Supporting Information

Lu *et al.* 10.1073/pnas.0800740105

SI Text

The determination of the state fidelity and the detection of entanglement are fundamental problems in quantum information. This task is usually difficult in experiments especially for systems with many particles. Instead of using quantum state tomography (1), which would require $\approx 4^N$ (N is the number of qubits) measurements, in our experiment, we use a method similar to entanglement witness (2, 3), which requires only a linear experimental effort.

In an experiment, one typically aims at the creation of some pure entangled multiqubit state $|\Phi\rangle$. The fidelity of the produced state, that is, to what extent the desired state was prepared, is given by $F = \langle\Phi|\rho_{\text{exp}}|\Phi\rangle$ and should ideally equal 1. For experimental implementations, it is necessary to decompose $|\Phi\rangle\langle\Phi|$ into a number of local von Neumann (or projective) measurements (4). The detailed constructions of the states $|V\rangle_l$, $|+\rangle_l$, $|R\rangle_l$,$|\Phi_5\rangle$ are as follows.

1. The four-photon QEEC codeword of $|V\rangle$ is

$$|V\rangle_l = \frac{1}{2}(|H\rangle_2|H\rangle_3 - |V\rangle_2|V\rangle_3)(|H\rangle_4|H\rangle_5 - |V\rangle_4|V\rangle_5).$$

We can decompose $|V\rangle_l$ into locally measurable observables

$$|V\rangle_l\langle V| = \frac{1}{16}(I + Z_2Z_3 - X_2X_3 - Y_2Y_3) \cdot (I + Z_4Z_5 - X_4X_5 - Y_4Y_5),$$

where I denotes the identity operator, and Z, X, Y are short for the Pauli matrix σ_z, σ_x, σ_y, respectively. To determine the expectation value of $|V\rangle_l\langle V|$, we need to take nine measurement settings, namely $X_2X_3X_4X_5$, $X_2X_3Y_4Y_5$, $X_2X_3Z_4Z_5$, $Y_2Y_3X_4X_5$, $Y_2Y_3Y_4Y_5$, $Y_2Y_3Z_4Z_5$, $Z_2Z_3X_4X_5$, $Z_2Z_3Y_4Y_5$, $Z_2Z_3Z_4Z_5$. Here, as used in the literature (2–4), a measurement setting refers to that which can be measured at the same time without changing the experimental setup.

Let us take the measurement setting $X_2X_3Z_4Z_5$ as an example to show how the expectation values of observables $X_2X_3Z_4Z_5$, $X_2X_3I_4I_5$, and $I_2I_3Z_4Z_5$ are derived from the experimental data. The spin observable Z corresponds to a measurement of the H/V linear polarization, and X (Y) corresponds to the analysis of $\pm 45°$ linear polarization (left/right circular polarization). For polarization analysis, half and quarter-wave plates (HWP, QWP) together with polarizers or PBSs are used. We use a programmable multichannel coincidence unit to register the fivefold coincidence events. Here, for the measurement of the four-photon QEEC codes, the signal of detector 1 (D_1) is used only as a trigger. In this setting, $X_2X_3Z_4Z_5$, we register the fivefold coincidence counts of the 16 different polarization combinations ($|+\rangle_2|+\rangle_3|H\rangle_4|H\rangle_5$, $|+\rangle_2|+\rangle_3|H\rangle_4|V\rangle_5$, $|+\rangle_2|+\rangle_3|V\rangle_4|H\rangle_5$, ... $|-\rangle_2|-\rangle_3|V\rangle_4|V\rangle_5$), each signaling the observation of an eigenstate of the observable $X_2X_3Z_4Z_5$ (also $X_2X_3I_4I_5$ and $I_2I_3Z_4Z_5$) with the corresponding eigenvalue of $v_j = +1$ or $v_j = -1$. From the probabilities of multiphoton detections $p_j, j = 1, 2, \ldots 16$, we can then compute the expectation values of the observables by $\Sigma_{j=1}^{16} p_j v_j$. The full experimental results for the state $|V\rangle_l$ is shown in Fig. S1.

