

## APPENDIX A

The derivation of eqs. (2) is detailed in Rocheleau et al. (2006) and described in brief below. The standard chemical kinetics applied to the reactions (1) yields the following differential equations characterising the temporal evolution of the metabolites' concentration:

$$\dot{[T_s]} = 2K_t[T_d] - k_T[pO][T_s] \quad (\text{A1a})$$

$$\dot{[T_d]} = -K_T[T_d] + k_T[pO][T_s] \quad (\text{A1b})$$

$$\dot{[L]} = k_L[pL][T_d], \quad (\text{A1c})$$

where the chemical equilibrium of reaction (1b) has been used through its equilibrium constant  $K_t$ . Considering that the total concentration of (single-)template can be written as  $[T_l] \equiv 2[T_d] + [T_s]$ , one can rewrite eqs. (A1) in terms of  $[T_l]$  as

$$\dot{[T_l]} = k_T[pO][T_s] \quad (\text{A2a})$$

$$\dot{[L]} = k_L[pL][T_d]. \quad (\text{A2b})$$

In addition, as the equilibrium constant satisfies  $K_t = [T_s]^2/[T_d]$ , the following equation holds:

$$2[T_s]^2 + K_t[T_s] - K_t[T_l] = 0,$$

implying that

$$\begin{aligned} [T_s] &= \frac{-K_t + \sqrt{K_t^2 + 8K_t[T_l]}}{4} \approx \frac{\sqrt{K_t[T_l]}2}{4} \\ [T_d] &= \frac{[T_l]}{2} + \frac{-K_t + \sqrt{K_t^2 + 8K_t[T_l]}}{4} \approx \frac{[T_l]}{2}, \end{aligned}$$

where the product inhibition has been taken into account as  $[T_s] \ll [T_d]$ . With the help of these approximations, eqs. (A2) can finally be rewritten as eqs. (2).

## APPENDIX B

Eqs. (2) implicitly assume that the protocell's volume can be approximated as being a constant. Thus one can also write eq. (2b) as

$$\frac{1}{V_A} \frac{dN_L}{dt} = C_L [T_l]. \quad (\text{B1})$$

If instead, the volume  $V_l$  is considered proportional to the number of lipid molecules, *i.e.*,  $V_l = Q \times N_L$ , with  $Q$  being the proportionality constant, then the volume must be considered as time-varying:

$$\frac{dV_l}{dt} = Q \times \frac{dN_L}{dt} \quad (\text{B2})$$

Thus, in the changing volume approach, the time derivative of the template concentration  $[T_l] \equiv N_T/V_l$  is given by:

$$\frac{d[T_l]}{dt} = \left. \frac{1}{V_l} \frac{dN_T}{dt} \right|_{V_l} - \frac{N_T}{V_l^2} \left. \frac{dV_l}{dt} \right|_{N_T} \quad (\text{B3})$$

The first term on the r.h.s of eq. (B3) is considered at constant volume and thus one must have:

$$\left. \frac{d[T_l]}{dt} \right|_{V_l} = \left. \frac{1}{V_l} \frac{dN_T}{dt} \right|_{V_l} \quad (\text{B4})$$

By using the definition of  $[T_l]$  and eqs. (B4) and (2a), both of which apply at constant volume, eq. (B3) can be rewritten as

$$\frac{d[T_l]}{dt} = C_T [T_l]^p - \frac{[T_l]}{V_l} \frac{dV_l}{dt} \quad (\text{B5})$$

Next, we consider the dividing protocells and their concentration,  $[A]$  in the entire experimental volume. As the lipid aggregates have a characteristic scale, there exists an average number  $m_0$  of lipids per average aggregate volume  $V_A$ . Thus, the growth rate of  $[A]$  is given by:

$$\begin{aligned}
\frac{d[A]}{dt} &= [A] \frac{d}{dt} (N_L/m_0) \\
&= \frac{[A]}{m_0} \frac{dN_L}{dt} \\
&= \frac{[A]}{Qm_0} \frac{dV_l}{dt}
\end{aligned} \tag{B6}$$

$$= \frac{C_L}{m_0} [T_g] \tag{B7}$$

where  $[T_g] = [A]V_a[T_l]$  and eq. (B1) have been used. One can recognise eq. (B7) as being eq. (7) with  $\gamma_A \equiv C_L/m_0$ . Applying eq. (B2), one can rewrite eq. (B6) as

$$\begin{aligned}
\frac{dA}{dt} &= \frac{[A]}{Qm_0} \frac{dV_l}{dt} \\
&= \frac{[A]}{V_A} \frac{dV_l}{dt}
\end{aligned} \tag{B8}$$

where we have used the definition of the average aggregate volume ( $V_A = Qm_0$ ). One can recognise the last equation as being eq. (5).

Finally, let us recover the evolution of the global concentration of templates. Taking the derivative of  $[T_g] = [A]V_A[T_l]$  yields

$$\frac{d[T_g]}{dt} = [T_l]V_A \frac{d[A]}{dt} \Big|_{[T_l]} + [A]V_A \frac{d[T_l]}{dt} \Big|_{[A]} \tag{B9}$$

Using eqs. (B8) and (B5) to substitute into the first and second terms on the r.h.s of eq. (B9), we have

$$\begin{aligned}
\frac{d[T_g]}{dt} &= [T_l] \frac{[A]}{Qm_0} \frac{dV_l}{dt} + [A]V_A \left( C_T [T_l]^p - \frac{[T_l]}{V_l} \frac{dV_l}{dt} \right) \\
&= [A]V_A C_T [T_l]^p - [A][T_l] \frac{dV_l}{dt} \left[ \frac{1}{V_l} - \frac{1}{Qm_0} \right]
\end{aligned} \tag{B10}$$

Considering again the approximation  $V_l \approx V_A$ , the bracketed r.h.s of eq. (B10) becomes zero. Then using the relation between  $[T_g]$  and  $[T_l]$ , one obtains

$$\frac{d[T_g]}{dt} = C_T [T_g] \left( \frac{[T_g]}{V_A [A]} \right)^{p-1} = \gamma_T [T_g] \left( \frac{[T_g]}{[A]} \right)^{p-1}$$

with  $\gamma_T \equiv C_T/V_A^{p-1}$ , which is eq. (6).