Technical Appendix

In this appendix, we discuss the mathematical properties of the *Ridit* score and *PRIDIT* analysis, using a simple example of ten hospitals and three measures in order to facilitate the use of this method. Note that Bross computes a similar table (Table 1 of his paper), a method that, as we have noted, is mathematically equivalent to ours (Bross 1958).

Denote the number of observations by N and the number of indicators by M. In Table 3, we show ten hospitals $(N = 10)$ indexed by row numbers $i \in \{1,2,\ldots 9,10\}$ with three quality indicators (*patients given assessment of left ventricular function* (LVF) for heart failure, *patients given beta blocker at arrival* for heart attack and hospital teaching status) indexed by column numbers $j \in \{1,2,3\}$. The scale for the clinical variables is continuous on $[0,1]$ where the value of the variable indicates the percent of successfully treatment of eligible patients with the indicated process measure. In other words, value (1,2) of the matrix of indicator values is 0.55, indicating that, in hospital 1, 55% of patients classified as having had a heart attack were given a beta blocker upon arrival. Teaching status is a binary variable that indicates whether a hospital was a Medicare Teaching Hospital (yes = 1).

After we have the indicator value for each measure, we then measure the empirical cumulative distribution for each measure. In other words, for each indicator j for hospital i, we determine the proportion of the sample to which the indicator value is greater than or equal. For LVF for hospital 1, 0.45 is greater than or equal to only 10% of all hospitals (i.e. hospital 1 itself) so the cumulative proportion is 0.10, whereas for hospital 4, 0.92 is greater than or equal to 90% of all hospitals so the cumulative proportion is 0.90. Obviously, indicator values can only take 0 or 1, so for teaching status cumulative proportion is either 0.70 or 1.00. We explain later why our choice of 0 for non-teaching and 1 for teaching (i.e. a proportion of 0.70 or 1.00 prospectively) does not mean that we have assumed that teaching hospitals are superior (or inferior) – merely that the knowledge of status is informative.

One feature of the calculation of the *Ridit* score is worth noting here. We are transforming based on rank, meaning that we are losing information about the magnitude of the difference between hospitals on different measures. In other words, whether hospital 1 scored 0.45, 0.47 or 0.20 on the LVF measure, we would still assign it a cumulative proportion of 0.10. From this point of view, then, we can see that, like any rank based or integer scoring type measure, *PRIDIT* could be sensitive to the number of observations and the distribution of measures. Given the large number of observations we have, this is not a concern in our study.

Next, we calculate the matrix of *Ridit*[1] scores B according to the following formula:

$$B_{ij} = F_j^-(i) - (1 - F_j(i)) \tag{1}$$

where $F_j^-(i)$ is the cumulative distribution function for the hospital ranked one spot below hospital i on indicator j ($F_1^-(1) = 0.00$, $F_2^-(10) = 0.90$, $F_3^-(2) = 0.70$) and $F_j(i)$ is the cumulative distribution function for the hospital i on indicator j ($F_1(1) = 0.10$, $F_2(10) = 1.00$, $F_3(2) = 1.00$). We show the *Ridit* scores in Table 4.

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[1] Brockett et al. uses the capitalized *RIDIT* terminology to distinguish their score from Bross' *Ridit* score. We will use *Ridit* to refer to the way in which we calculate the score. Our calculation, which is the same as Brockett et al., has a one-to-one mapping onto Bross' original measure, but makes the calculation and exposition of *PRIDIT* simpler.

Once we have the B matrix, we proceed with Principal Components Analysis to get the eigenvalues $\vec{\lambda}$ and eigenvectors $\vec{x}$ of the matrix. We used the first eigenvalue and eigenvectors as our main results and the second eigenvalue and eigenvector as our alternate specification. We determine a vector of weights w, which indicate the importance of each variable in detecting the outcome of interest, using the following formula:

$$w = \sqrt{\lambda}\vec{x} \tag{2}$$

In this case, the eigenvalues are $\lambda = \begin{pmatrix}1.6335 & 1.1016 & 0.2649\end{pmatrix}$, the eigenvectors are $x = \begin{pmatrix} 0.7061 & -0.2512 & -0.6621 \\ 0.7081 & 0.2416 & 0.6635 \\ 0.0067 & 0.9373 & -0.3485 \end{pmatrix}$, and the first eigenvalue and eigenvector are $\lambda = 1.6335, \bar{\mathrm{x}} = \begin{pmatrix}0.7061 & 0.7081 & 0.0067\end{pmatrix}$.[2] We show the weights w in Table 5.

The last step is to normalize the B matrix so that all *PRIDIT* scores lie on the range [-1,1]. We normalize the B matrix to generate $Bnorm$ as follows:

$$Bsq = B'B \tag{3}$$

$$bsq = diag(Bsq) \tag{4}$$

$$b = \sqrt{bsq} \tag{5}$$

$$Bnorm_{ij} = B_{ij} / b_j \tag{6}$$

Items (3)-(5) are in Table 5, while item (6) is in Table 6.

Finally, we determine a vector of *PRIDIT* scores s, which indicate how each observation scores overall on the outcome of interest, by:

[2] When calculating the principal components, we scale the variables to have unit variance before the analysis takes place.

$$s = \frac{w'Bnorm_{ij}}{\lambda} \tag{7}$$

We show the vector of scores s and associated ranks in Table 7.

