

# The impact of prophylaxis of healthcare workers on influenza pandemic burden

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## 1 The mathematical model

We assumed that the transmission of infection can occur through contacts between susceptible and infected individuals, in which mass action incidence is used for sake of simplicity. Previous work (Arino *et al.*, 2006) shows that this simplification still provides a good approximation to a more realistic situation in which other incidence functions may be used. Since the period of a pandemic is expected to be short, we ignore the effect of birth and natural death rates on the transmission dynamics of infection. It is assumed that infected individuals under treatment can contribute to disease transmission only through contacts with their healthcare providers. Antiviral treatment is assumed to reduce the infectious period and infectiousness of a clinical case from when treatment is initiated (Ferguson *et al.*, 2005, 2003). We considered only susceptible healthcare workers for a prophylactic treatment that reduces susceptibility, infectiousness if infection occurs, and the probability of developing clinical symptoms. These assumptions, with the model structure described in the main paper (Figure 1), lead to the following sets of deterministic equations:

(a) *General population:*

$$\begin{aligned} S' &= -\beta QS, \\ E' &= \beta QS - \mu_E E, \\ A' &= (1-p)\mu_E E - \mu_A A, \\ P' &= p\mu_E E - \mu_P P, \\ I' &= \mu_P P - \mu_I I, \\ I'_U &= (1-\rho)\mu_I I - (\mu_U + d_U)I_U, \\ I'_T &= \rho\mu_I I - (\mu_T + d_T)I_T, \\ R' &= \mu_A A + \mu_U I_U + \mu_T I_T, \end{aligned} \tag{1}$$

(b) *HCWs without prophylaxis:*

$$\begin{aligned}
S'_H &= -[\beta Q + \beta_H Q_H]S_H, \\
E'_H &= [\beta Q + \beta_H Q_H]S_H - \mu_E E_H, \\
A'_H &= (1-p)\mu_E E_H - \mu_A A_H, \\
P'_H &= p\mu_E E_H - \mu_P P_H, \\
I'_H &= \mu_P P_H - \mu_I I_H, \\
I'_{HU} &= (1-\rho_H)\mu_I I_H - (\mu_U + d_U)I_{HU}, \\
I'_{HT} &= \rho_H \mu_I I_H - (\mu_T + d_T)I_{HT}, \\
R'_H &= \mu_A A_H + \mu_U I_{HU} + \mu_T I_{HT},
\end{aligned} \tag{2}$$

(c) *HCWs with prophylaxis:*

$$\begin{aligned}
S'_{HP} &= -(1-\alpha_S)[\beta Q + \beta_H Q_H]S_{HP}, \\
E'_{HP} &= (1-\alpha_S)[\beta Q + \beta_H Q_H]S_{HP} - \mu_E E_{HP}, \\
A'_{HP} &= (1-p_P)\mu_E E_{HP} - \mu_A A_{HP}, \\
P'_{HP} &= p_P \mu_E E_{HP} - \mu_P P_{HP}, \\
I'_{HP} &= \mu_P P_{HP} - \mu_I I_{HP}, \\
I'_{HPU} &= (1-\rho_H)\mu_I I_{HP} - (\mu_U + d_U)I_{HPU}, \\
I'_{HPT} &= \rho_H \mu_I I_{HP} - (\mu_T + d_T)I_{HPT}, \\
R'_{HP} &= \mu_A A_{HP} + \mu_U I_{HPU} + \mu_T I_{HPT},
\end{aligned} \tag{3}$$

where “ ’ ” denotes the derivative with respect to the time,

$$\begin{aligned}
Q &= \delta_A A + \delta_P P + I + \delta_U I_U + \delta_A A_H + \delta_P P_H + I_H + \delta_U I_{HU} \\
&\quad + \alpha_P (\delta_A A_{HP} + \delta_P P_{HP} + I_{HP} + \delta_U I_{HPU}), \\
Q_H &= \delta_T (I_T + I_{HT} + \alpha_P I_{HPT}) + \delta_A A_H + \delta_P P_H \\
&\quad + \alpha_P (\delta_A A_{HP} + \delta_P P_{HP}),
\end{aligned}$$

