APPENDIX A: The DCOH model

The principle of this method is to model two signals *X* and *Y* by a bivariate AR model, as described by equation (A1).

$$\begin{pmatrix} X(k) \\ Y(k) \end{pmatrix} = \sum_{i=1}^{L} \underbrace{\begin{pmatrix} a_{xx_i} & a_{xy_i} \\ a_{yx_i} & a_{yy_i} \end{pmatrix}}_{\mathbf{A}_i} \begin{pmatrix} X(k-i) \\ Y(k-i) \end{pmatrix} + \underbrace{\begin{pmatrix} b_{xx} & b_{xs} & 0 \\ 0 & b_{ys} & b_{yy} \end{pmatrix}}_{\mathbf{B}} \begin{pmatrix} W_x(k) \\ W_s(k) \\ W_y(k) \end{pmatrix} \quad \text{(A1)}$$

k is the current sample, $\mathbf{A}_i$ is the 2-by-2 AR coefficients matrix at lag *i* which off-diagonal elements represent the cross-terms between the two channels and *L* is the model order. W_x, W_s and W_y are three white noise sources, mutually independent with zero mean and unit variance, where W_x and W_y are independent components and W_s is a common source coming from the rest of the brain. **B** is a 2-by-3 matrix of weighting factors.

By Fourier transform of (A1), we obtain:

$$\begin{pmatrix} X(f) \\ Y(f) \end{pmatrix} = \underbrace{\begin{pmatrix} 1-\sum_{i=1}^{L} a_{xx_i} e^{-j2\pi fi\Delta t} & -\sum_{i=1}^{L} a_{xy_i} e^{-j2\pi fi\Delta t} \\ -\sum_{i=1}^{L} a_{yx_i} e^{-j2\pi fi\Delta t} & 1-\sum_{i=1}^{L} a_{yy_i} e^{-j2\pi fi\Delta t} \end{pmatrix}^{-1}}_{=(\mathbf{I}-\mathbf{A}(f))^{-1}} \begin{pmatrix} b_{xx} & b_{xs} & 0 \\ 0 & b_{ys} & b_{yy} \end{pmatrix} \begin{pmatrix} W_x(f) \\ W_s(f) \\ W_y(f) \end{pmatrix},$$

$$= \begin{pmatrix} A_{xx}(f) & A_{xy}(f) \\ A_{yx}(f) & A_{yy}(f) \end{pmatrix} \begin{pmatrix} b_{xx} & b_{xs} & 0 \\ 0 & b_{ys} & b_{yy} \end{pmatrix} \begin{pmatrix} W_x(f) \\ W_s(f) \\ W_y(f) \end{pmatrix}, \quad \text{(A2)}$$

$$= \begin{pmatrix} H_{xx}(f) & H_{xs}(f) & H_{xy}(f) \\ H_{yx}(f) & H_{ys}(f) & H_{yy}(f) \end{pmatrix} \begin{pmatrix} W_x(f) \\ W_s(f) \\ W_y(f) \end{pmatrix},$$

where **I** is the 2-by-2 identity matrix, $\mathbf{A}(f) = \sum_{i=1}^{L} \mathbf{A}_i e^{-j2\pi fi\Delta t}$ and where $\mathrm{E}[W_k(f)]^2 = 1, (k = x, s, y)$ and $\mathrm{E}[W_k(f) W_m(f)] = 0, (k \neq m; k, m = x, s, y)$ are assumed. Coefficients $A_{km}(f)$ $(k, m = x, y)$ are transfer functions of each route and $H_{km}(f)$ $(k, m = x, s, y)$ are the system transfer functions. Equation (A2) leads to the linear generation model of the physiological signal recorded from the auditory cortex as shown in Fig. A1.

Then three directed coherences can be defined from W_x, W_s and W_y to signal X at each frequency f, by equations (A3), (A4) and (A5) respectively.

$$DCOH_{xx}(f) = \frac{H_{xx}(f)}{\sqrt{\sum_{m=x,s,y} \left|H_{xm}(f)\right|^2}} \qquad \text{(A3)}$$

$$DCOH_{xs}(f) = \frac{H_{xs}(f)}{\sqrt{\sum_{m=x,s,y} \left|H_{xm}(f)\right|^2}} \qquad \text{(A4)}$$

$$DCOH_{xy}(f) = \frac{H_{xy}(f)}{\sqrt{\sum_{m=x,s,y} \left|H_{xm}(f)\right|^2}} \qquad \text{(A5)}$$

The directed coherences for signal Y can be written in similar but symmetrical way.

