone contains n members, each of which are detected with efficiency s , then the probability of detecting exactly a members of that clone is given by the binomial distribution, $b(a, n, s)$. Therefore the number of clones containing a members, $G(n = a, k, s)$ will be the sum of the contributions of the “ a ” term of the binomial distributions for all clones of size n , where $n \geq a$.

$$G(n = a, k, s) = \sum_{n=a}^c b(a, n, s) \cdot F(n, k)$$

Clonal Complexity. The observed clonal complexity of the population, C , is defined as the total number of clones detected divided by the total number of cells detected. The clonal complexity predicted by the model is:

$$C(k, s) = \sum_{n=1}^c [G(n, k, s) / c \cdot s]$$

Predicted values for the observed clonal complexities over a range of values of k and s are shown in Table 6.