and the parameters are defined as

- $\beta$ : baseline transmission rate in the general population;
- $\beta_H$ : baseline transmission rate within the healthcare setting;
- $\mu_E$ : progression rate of exposed to pre-symptomatic infection;
- $\mu_P$ : progression rate of pre-symptomatic to symptomatic infection (stage 1);
- $\mu_I$ : progression rate of stage 1 to stage 2 symptomatic infection;
- $\mu_A$ : recovery rate of asymptomatic infection;
- $\mu_U$ : recovery rate of untreated symptomatic infection;
- $\mu_T$ : recovery rate of treated symptomatic infection;
- $d_U$ : death rate of untreated symptomatic infection;
- $d_T$ : death rate of treated symptomatic infection;
- $\delta_P$ : reduction in transmission of pre-symptomatic infection;
- $\delta_A$ : reduction in transmission of asymptomatic infection;

$\delta_U$ : reduction in transmission of untreated symptomatic infection;  
 $\delta_T$ : reduction in transmission of treated symptomatic infection;  
 $\alpha_S$ : reduction in susceptibility due to prophylaxis;  
 $\alpha_P$ : reduction in infectiousness due to prophylaxis;  
 $p$ : probability of developing symptoms without prophylaxis;  
 $p_P$ : probability of developing symptoms with prophylaxis;  
 $\rho$ : treatment level of the general population;  
 $\rho_H$ : treatment level of healthcare workers.

An average incubation period ( $1/\mu_E$ ) of 1.25 days (within the estimated range  $1.48 \pm 0.48$  days) is associated with exposed cases (Ferguson *et al.*, 2003), after which infected individuals either develop clinical symptoms or undergo an asymptomatic phase for the entire course of infection. Since influenza can be transmitted before symptoms appear, we considered a relatively short pre-symptomatic infection ( $1/\mu_P$ ) as an extension to previous work (Arino *et al.*, 2006). This is consistent with the assumption made in a recent modeling study for a 0.25-day delay since the onset of clinical symptoms until diagnosis of the infected case (Ferguson *et al.*, 2003). Based on the infectiousness profile (Ferguson *et al.*, 2003), we divided the symptomatic infection into two stages including 1-day period ( $1/\mu_I$ ) for high viral shedding and a period ( $1/\mu_U$ ) of 2.85 days for low viral shedding. This corresponds to a mean infectiousness period of 4.1 days (including pre-symptomatic stage) for the clinical infection (Ferguson *et al.*, 2005, 2003; Longini *et al.*, 2004, 2005). We used the same infectious period ( $1/\mu_A$ ) for the asymptomatic infection. It is assumed that stage 1 symptomatic infection provides the window of opportunity for an effective treatment with maximum response of infected individuals to antiviral drugs. Antiviral treatment is assumed to reduce the infectious period ( $1/\mu_T$ ) by 1.5 days, and infectiousness by 60% from when treatment is initiated (Ferguson *et al.*, 2005, 2003). Prophylactic treatment of healthcare workers is assumed to reduce susceptibility to infection by 30%, infectiousness if infection occurs by 60%, and the probability of developing clinical symptoms by 65% (Ferguson *et al.*, 2005, 2003).

