Supplementary material online

Performance of LCA models in free-response paradigm for multiple alternatives with all accumulators receiving equal noise

Figures S1*a* and S1*b* compare the performance of bounded and unbounded-linear models for levels of inputs I_1 and I_2 used in Figures 3*a* and 3*b*, respectively. In the case of larger inputs (relative to the noise-variance, *c*) of Figure S1*a*, the bounded model outperforms the unbounded model, while in the case of lower inputs of Figure S1*b* the bounded model has longer DTs than the unbounded model. To help understand this difference, Figures S1*c* and S1*d* show sample time courses of accumulators' activity for the same parameters used in Figures S1*a* and S1*b*. Note that in Figure S1*c*, accumulators y_1 and y_2 grow faster (due to larger input) than in Figure S1*d*. Hence in Figure S1*c*, the other accumulators y_3, y_4 and y_5 receive larger inhibition from y_1 and y_2 than in Figure S1*d*. As a result, in Figure S1*c* the activity levels of accumulators y_3, y_4 and y_5 are very close to zero and they are unlikely to compete with y_1 and y_2, while in Figure S1*d* the activity levels of accumulators y_3, y_4 and y_5 are higher and compete with y_1 and y_2.

This analysis may suggest that in the case of Figure S1*b*, the balance of inhibition and decay is not optimizing the performance, but rather it may be more profitable to increase the inhibition parameter *w*, which would increase inhibition of accumulators y_3, y_4, y_5 and thus prevent them from the competition with y_1 and y_2. Figure S1*e* shows that this reasoning is indeed correct, as for $N = 5$ alternatives, increasing inhibition to a certain point decreases DT of the bounded LCA model, and DT of the bounded LCA model for the value of the inhibition optimizing performance decreases below DT of the race model.

The input/noise ratio used in Figure S1*b* is similar to that used in simulations by (McMillen & Holmes, 2006) (end of Subsection 2.7), where the bounded LCA model is slower than the unbounded model at an equivalent ER. This corresponds to a low input/noise ratio. When the input noise ratio increases, corresponding to the case of Figures 5*a*, 5*b* and S1*a*, the nonlinear bounded model is faster than the unbounded model at an equivalent ER. It seems likely that the latter reflects better choice situations of the type illustrated in Figure 6, where there is strong evidence for a subset of the alternatives.