DCOH is based on the assumption that a time delay exists in the cross route of the model shown in Fig. A1 and like the ordinary coherence its values are in the range [0-1].

To obtain the DCOH, AR model order, coefficients and weighting factors have to be determined. AR coefficients are determined by the Matlab ARX routine, performing a least squares estimation by QR-factorization. The weighting factors are determined, knowing that the residual power matrix $\mathbf{P_0}$ is as described by equation (A6) (Wang and Yunokuchi, 2002).

$$\mathbf{P_0} = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{yx} & \varepsilon_{yy} \end{pmatrix} = \begin{pmatrix} b_{xx}^2 + b_{xs}^2 & b_{xs} \cdot b_{ys} \\ b_{xs} \cdot b_{ys} & b_{yy}^2 + b_{ys}^2 \end{pmatrix} \qquad \text{(A6)}$$

As $\varepsilon_{xy} = \varepsilon_{yx}$, (A6) leads to an underdetermined set of equations. So, a further physiological assumption is made that the neuronal populations underneath the leads where X and Y have been recorded receive the same quantity of information from the common source W_s, *i.e.*

$$\frac{|b_{xx}|}{|b_{xs}|} = \frac{|b_{yy}|}{|b_{ys}|}. \qquad \text{(A7)}$$

The four variables can be computed thanks to the equations (A8) to (A11).

$$b_{xx} = \sqrt{\varepsilon_{xx} \cdot \left(1 - \left|\frac{\varepsilon_{xy}}{\sqrt{\varepsilon_{xx} \cdot \varepsilon_{yy}}}\right|\right)} \qquad \text{(A8)}$$

$$b_{yy} = \sqrt{\varepsilon_{yy} \cdot \left(1 - \left|\frac{\varepsilon_{xy}}{\sqrt{\varepsilon_{xx} \cdot \varepsilon_{yy}}}\right|\right)} \qquad \text{(A9)}$$

$$|b_{xs}| = \sqrt{\varepsilon_{xx} \cdot \left|\frac{\varepsilon_{xy}}{\sqrt{\varepsilon_{xx} \cdot \varepsilon_{yy}}}\right|} \qquad \text{(A10)}$$

$$|b_{ys}| = \sqrt{\varepsilon_{yy} \cdot \left|\frac{\varepsilon_{xy}}{\sqrt{\varepsilon_{xx} \cdot \varepsilon_{yy}}}\right|} \qquad \text{(A11)}$$

Here the signs of b_{xs} and b_{ys} are determined knowing $\varepsilon_{xy} = b_{xs}.b_{ys}$ sign.

These two steps are iterated for each model order value in the range [L_{min}, L_{max}]. Then, the model order value is chosen so that the AR power spectra of *X* and *Y* fit at best the peak at the modulation frequency in *X* and *Y* power spectral densities estimated by FFT. The AR power spectra of *X* and *Y* can be computed thanks to the cross route transfer functions $A_{km}(f)$ $(k, m = x, y)$ as follows:

$$\mathbf{S}(f) = \begin{pmatrix} S_{xx}(f) & S_{xy}(f) \\ S_{yx}(f) & S_{yy}(f) \end{pmatrix} = \begin{pmatrix} A_{xx}(f) & A_{xy}(f) \\ A_{yx}(f) & A_{yy}(f) \end{pmatrix} \cdot \mathbf{P_0} \cdot \begin{pmatrix} A_{xx}(f) & A_{xy}(f) \\ A_{yx}(f) & A_{yy}(f) \end{pmatrix}^{\mathrm{T}} \qquad \text{(A12)}$$

where **S**(*f*) is the cross-power spectral density matrix at frequency *f*, $S_{xx}(f)$ (resp. $S_{yy}(f)$) is the auto-power spectral density of *X* (resp. *Y*) at frequency *f* and $S_{xy}(f)$ and $S_{yx}(f)$ are cross-power spectral densities of *X* and *Y* at frequency *f*. The superscript T stands for transpose. An example is illustrated on Fig. A2 on the two signals H3 and H4 taken in their oscillatory part. Fig. A2 shows that a 70-order bivariate AR model fits the frequency peaks at MF = 16 Hz of the two 1024-point FFT power spectral densities (using zero-padding) of the oscillatory parts of H3 and H4.

REFERENCES

Wang G, Yunokuchi K (2002) Causality of frontal and occipital alpha activity revealed by directed coherence. IEICE T Inf Syst E85-D: 1334-1340.

