**SI Appendix**

**Model Equations**

In the following equations, secondary parameters (Table S4) and the force of infection (Eq. **1**) are denoted by lower-case letters, sub-populations (Table S3) are denoted by capital letters, and outputs from other equations are denoted by longer terms started with a capital letter. Time-dependent parameters are denoted by (*t*), referring to the value at the present time step, or (*t+1*) to the value at the next time step. Generic subscripts are used as follows: *h* = HIV status (+, positive; –, negative), *d* = drug susceptibility (*n*, non-resistant; *m*, MDR; *x*, XDR), *i* = infectivity (*o*, no active TB; *a*, highly infective; *l*, less infective), and *t* = treatment status (*0* = never treated; *1* = previously treated).

**Force of infection, $f_d(t)$.**

$$f_d(t) = \sum_i \left\{ \lambda_{di} \times \sum_h \sum_t \left[ PD_{hdit}(t) + PT_{hdit}(t) + PTC_{hdit}(t) + U_{hdit}(t) \right] \right\} \quad . \qquad [\mathbf{1}]$$

**Flow due to HIV infection, *Hivflow*(*t*).**

$$\text{If } h = +\text{, then, for population } X_{+dit}(t),\ Hivflow_{+}(t) = X_{-dit}(t) \times ts \times hiv \qquad [\mathbf{2}]$$

$$\text{If } h = -\text{, then, for population } X_{-dit}(t),\ Hivflow_{-}(t) = -X_{-dit}(t) \times ts \times hiv \text{ .}$$

**Flow due to conversion from less to highly infectious TB, *Infflow*(*t*).**

If $i = a$, then for population $X_{hdat}(t)$, $Infflow_a(t) = X_{hdlt}(t) \times ts \times sc$ **[3]**

If $i = l$, then for population $X_{hdlt}(t)$, $Infflow_l(t) = -X_{hdlt}(t) \times ts \times sc$ .

**Flow due to mortality and aging to 50th birthday, *Mortflow*(*t*)**

For population $X_{hdit}(t)$, $Mortflow_{hi}(t) = X_{hdit}(t) \times ts \times m_{hi}$. **[4]**

Note that $i = o$ for populations without an infectivity designation.

**Number of susceptible patients, *S*(*t*)**

$$S_h(t+1) = S_h(t) + a\left[\left(1 - 0.98^{15}\right) \times \sum_h \sum_d \sum_i \sum_t Mortflow_{hdit}(t)\right] - S_h(t) \times ts \times \left(\sum_d f_d(t)\right) - Mortflow_{ho}(t) + Hivflow_h(t), \quad [5]$$

where $a = 1$ for $S_-$ and $a = 0$ for $S_+$.

**Number of latently infected patients, *L*(*t*).**

$$L_{hd}(t+1) = L_{hd}(t) + a\left[0.98^{15} \times \sum_h \sum_d \sum_i \sum_t Mortflow_{hdit}(t)\right] + \left[S_h(t) \times f_d(t) \times (1 - rp_h)\right] + \left\{ f_d(t) \times ts \times \sum_d \left[L_{hd}(t) \times ps_h \times (1 - rp_h)\right]\right\} - L_{hd}(t) \times ts \times \left\{er_h + ps_h \times \left[\sum_d f_d(t)\right]\right\} - Mortflow_{ho}(t) + Hivflow_h(t), \quad [6]$$

where $a = 1$ for $L_{-d}$ and $a = 0$ for $L_{+d}$.

**Number of patients with active TB before presenting for diagnosis, *PD*(*t*).**

$$PD_{hdit}(t+1) = PD_{hdit}(t) + [S_h(t) \times f_d(t) \times rp_h] + (1-b) \times \left\{ \sum_d [L_{hd}(t)] \times ts \times ip_{hi} \times [er_h + ps_h \times rp_h \times f_d(t)] \right\}$$
$$+ (b \times ts \times ip_{hi}) \times \left\{ [rel \times F_{hd}(t) \times (1-dr)] + [rel \times F_{hd-1}(t) \times dr] + [rp_h \times f_d(t)] \times \left[ C_h(t) + ps_h \times \sum_d F_{hd}(t) \right] \right\}$$
$$- [PD_{hdit}(t) \times ts / pd] - Mortflow_{ho}(t) + Hivflow_h(t) + Infflow_i(t), \quad [7]$$

where $b$ = 0 for $PD_{hdi0}$ and $b$ = 1 for $PD_{hdi1}$.