Since the transmission rate within healthcare settings is influenced by the presence of asymptotically (or even clinically) infected HCWs, susceptible HCWs may be exposed to even higher level of exposure (Low & Wilder-Smith, 2005; Nguyen-Van-Tam *et al.*, 1999). This explains the appearance of the terms  $A_H$ ,  $P_H$ ,  $A_{HP}$  and  $P_{HP}$  in the force of infection  $Q_H$ . Parameters for the level of treatment, as explained in the main paper, depend on the availability of HCWs at time  $t$ . Let  $\nu$  and  $\nu_H$  represent the fractions of clinical cases in the GP and HCWs, respectively, which are effectively treated within a naïve healthcare system. We define  $\rho = \theta\nu$  and  $\rho_H = \theta\nu_H$  where  $\theta$  is given by the expression (4) in the main paper. The parameter  $\theta(t)$  represents the proportion of HCWs who provide care at time  $t$  during a pandemic (including susceptible, exposed, asymptomatic, pre-symptomatic, and recovered individuals), except those who are infected and clinically recognizable.

Thus, at the onset of epidemic,  $\theta = 1$  and the treatment parameters reduce to  $\rho = \nu$  and  $\rho_H = \nu_H$ . For the purpose of simulations, we assumed that 90% of clinical cases in the GP (60% of infections) and 100% of those among HCWs are detected and effectively treated (Ferguson *et al.*, 2003), corresponding to  $\nu = 0.9$  and  $\nu_H = 1$ , when infection is introduced into the population.

## 2 Control reproduction number

Here, we will treat parameters  $\rho$  and  $\rho_H$  as constants by considering  $\theta = 1$ , which is sufficient for the calculation of the control reproduction number. This calculation involves the linearization of the model at the disease free equilibrium, which coincides with the original model when  $\rho = \nu$  and  $\rho_H = \nu_H$  are constants. Also, our need for the final size equation is to establish a relationship between the basic (or control) reproduction number and the final size of the susceptible population when the epidemic dies out. For this purpose, it again suffices to assume constant treatment levels.

We may follow a previous approach (van den Driessche & Watmough, 2002) to calculate the control reproduction number ( $\mathcal{R}_c$ ). We first simplify the model by introducing new change of variables as follows:

$$x = \begin{bmatrix} S \\ S_H \\ S_{HP} \end{bmatrix}, \quad y = \begin{bmatrix} E \\ E_H \\ E_{HP} \end{bmatrix}, \quad z_P = \begin{bmatrix} P \\ P_H \\ P_{HP} \end{bmatrix}, \quad z_A = \begin{bmatrix} A \\ A_H \\ A_{HP} \end{bmatrix},$$

$$i = \begin{bmatrix} I \\ I_H \\ I_{HP} \end{bmatrix}, \quad i_U = \begin{bmatrix} I_U \\ I_{HU} \\ I_{HPU} \end{bmatrix}, \quad i_T = \begin{bmatrix} I_T \\ I_{HT} \\ I_{HPT} \end{bmatrix},$$

and

$$f(x, y, z_P, z_A, i, i_U, i_T) = \begin{bmatrix} \beta QS \\ (\beta Q + \beta_H Q_H) S_H \\ \alpha_S (\beta Q + \beta_H Q_H) S_{HP} \end{bmatrix}.$$

Then the model (1)-(3) can be written as the following system of equations

$$\begin{aligned} x' &= -f(x, y, z_P, z_A, i, i_U, i_T), \\ y' &= f(x, y, z_P, z_A, i, i_U, i_T) - V_E y, \\ z_P' &= V_{EP} y - V_P z_P, \\ z_A' &= V_{EA} y - V_A z_A, \\ i' &= V_P z_P - V_I i, \\ i_U' &= V_U i - V_{IU} i_U, \\ i_T' &= V_T i - V_{IT} i_T, \end{aligned} \tag{4}$$