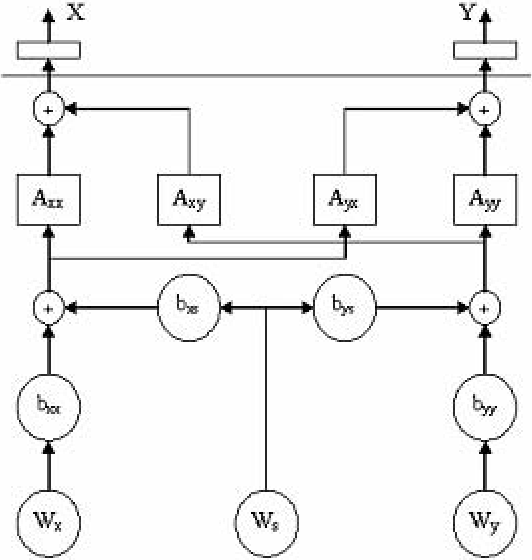

Figure A1. Linear generation model of signals X and Y

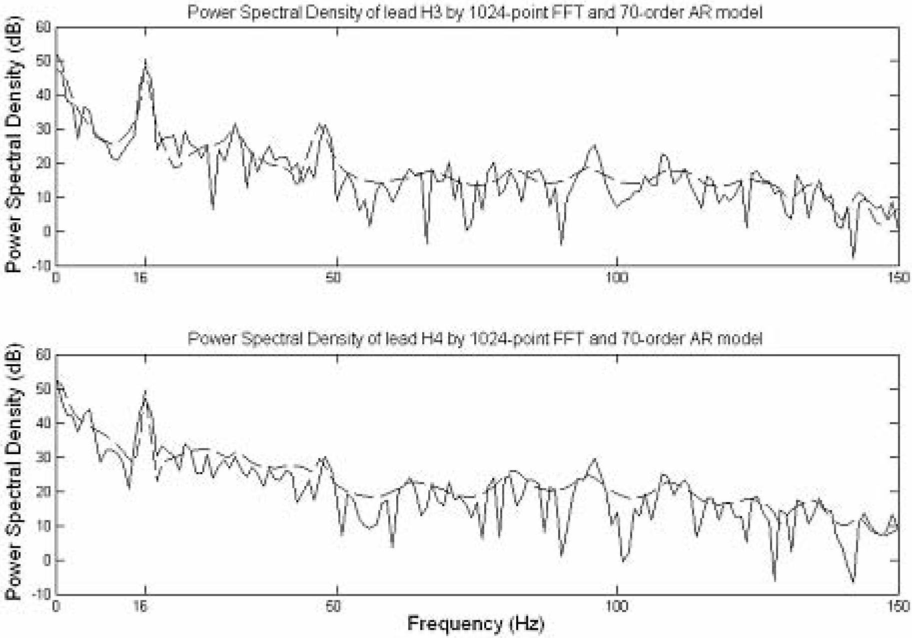

Figure A2. On the top: Power Spectral Densities of lead H3 by 1024-point FFT (solid) and by 70-order bivariate AR model between leads H3 and H4 (dashed) on the range 0-150 Hz. On the bottom: Power Spectral Densities of lead H4 by 1024-point FFT (solid) and by 70-order bivariate AR model between leads H3 and H4 (dashed) on the range 0-150 Hz

	T11	T12	T13	T14	T15	H10	H11	H12	H13	H14	H15
T11	-	0.62	0.61	0.19	0.77	0.80	0.75	0.23	0.55	0.38	0.77
T12	0.50	-	0.21	0.45	0.15	0.75	0.16	0.33	0.33	0.32	0.15
T13	0.62	0.43	-	0.53	0.42	0.59	0.34	0.15	0.20	0.25	0.39
T14	0.22	0.68	0.78	-	0.56	0.58	0.73	0.66	0.18	0.48	0.72
T15	0.03	0.81	0.61	0.10	-	0.71	0.70	0.47	0.52	0.44	0.23
H10	0.21	0.50	0.63	0.41	0.48	-	0.61	0.57	0.41	0.44	0.44
H11	0.32	0.85	0.62	0.17	0.59	0.51	-	0.46	0.24	0.40	0.53
H12	0.44	0.80	0.52	0.15	0.56	0.60	0.82	-	0.15	0.42	0.79
H13	0.44	0.77	0.84	0.08	0.72	0.76	0.68	0.56	-	0.60	0.62
H14	0.41	0.78	0.75	0.02	0.84	0.85	0.81	0.28	0.77	-	0.71
H15	0.04	0.84	0.70	0.08	0.76	0.87	0.80	0.42	0.75	0.67	-

Table A1. DCOH modules at the frequency MF = 8 Hz between all the leads located in the PAC of Case 20. The corresponding AEPs are taken in the time window between 200 to 1000 ms post-stimulus onset. This table follows the reading convention of source (column) to target (row). DCOH modules superior to 0.6 are highlighted in grey.