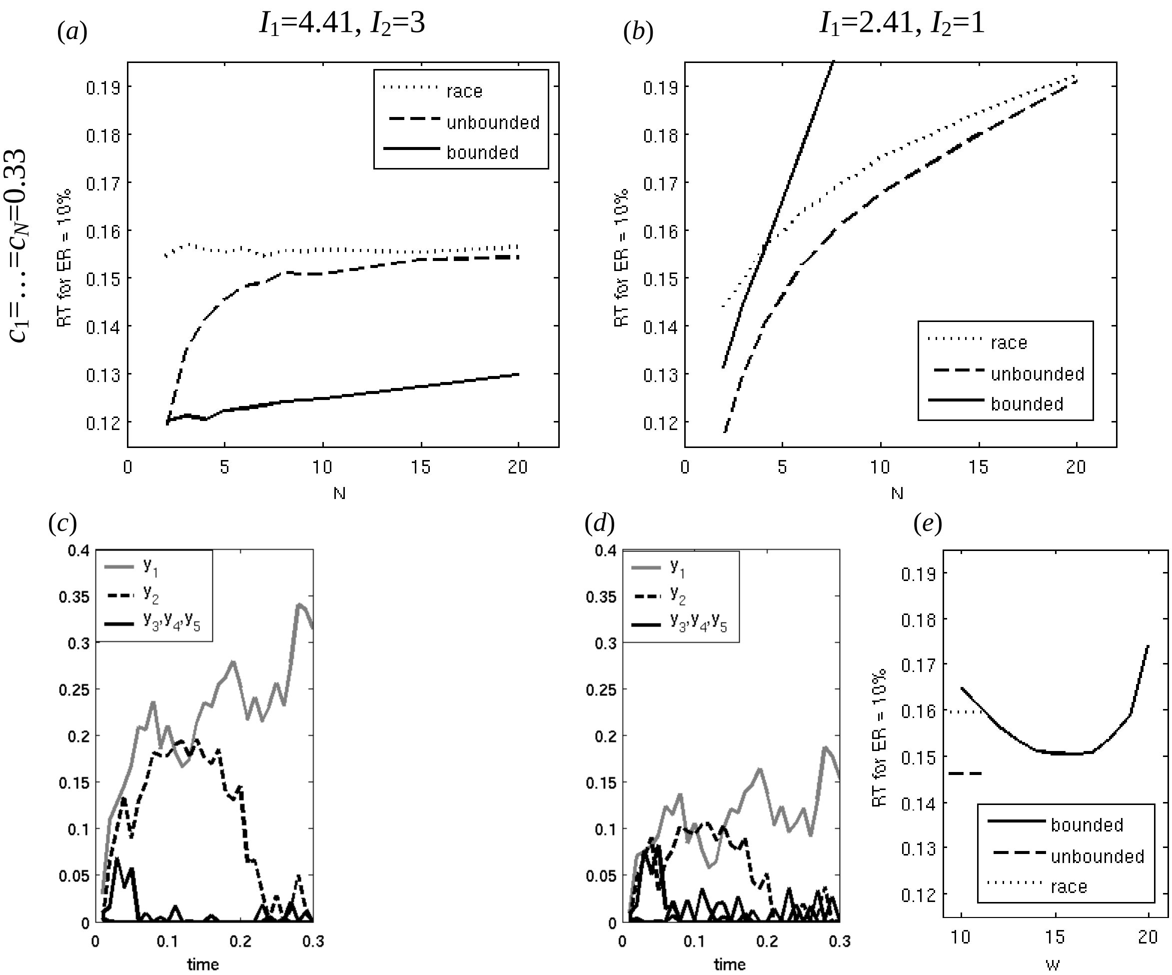

Figure S1. Performance and dynamics of choice models with only two accumulators receiving inputs with positive mean and all accumulators receiving equal noise. Methods of simulations as in Figure 5. (*a*), (*b*) Decision time (DT) for a threshold resulting in error rate of 10% of different choice models as a function of the number of alternatives N (shown on x-axis). Three models are shown: the race model, the unbounded (i.e. linear) LCA model, and the bounded LCA model (see key). The parameters of the LCA model are equal to $w = k = 10$. The parameters of the two first inputs were chosen such that $c_1 = c_2 = 0.33$, I_1-$I_2 = 1.41$ (as in Figure 5); the other accumulator received input with mean equal to 0, $I_3 = \ldots = I_N = 0$, and standard deviation equal to that of the first two inputs, $c_3 = \ldots = c_N = 0.33$. The panels differ in the total mean input to the first two accumulators: in panel (*a*) $I_2 = 3$, while in panel (*b*) $I_2 = 1$. (*c*), (*d*) Examples of the evolution of the bounded LCA model, showing y_i as functions of time for the same parameters as in panels (*a*), (*b*) respectively, and for $N = 5$ alternatives. Panels (*c*) and (*d*) were simulated for the same initial seed of the random number generator hence in both cases the networks received exactly the same inputs. (*e*) DT for a threshold resulting in error rate of 10% of unbalanced bounded LCA model as a function of inhibition (shown on x-axis). The model was simulated for the same parameters of inputs as in panel (*b*), and for $N = 5$ alternatives. In the simulations, the sum of decay and inhibition was kept constant at $w + k = 20$, while the relative values of decay and inhibition was different for each data point. Thus the most left data point correspond to the balanced model ($w = k = 10$) and shows the same value as in panel (*b*) for $N =$ 5. The short dashed and dotted lines on the left of the panel indicate the DT of the balanced unbounded LCA and race models.

A stronger version of St Petersburg paradox

One may contend, that it is possible to construct a new St Petersburg paradox, which will not be solved even by such diminishing-return value function (as long as it is unbounded), by increasing the gains for the head in i^{th} toss to 2^{2^i} (Weirich, 1984). Although the price that people would offer for such a game is still not infinite (though much larger than the one in the classical St Petersburg), this is likely to reflect the fact that people are right to doubt that such gains would be paid, or that there are practical bounds on the number of tosses (Jeffrey, 1983).

