Supporting Information

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SI Text

NA NG

S A NO

The Response of a Spherical Cap to Point Indentation. See Movie S1, Fig. 1 in the main text, and Fig. S1). Movie S1 shows distribution of the elastic energy density (A), deformed configuration (B), and force-indentation response of the shell (C) as the indentation increases. The elastic shell has the following geometrical parameters: t/R = 0.005, $\alpha = 120^\circ$, R = 1 m (see Fig. 1B for definitions)

Effect of Shell Thickness on the Response of a Spherical Cap to Point Indentation. To understand how the thickness of the shell affects the nonlinear response of the spherical cap, we resorted to a series of numerical simulations for the indentation problem.



Fig. 51. Response of an elastic spherical cap under point indentation. (See also Fig. 1.) (*A*) Critical indentation associated with bifurcation of each buckled pattern in shells with various normalized thickness, *t/R*. The results presented here are examined by developing computational models that invoke periodicity and symmetry associated with each pattern, which suppress some of the buckling patterns (not discussed for the sake of brevity). The broken lines show the best fit to each set of numerical data in the form of $Z'_c = C(t/R)^{\alpha}$, where the values of *C* and α are as follows: $C \approx 14.5$, $\alpha \approx 1$ (for axisymmetric); $C \approx 6.6$, $\alpha \approx 0.65$ (for n = 3); $C \approx 3.8$, $\alpha \approx 0.4$ (for n = 4); $C \approx 3.7$, $\alpha \approx 0.32$ (for n = 5). Note that the indentation associated with breaking the symmetry depends linearly on the thickness, which is in agreement with the analytical prediction [Jellett JH (1855) On the properties of inextensible surfaces. *Trans R Irish Acad* 22:343–374]. (*B*) Dependence of the average radius, R_c on the shell normalized thickness, *t/R*, which approximately follows $R_c/R \approx C_R(t/R)^{0.44}$, where $C_R \approx 2.5-3.1$, depending on the number of vertices. The broken lines correspond to $C_R = 2.5$. Note that the total length of the vertices is always $\sim 2\pi R_c$.





Movie S1 (MOV)