PAC P	-	0.34	0.48	0.47	0.32	0.36	0.29
PT P	0.71	-	0.62	0.66	0.55	0.50	0.45
Insula H	0.64	0.37	-	0.52	0.55	0.47	0.37
PAC H	0.62	0.30	0.64	-	0.42	0.43	0.36
PT H	0.80	0.36	0.62	0.65	-	0.44	0.40
SAC T	0.72	0.42	0.58	0.68	0.60	-	0.44
A22 T	0.76	0.39	0.69	0.68	0.52	0.56	-
	PAC P	**PT P**	**Insula H**	**PAC H**	**PT H**	**SAC T**	**A22 T**

Table A2. Means of DCOH modules over all the modulation frequencies (MF = 4, 8, 16 and 32 Hz) between 7 zones of the auditory cortex of Case 3. The corresponding AEPs are taken in the time window between 200 to 1000 ms post-stimulus onset. This table follows the reading convention of source (column) to target (row). DCOH modules superior to 0.6 are highlighted in grey.

PAC P	-	0.42	0.49	0.39	0.77	0.33	0.37	0.20
PT P	0.73	-	0.72	0.68	0.89	0.52	0.27	0.22
Insula H	0.60	0.38	-	0.32	0.61	0.56	0.46	0.35
PAC H	0.68	0.33	0.72	-	0.74	0.33	0.54	0.28
HS H	0.38	0.38	0.73	0.63	-	0.37	0.43	0.29
PT H	0.77	0.33	0.60	0.61	0.79	-	0.39	0.11
SAC T	0.69	0.62	0.49	0.58	0.81	0.59	-	0.40
A22 T	0.69	0.69	0.59	0.66	0.88	0.51	0.42	-
	PAC P	**PT P**	**Insula H**	**PAC H**	**HS H**	**PT H**	**SAC T**	**A22 T**

Table A3. DCOH modules between the 8 zones of Case 3 at MF = 8 Hz. The corresponding AEPs are taken in the time window between 200 to 1000 ms post-stimulus onset. This table follows the reading convention of source (column) to target (row). DCOH modules superior to 0.6 are highlighted in grey.

PAC P	-	0.22	0.65	0.44	0.93	0.48	0.35	0.34
PT P	0.77	-	0.69	0.58	0.79	0.66	0.47	0.47
Insula H	0.57	0.20	-	0.51	0.85	0.44	0.53	0.36
PAC H	0.66	0.32	0.68	-	0.85	0.56	0.45	0.42
HS H	0.21	0.18	0.46	0.32	-	0.39	0.23	0.25
PT H	0.80	0.20	0.70	0.49	0.84	-	0.45	0.51
SAC T	0.74	0.20	0.51	0.64	0.89	0.61	-	0.52
A22 T	0.78	0.20	0.74	0.57	0.90	0.41	0.53	-
	PAC P	**PT P**	**Insula H**	**PAC H**	**HS H**	**PT H**	**SAC T**	**A22 T**

Table A4. DCOH modules between the 8 zones of Case 3 at MF = 16 Hz. The corresponding AEPs are taken in the time window between 200 to 1000 ms post-stimulus onset. This table follows the reading convention of source (column) to target (row). DCOH modules superior to 0.6 are highlighted in grey.

PAC P	-	0.21	0.39	0.41	0.41	0.22	0.30	0.24
PT P	0.78	-	0.75	0.87	0.48	0.66	0.55	0.46
Insula H	0.82	0.29	-	0.58	0.05	0.50	0.37	0.23
PAC H	0.69	0.21	0.72	-	0.62	0.36	0.22	0.27
HS H	0.76	0.32	0.86	0.58	-	0.66	0.69	0.45
PT H	0.84	0.38	0.74	0.85	0.46	-	0.45	0.37
SAC T	0.71	0.41	0.82	0.85	0.28	0.73	-	0.46
A22 T	0.79	0.40	0.88	0.82	0.31	0.71	0.63	-
	PAC P	**PT P**	**Insula H**	**PAC H**	**HS H**	**PT H**	**SAC T**	**A22 T**

Table A5. DCOH modules between the 8 zones of Case 3 at MF = 32 Hz. The corresponding AEPs are taken in the time window between 200 to 1000 ms post-stimulus onset. This table follows the reading convention of source (column) to target (row). DCOH modules superior to 0.6 are highlighted in grey.