2. The encoded state of $|+\rangle = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$ is

$$|+\rangle_l = \frac{1}{\sqrt{2}}(|H\rangle_2|H\rangle_3|H\rangle_4|H\rangle_5 + |V\rangle_2|V\rangle_3|V\rangle_4|V\rangle_5).$$

The decomposition of $|+\rangle_l$ is:

$$|+\rangle_l\langle +| = \frac{1}{2}(|HHHH\rangle_{2345}\langle HHHH| + |VVVV\rangle_{2345}\langle VVVV|) + \frac{1}{8}\left(X_2X_3X_4X_5 + Y_2Y_3Y_4Y_5 - \left(\frac{X+Y}{\sqrt{2}}\right)^{\otimes 4} - \left(\frac{X-Y}{\sqrt{2}}\right)^{\otimes 4}\right). \quad [4]$$

Here, we need to take five measurement settings, namely $Z_2Z_3Z_4Z_5$, $X_2X_3X_4X_5$, $Y_2Y_3Y_4Y_5$, $((X+Y)/\sqrt{2})^{\otimes 4}$, and $((X-Y)/\sqrt{2})^{\otimes 4}$. $(|HHHH\rangle_{2345}\langle HHHH| + |VVVV\rangle_{2345}\langle VVVV|$}can be calculated as the population of the sum of $HHHH$ and $VVVV$ events. The spin operator $(X+Y)/\sqrt{2}((X-Y)/\sqrt{2})$ corresponds to a measurement in the polarization basis $(H \pm e^{i\pi/4}V)/\sqrt{2}((H \pm e^{-i\pi/4}V)/\sqrt{2})$. The experimental results for the state $|+\rangle_l$ is shown in Fig. S2.

3. The encoded state of $|R\rangle = \frac{1}{\sqrt{2}}(|H\rangle + i|V\rangle)$ is

$$|R\rangle_l = \frac{1}{2\sqrt{2}}[(|H\rangle_2|H\rangle_3 + |V\rangle_2|V\rangle_3)(|H\rangle_4|H\rangle_5 + |V\rangle_4|V\rangle_5) + i(|H\rangle_2|H\rangle_3 - |V\rangle_2|V\rangle_3)(|H\rangle_4|H\rangle_5 - |V\rangle_4|V\rangle_5)].$$

The decomposition of $|R\rangle$ is:

$$|R\rangle_l\langle R| = \frac{1}{4}[(|HHHH\rangle\langle HHHH| + |VVVV\rangle\langle VVVV| + |HHVV\rangle\langle HHVV| + |VVHH\rangle\langle VVHH|)] + \frac{1}{16}(X_2X_3X_4X_5 + Y_2Y_3Y_4Y_5 - X_2X_3Y_4Y_5 - Y_2Y_3X_4X_5) - \frac{1}{16}[(Z_2 + Z_3)(Y_4X_5 + X_4Y_5) + (Y_2X_3 + X_2Y_3)(Z_4 + Z_5)].$$

The experimental results for the state $|R\rangle_l$ is shown in Fig. S3.

4. The five-photon cluster state can be written as

$$|\Phi_5\rangle = \frac{1}{2}[|H\rangle_1(|HHHH\rangle_{2345} + |VVVV\rangle_{2345}) + |V\rangle_1(|HHVV\rangle_{2345} + |VVHH\rangle_{2345})].$$