where

$$\begin{aligned}
V_E &= \mu_E \mathbb{I}; & V_P &= \mu_P \mathbb{I}; & V_A &= \mu_A \mathbb{I}; & V_I &= \mu_I \mathbb{I}; & V_{IU} &= (\mu_U + d_U) \mathbb{I}; \\
V_{IT} &= (\mu_T + d_T) \mathbb{I}; & V_{EP} &= V_E - V_{EA}; & V_U &= V_I - V_T; \\
V_{EA} &= \mu_E \begin{bmatrix} 1-p & 0 & 0 \\ 0 & 1-p & 0 \\ 0 & 0 & 1-p_P \end{bmatrix}; & V_T &= \mu_I \begin{bmatrix} \rho & 0 & 0 \\ 0 & \rho_H & 0 \\ 0 & 0 & \rho_H \end{bmatrix},
\end{aligned}$$

and  $\mathbb{I}$  is the identity matrix of size  $3 \times 3$ . Let  $X = [S, S_H, \alpha_S S_{HP}]^t$  and  $Y = [0, S_H, \alpha_S S_{HP}]^t$ . We now define square matrices  $F$  and  $V$  as

$$F = \begin{bmatrix} F_E & F_P & F_A & F_I & F_{IU} & F_{IT} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad V = \begin{bmatrix} V_E & 0 & 0 & 0 & 0 & 0 \\ -V_{EP} & V_P & 0 & 0 & 0 & 0 \\ -V_{EA} & 0 & V_A & 0 & 0 & 0 \\ 0 & -V_P & 0 & V_I & 0 & 0 \\ 0 & 0 & 0 & -V_U & V_{IU} & 0 \\ 0 & 0 & 0 & -V_T & 0 & V_{IT} \end{bmatrix},$$

where  $F_E = O_{3 \times 3}$  zero matrix, and

$$\begin{aligned}
F_P &= \beta X \begin{bmatrix} 1 & 1 & \alpha_P \end{bmatrix} \delta_P + \beta_H Y \begin{bmatrix} 0 & 1 & \alpha_P \end{bmatrix} \delta_P, \\
F_A &= \beta X \begin{bmatrix} 1 & 1 & \alpha_P \end{bmatrix} \delta_A + \beta_H Y \begin{bmatrix} 0 & 1 & \alpha_P \end{bmatrix} \delta_A, \\
F_I &= \beta X \begin{bmatrix} 1 & 1 & \alpha_P \end{bmatrix}, \\
F_{IU} &= \beta X \begin{bmatrix} 1 & 1 & \alpha_P \end{bmatrix} \delta_U, \\
F_{IT} &= \beta_H Y \begin{bmatrix} 1 & 1 & \alpha_P \end{bmatrix} \delta_T.
\end{aligned}$$

Then, the control reproduction number is the spectral radius of the matrix  $FV^{-1}$  (van den Driessche & Watmough, 2002). Without prophylactic treatment, and considering HCWs in the general population, we have

$$F = \beta S_0 \begin{bmatrix} 0 & \delta_P & \delta_A & 1 & \delta_U & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$V = \begin{bmatrix} \mu_E & 0 & 0 & 0 & 0 & 0 \\ -p\mu_E & \mu_P & 0 & 0 & 0 & 0 \\ -(1-p)\mu_E & 0 & \mu_A & 0 & 0 & 0 \\ 0 & -\mu_P & 0 & \mu_I & 0 & 0 \\ 0 & 0 & 0 & -(1-\rho)\mu_I & \mu_U + d_U & 0 \\ 0 & 0 & 0 & -\rho\mu_I & 0 & \mu_T + d_T \end{bmatrix}.$$

A simple calculation yields the following expression

$$\mathcal{R}_c = \beta S_0 \left( \frac{(1-p)\delta_A}{\mu_A} + \frac{p\delta_P}{\mu_P} + \frac{p}{\mu_I} + \frac{p(1-\rho)\delta_U}{\mu_U + d_U} \right). \quad (5)$$

In the absence of antiviral treatment ( $\rho = 0$ ),  $\mathcal{R}_c$  reduces to the expression (5) in the main paper for the basic reproduction number.