**Number of active TB patients awaiting treatment based on diagnosis not involving culture, *PT*(*t*).**

$$PT_{hdit}(t+1) = PT_{hdit}(t) + \{PD_{hdit}(t) \times ts \times cdr_i \times tx / pd\} + \{(ts \times redx_h) \times (U_{hdit} \times cdr_i \times tx)\}$$
$$- [PT_{hdit}(t) \times ts / dd] - Mortflow_{hi}(t) + Hivflow_h(t) + Infflow_i(t). \quad [8]$$

**Number of active TB patients awaiting treatment based on culture result, *PTC*(*t*).**

$$PTC_{hdit}(t+1) = PTC_{hdit}(t) + \{PD_{hdit}(t) \times ts \times cx_t \times (cdrc_i - cdr_i) \times tx \times (1 - ltf) / pd\}$$
$$+ \{(ts \times redx_h) \times [U_{hdit} \times (cdrc_i - cdr_i) \times cx_t \times tx \times (1 - ltf)]\} \quad [9]$$
$$- [PTC_{hdit}(t) \times ts / ddc] - Mortflow_{hi}(t) + Hivflow_h(t) + Infflow_i(t).$$

**Number of patients with untreated active TB, *U*(*t*).**

$$
\begin{aligned}
&U_{hdit}(t+1)=U_{hdit}(t)+\{PD_{hdit}(t)\times ts\times[1-tx\times[(cdrc_i-cdr_i)\times cx_t\times(1-ltf)+cdr_i]]/pd\}\\
&+(b\times ts)\times\left\{\begin{aligned}&\sum_t[PT_{hdit}(t)\times(1-e\times cx_t\times dst_t\times g)\times(1-act_d)/dd]\\&+\sum_t[PTC_{hdit}(t)\times(1-act_d)\times(1-e\times dst_t\times g)/ddc]\end{aligned}\right\} \qquad [\mathbf{10}]\\
&-U_{hdit}(t)\times ts\times redx_h\times\{(cdr_i\times tx)+[(cdrc_i-cdr_i)\times cx_t\times(1-ltf)\times tx]\}\\
&-Mortflow_{hi}(t)+Hivflow_h(t)+Infflow_i(t),
\end{aligned}
$$

where $b = 0$ for $U_{hdi0}$ and $b = 1$ for $U_{hdi1;}$ $e = 0$ for $U_{hnit}$, $e = 1$ for $U_{hmit}$ or $U_{hxit}$; and $g = 1$ for $U_{hnit}$ or $U_{hmit}$, $g = q$ for $U_{hxit}$.

**Number of patients cured of prior active TB, *C*(*t*).**

$$
\begin{aligned}
&C_h(t+1)=C_h(t)+\sum_d\sum_i\sum_t\{[PT_{hdit}(t)\times ts\times act_d\times cr/dd]+[PTC_{hdit}(t)\times ts\times act_d\times cr/ddc]\}\\
&+\{DST_{hm}(t)\times ts\times cr/dstd\}-C_h(t)\times ts\times\left[rp_h\times\sum_d f_d(t)\right]-Mortflow_{ho}(t)+Hivflow_h(t). \qquad [\mathbf{11}]
\end{aligned}
$$

**Number of patients receiving inactive therapy that will be changed on the basis of DST result, *DST*(*t*).**

$$
\begin{aligned}
&DST_{hd}(t+1)=DST_{hd}(t)\\
&+ts\times(1-act_d)\times\sum_i\sum_t\{[PT_{hdit}(t)\times cx_t\times dst_t\times g/dd]+[PTC_{hdit}(t)\times dst_t\times g/ddc]\} \qquad [\mathbf{12}]\\
&-DST_{hd}(t)\times ts/dstd-Mortflow_{ha}(t)\times ip_{ha}-Mortflow_{hl}(t)\times ip_{hl}+Hivflow_h(t),
\end{aligned}
$$

where $g = 1$ for $DST_{hm}$ and $g = q$ for $DST_{hx}$; note that $DST_{hn}$ does not exist.

**Number of patients treated for active TB but not cured, *F*(*t*)**

$$F_{hd}(t+1)=F_{hd}(t)+\sum_{i}\sum_{t}\{[PT_{hdit}(t)\times ts\times act_{d}\times(1-cr)/dd]+[PTC_{hdit}(t)\times ts\times act_{d}\times(1-cr)/ddc]\}$$
$$+[DST_{hd}(t)\times ts\times(1-cr)\times(1-g)/dstd]+\left\{f_{d}(t)\times ts\times\sum_{d}[ps_{h}\times(1-rp_{h})\times F_{hd}(t)]\right\} \quad [\mathbf{13}]$$
$$-F_{hd}(t)\times ts\times\left\{rel+\left[ps_{h}\times\sum_{d}f_{d}(t)\right]\right\}-Mortflow_{ho}(t)+Hivflow_{h}(t),$$

where $g = 0$ for $F_{hn}$ or $F_{hm}$, and $g = q$ for $F_{hx}$.

**Number of patients under respiratory isolation for XDR-TB, *Q*(*t*).**

$$Q_{hi}(t+1)=Q_{hi}(t)+[q\times DST_{hx}(t)\times ip_{hi}\times ts/dstd]-Mortflow_{hi}(t)+Infflow_{i}(t). \quad [\mathbf{14}]$$