As we can see from the ranks, Hospital 10 is the best and Hospital 1 is the worst. This should not be a surprise since Hospital 10 dominates all hospitals on the two clinical measures and all hospitals dominate Hospital 1 on the two clinical measures. In this example, teaching status is minimally informative about hospital quality. Teaching hospitals are of varying quality in this example (they are above average on LVF but above average on Beta Blockers), which leads to the weight on teaching facility being so low, and in this example excluding teaching status would not affect the result.

In addition, we see that the sign on teaching hospital is positive, meaning that teaching hospitals are associated with higher quality. If we instead coded teaching status as 0 and non-teaching as 1, then the sign would have changed and the result would have stayed the same. This is the sense in which our choice of how we code binary variables does not indicate an assumption about the direction of the association between teaching status and hospital quality.

Figure Legends - Appendix

Table 3: Sample Quality Measures and Cumulative Proportions
Table 4: *Ridit* Scores and Cumulative Proportions
Table 5: Principal Component Analysis Calculation Measures – w, Bsq, bsq, b
Table 6: Principal Component Analysis Calculation Measures – $Bnorm$
Table 7: *PRIDIT* Scores by Quality Measure and in Total

Tables – Appendix

	LVF		Beta Blocker		Teaching Status	
Hospital Number	Indicator Value	Cumulative Proportion	Indicator Value	Cumulative Proportion	Indicator Value	Cumulative Proportion
1	0.45	0.10	0.55	0.10	0	0.70
2	0.48	0.20	0.68	0.20	1	1.00
3	0.70	0.60	0.88	0.40	0	0.70
4	0.92	0.90	0.93	0.50	0	0.70
5	0.80	0.80	1.00	1.00	1	1.00
6	0.65	0.40	1.00	1.00	1	1.00
7	0.64	0.30	0.97	0.60	0	0.70
8	0.73	0.70	1.00	1.00	0	0.70
9	0.70	0.60	0.73	0.30	0	0.70
10	1.00	1.00	1.00	1.00	0	0.70

Table 3

	LVF		Beta Blocker		Teaching Status	
Hospital Number	B_{ij} (*Ridit*)	Cumulative Proportion	B_{ij} (*Ridit*)	Cumulative Proportion	B_{ij} (*Ridit*)	Cumulative Proportion
1	0.90	0.10	0.90	0.10	0.30	0.70
2	0.70	0.20	0.70	0.20	-0.70	1.00
3	0.00	0.60	0.30	0.40	0.30	0.70
4	-0.70	0.90	0.10	0.50	0.30	0.70
5	-0.50	0.80	-0.60	1.00	-0.70	1.00
6	0.30	0.40	-0.60	1.00	-0.70	1.00
7	0.50	0.30	-0.10	0.60	0.30	0.70
8	-0.30	0.70	-0.60	1.00	0.30	0.70
9	0.00	0.60	0.50	0.30	0.30	0.70
10	-0.90	1.00	-0.60	1.00	0.30	0.70
Sum	0.00		0.00		0.00	

Table 4

Formula Number	(2)	(3)	(4)	(5)
Calculation Item	*w*	*Bsq*	*bsq*	*b*
	$\begin{pmatrix} 0.9024 \\ 0.9050 \\ 0.0086 \end{pmatrix}$	$\begin{pmatrix} 3.28 & 2.02 & -0.50 \\ 2.02 & 3.10 & 0.50 \\ -0.50 & 0.50 & 2.10 \end{pmatrix}$	$\begin{pmatrix} 3.28 \\ 3.10 \\ 2.10 \end{pmatrix}$	$\begin{pmatrix} 1.8111 \\ 1.7607 \\ 1.4491 \end{pmatrix}$

Table 5

	LVF	Beta Blocker	Teaching Status
Hospital Number (i)	$Bnorm_{i1}$	$Bnorm_{i2}$	$Bnorm_{i3}$
1	-0.4969	-0.5112	-0.2070
2	-0.3865	-0.3976	0.4830
3	0.0000	-0.1704	-0.2070
4	0.3865	-0.0568	-0.2070
5	0.2761	0.3408	0.4830
6	-0.1656	0.3408	0.4830
7	-0.2761	0.0568	-0.2070
8	0.1656	0.3408	-0.2070
9	0.0000	-0.2840	-0.2070
10	0.4969	0.3408	-0.2070
Sum	0.0000	0.0000	0.0000

Table 6

	LVF	Beta Blocker	Teaching Status	Total Score	Overall Rank
Hospital Number (i)	s_{i1}	s_{i2}	s_{i3}	s $(s_{i1}+s_{i2}+s_{i3})$	
1	-0.2745	-0.2832	-0.0011	-0.5588	10
2	-0.2135	-0.2203	0.0025	-0.4313	9
3	0.0000	-0.0944	-0.0011	-0.0955	6
4	0.2135	-0.0315	-0.0011	0.1809	4
5	0.1525	0.1888	0.0025	0.3438	2
6	-0.0915	0.1888	0.0025	0.0998	5
7	-0.1525	0.0315	-0.0011	-0.1221	7
8	0.0915	0.1888	-0.0011	0.2792	3
9	0.0000	-0.1573	-0.0011	-0.1584	8
10	0.2745	0.1888	-0.0011	0.4622	1
Sum	0.0000	0.0000	-0.0002	-0.0002	

Table 7