### 3 Final size relation

At the beginning of the epidemic, we assume that

$$S(0) + S_H(0) + S_{HP}(0) + I(0) + I_H(0) + I_{HP}(0) = C,$$

and other variables are zero, leading to

$$y(0) = z_P(0) = z_A(0) = i_U(0) = i_T(0) = 0.$$

For simplicity, we denote  $\lim_{t \rightarrow \infty} g(t)$  and  $\int_0^\infty g(s)ds$  by  $g(\infty)$  and  $\hat{g}$ , respectively, for any non-negative integrable function  $g$  defined on the interval  $0 \leq t \leq \infty$ . It can be easily shown that all compartments of exposed, pre-symptomatic, symptomatic (treated or untreated), and asymptomatic infection tend to 0 as  $t \rightarrow \infty$ . Integrating  $i'_T, i'_U, i', z'_A$  and  $z'_P$  of the model (4) leads to

$$\hat{i}_T = V_{IT}^{-1} V_T \hat{i}, \quad (6)$$

$$\hat{i}_U = V_{IU}^{-1} V_U \hat{i}, \quad (7)$$

$$\hat{i} = V_I^{-1} V_P \hat{z}_P + V_I^{-1} i(0), \quad (8)$$

$$\hat{z}_A = V_A^{-1} V_{EA} \hat{y}, \quad (9)$$

$$\hat{z}_P = V_P^{-1} V_{EP} \hat{y}, \quad (10)$$

Also, integrating the sum of  $x'$  and  $y'$  gives  $\hat{y} = V_E^{-1}(x(0) - x(\infty))$ , and hence

$$\hat{z}_P = V_P^{-1} V_{EP} V_E^{-1}(x(0) - x(\infty)), \quad (11)$$

$$\hat{z}_A = V_A^{-1} V_{EA} V_E^{-1}(x(0) - x(\infty)), \quad (12)$$

$$\hat{i} = V_I^{-1} V_{EP} V_E^{-1}(x(0) - x(\infty)) + V_I^{-1} i(0), \quad (13)$$

Letting

$$W_1 = [1 \ 0 \ 0] \left( \delta_P V_P^{-1} V_{EP} + \delta_A V_A^{-1} V_{EA} + V_I^{-1} V_{EP} + \delta_U V_{IU}^{-1} V_U V_I^{-1} V_{EP} \right) V_E^{-1},$$

$$W_2 = [0 \ 1 \ \alpha_P] \left( \delta_P V_P^{-1} V_{EP} + \delta_A V_A^{-1} V_{EA} + V_I^{-1} V_{EP} + \delta_U V_{IU}^{-1} V_U V_I^{-1} V_{EP} \right) V_E^{-1},$$

$$W_3 = [1 \ 0 \ 0] \left( 1 + \delta_U V_{IU}^{-1} V_U \right) V_I^{-1} i(0),$$

$$W_4 = [0 \ 1 \ \alpha_P] \left( 1 + \delta_U V_{IU}^{-1} V_U \right) V_I^{-1} i(0),$$

$$W_5 = [0 \ 1 \ \alpha_P] \left( \delta_P V_P^{-1} V_{EP} + \delta_A V_A^{-1} V_{EA} + \delta_T V_{IT}^{-1} V_T V_I^{-1} V_{EP} \right) V_E^{-1},$$

$$W_6 = [1 \ 1 \ \alpha_P] \delta_T V_{IT}^{-1} V_T V_I^{-1} i(0),$$

it follows from (6)-(13) that

$$\begin{aligned}\hat{Q} &= (W_1 + W_2)(x(0) - x(\infty)) + W_3 + W_4, \\ \hat{Q}_H &= W_5(x(0) - x(\infty)) + W_6,\end{aligned}$$