Weber law and logarithmic utility function

The Weber law, states that in order to discriminate between two stimuli, x, and $x+dx$, the value of dx needs to be proportional to x, itself. This Section shows that one can satisfy this law, by assuming that there are neural representations that transform their inputs x (which corresponds to objective value) under a logarithmic type of nonlinearity $u(x)=\log(x)$ and that the output is subject to additional independent noise of constant variance c^2. This will require to show that for dx proportional to x, the ability to discriminate the two sample x, and $x+dx$ is a constant (which depends on c^2).

Consider a choice between two alternatives with objective values x_1 and $x_2=x_1+dx$. Their subjective values are equal to $y_i=u(x_i)+N(0,c)$, where $N(0,c)$ denotes a number sampled from normal distribution with mean 0 and standard deviation c. We now show that if the difference between x_1 and x_2 is proportional to x_1 (let ε denote the proportionality constant so that $dx=\varepsilon x_1$), then the probability of choosing second (i.e., correct) alternative is independent of objective values x_1 and x_2, i.e., the Weber law is satisfied. The probability of choosing second alternative is equal to:

$$
\begin{aligned}
&P(y_1 < y_2) = P(u(x_1) + N(0,c) < u(x_2) + N(0,c)) = \\
&= P\left(N(0,1) < \frac{u(x_2) - u(x_1)}{\sqrt{2}c}\right) = \Phi\left(\frac{\log(x_1 + \varepsilon x_1) - \log(x_1)}{\sqrt{2}c}\right) = \Phi\left(\frac{\log(1+\varepsilon)}{\sqrt{2}c}\right)
\end{aligned}
$$

where Φ denotes the normal standard cumulative distribution function. Note that the probability does not depend on either x_1 or x_2.

Derivation of probability of choosing the sure alternative

If we assume for simplicity that the boundaries are not present, then the balanced LCA model in the interrogation paradigm chooses the alternative which has received more input by time T. Hence let us calculate the probability that the input to the unit corresponding to the sure alternative (until time T) is larger than to the risky alternative.

First let us denote the inputs to units corresponding to the sure and the risky alternatives at time t by s_t and r_t. Let us calculate their distributions. s_t come from the normal distribution with mean $I_0+u(W)$ and standard deviation SD:

$$s_t \sim N(I_0+u(W), \mathrm{SD})$$

The mean of r_t is equal to $I_0 + u(W/p)p$. The variance of r_t is equal to $u^2(W/p)p - u^2(W/p)p^2 + SD^2$.

Let us denote the input until time T to the units by S and R. According to the central limit theorem they come from the following distributions:

$$S = \sum_{t=1}^{T} s_t = N\left(T\left(I_0 + u(W)\right), \sqrt{T}SD\right)$$

$$R = \sum_{t=1}^{T} r_t = N\left(T\left(I_0 + u(W/p)p\right), \sqrt{T\left(u^2(W/p)p(1-p) + SD^2\right)}\right)$$

Hence the probability of choosing the sure alternative is:

$$P(R < S) = P\left(N\left(Tu(W/p)p - Tu(W), \sqrt{T\left(u^2(W/p)p(1-p) + 2SD^2\right)}\right) < 0\right) =$$

$$= P\left(N(0,1) < \frac{Tu(W) - Tu(W/p)p}{\sqrt{T\left(u^2(W/p)p(1-p) + 2SD^2\right)}}\right) = \Phi\left(\sqrt{T}\,\frac{u(W) - u(W/p)p}{\sqrt{u^2(W/p)p(1-p) + 2SD^2}}\right)$$

where Φ denotes the normal standard cumulative distribution function.

Hence in summary, the probability of choosing the sure alternative increases with square root of deliberation time until it saturates.

References

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McMillen, T., & Holmes, P. (2006). The dynamics of choice among multiple alternatives. *Journal of Mathematical Psychology, 50*, 30-57.

Weirich, P. (1984). The St. Petersburg Gamble and Risk. *Theory and Decision, 17*, 193-202.