Integrating the equations for  $S'$ ,  $S'_H$  and  $S'_{HP}$  gives

$$\log S(0) - \log S(\infty) = \int_0^\infty \beta Q(t) dt = \beta \hat{Q}, \quad (14)$$

$$\log S_H(0) - \log S_H(\infty) = \int_0^\infty \beta Q(t) dt + \int_0^\infty \beta_H Q_H dt = \beta \hat{Q} + \beta_H \hat{Q}_H, \quad (15)$$

and

$$\log S_{HP}(0) - \log S_{HP}(\infty) = \int_0^\infty \beta \alpha_S Q(t) dt + \int_0^\infty \beta_H \alpha_S Q_H dt \quad (16)$$

$$= \alpha_S (\log S_H(0) - \log S_H(\infty)), \quad (17)$$

which lead to a relationship between the initial and final sizes of the susceptible HCWs with and without prophylaxis as

$$\frac{S_{HP}(\infty)}{S_{HP}(0)} = \left( \frac{S_H(\infty)}{S_H(0)} \right)^{(1-\alpha_S)}. \quad (18)$$

The above equations in (14), (15) and (18) can be used to evaluate  $S(\infty)$ ,  $S_H(\infty)$  and  $S_{HP}(\infty)$  in terms of the model parameters, and determine the values of  $\hat{y}$ ,  $\hat{z}_P$ ,  $\hat{z}_A$ ,  $\hat{i}$ ,  $\hat{i}_U$  and  $\hat{i}_T$ .

In the general population, assuming that  $W_3$  is small enough to be neglected, the final size relation is given by

$$\log \frac{S_0}{S_\infty} = \frac{\mathcal{R}_c}{S_0} (S_0 - S_\infty), \quad (19)$$

where  $\mathcal{R}_c$  is defined in (5) and  $S_\infty$  is the size of susceptible population when the epidemic dies out (Arino *et al.*, 2006). In the absence of antiviral treatment, this relation reduces to (6) in the main paper, which allows us to calculate  $\mathcal{R}_0$  from the assumed attack rate  $p(1 - S_\infty/S_0)$ . For the particular case of 30% clinical attack rate in the general population, it follows that  $S_\infty/S_0 = 0.5522$ , assuming that 67% of infected individuals develops clinical symptoms. Taking into account equation (6) in the main paper, we obtain the value of  $\mathcal{R}_0 = 1.3261$ . Thus, with the parameter values given in Table 1 of the main paper, the expression for  $\mathcal{R}_0$  provides an estimation of  $\beta = 1.2214/S_0$  for the baseline transmission rate of infection. We used the final size relation to estimate a range of  $\beta$  according to a range of 25%–35% clinical attack rate in the GP. To determine the transmission rate in the healthcare setting, we ran the simulations (given  $\beta$  for the GP) in the absence of treatment

and prophylaxis, in order to estimate values of  $\beta_H$  that correspond to a range of attack rates among HCWs given by the relation  $p(1 - S_\infty/S_0)$ . Using  $S_\infty$  and  $S_0$  for HCWs, and assuming attack rates of 30% and 45% in the GP and HCWs, respectively, simulations provide an estimation of  $\beta_H = 4.9845/(S_H(0) + S_{HP}(0))$ .

We used the estimated ranges of  $\beta$  and  $\beta_H$  for performing an uncertainty analysis of the model with a range of 25% – 35% for the reduction in susceptibility due to prophylaxis ( $\alpha_s$ ). With the parameter values given in Table 1, we run the simulations for samples generated by the LHS technique to calculate 1000 values of the averaged control reproduction number ( $\mathcal{R}_a$ ) using the final size relation (19), as prophylaxis coverage of HCWs increases. Box plots in Figure 3 of the main paper provide a range of variations in  $\mathcal{R}_a$  according to the changes in the clinical attack rates and susceptibility of HCWs with prophylaxis. Figure 4 in the main paper shows the ranges of variations in the total number of clinical infections and deaths during the entire course of an outbreak. Our analysis shows that the predicted outbreak dynamics is substantially affected by transmission rates; and the effectiveness of healthcare worker prophylaxis reduces as these rates increase.

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