The decomposition of $|\Phi_5\rangle$ is:

$$|\Phi_5\rangle\langle\Phi_5| = \frac{1}{4}(|HHHHH\rangle\langle HHHHH| + |HVVVV\rangle\langle HVVVV| + |VHHVV\rangle\langle VHHVV| + |VVVHH\rangle\langle VVVHH|) + \frac{1}{16}[I_1X_2X_3X_4X_5 + I_1Y_2Y_3Y_4Y_5$$

$$- |H\rangle_1\langle H|\left(\frac{X+Y}{\sqrt{2}}\right)^{\otimes 4} - |H\rangle_1\langle H|\left(\frac{X-Y}{\sqrt{2}}\right)^{\otimes 4}$$
$$- |V\rangle_1\langle V|\left(\frac{X+Y}{\sqrt{2}}\right)^{\otimes 2}_{23}\left(\frac{X-Y}{\sqrt{2}}\right)^{\otimes 2}_{45}$$
$$- |V\rangle_1\langle V|\left(\frac{X-Y}{\sqrt{2}}\right)^{\otimes 2}_{23}\left(\frac{X+Y}{\sqrt{2}}\right)^{\otimes 2}_{45} + (|HH\rangle_{23}\langle HH|$$
$$+ |VV\rangle_{23}\langle VV|)X_1X_4X_5 - (|HH\rangle_{23}\langle HH|$$
$$+ |VV\rangle_{23}\langle VV|)X_1Y_4Y_5 + X_1X_2X_3(|HH\rangle_{45}\langle HH|$$
$$+ |VV\rangle_{45}\langle VV|) - X_1Y_2Y_3(|HH\rangle_{45}\langle HH|$$
$$+ |VV\rangle_{45}\langle VV|) - (|HH\rangle_{23}\langle HH|$$
$$- |VV\rangle_{23}\langle VV|)Y_1X_4Y_5 - (|HH\rangle_{23}\langle HH|$$
$$- |VV\rangle_{23}\langle VV|)Y_1Y_4X_5 - Y_1X_2Y_3(|HH\rangle_{45}\langle HH|$$
$$- |VV\rangle_{45}\langle VV|) - Y_1Y_2X_3(|HH\rangle_{45}\langle HH|$$
$$- |VV\rangle_{45}\langle VV|).$$

The observables (M_1, M_2.,.M_{14}) listed in Fig. 5*B* are as follows:

$$M_1 = I_1X_2X_3X_4X_5,$$
$$M_2 = I_1Y_2Y_3Y_4Y_5,$$
$$M_3 = |H\rangle_1\langle H|\left(\frac{X+Y}{\sqrt{2}}\right)^{\otimes 4},$$
$$M_4 = |H\rangle_1\langle H|\left(\frac{X-Y}{\sqrt{2}}\right)^{\otimes 4},$$
$$M_5 = |V\rangle_1\langle V|\left(\frac{X+Y}{\sqrt{2}}\right)^{\otimes 2}_{23}\left(\frac{X-Y}{\sqrt{2}}\right)^{\otimes 2}_{45},$$
$$M_6 = |V\rangle_1\langle V|\left(\frac{X-Y}{\sqrt{2}}\right)^{\otimes 2}_{23}\left(\frac{X+Y}{\sqrt{2}}\right)^{\otimes 2}_{45},$$
$$M_7 = X_1X_2X_3(|HH\rangle_{45}\langle HH| + |VV\rangle_{45}\langle VV|),$$
$$M_8 = (|HH\rangle_{23}\langle HH| + |VV\rangle_{23}\langle VV|)X_1X_4X_5,$$
$$M_9 = (|HH\rangle_{23}\langle HH| + |VV\rangle_{23}\langle VV|)X_1Y_4Y_5,$$
$$M_{10} = X_1Y_2Y_3(|HH\rangle_{45}\langle HH| + |VV\rangle_{45}\langle VV|),$$
$$M_{11} = Y_1X_2Y_3(|HH\rangle_{45}\langle HH| - |VV\rangle_{45}\langle VV|),$$
$$M_{12} = Y_1Y_2X_3(|HH\rangle_{45}\langle HH| - |VV\rangle_{45}\langle VV|),$$
$$M_{13} = (|HH\rangle_{23}\langle HH| - |VV\rangle_{23}\langle VV|)Y_1Y_4X_5,$$
$$M_{14} = (|HH\rangle_{23}\langle HH| - |VV\rangle_{23}\langle VV|)Y_1X_4Y_5.$$

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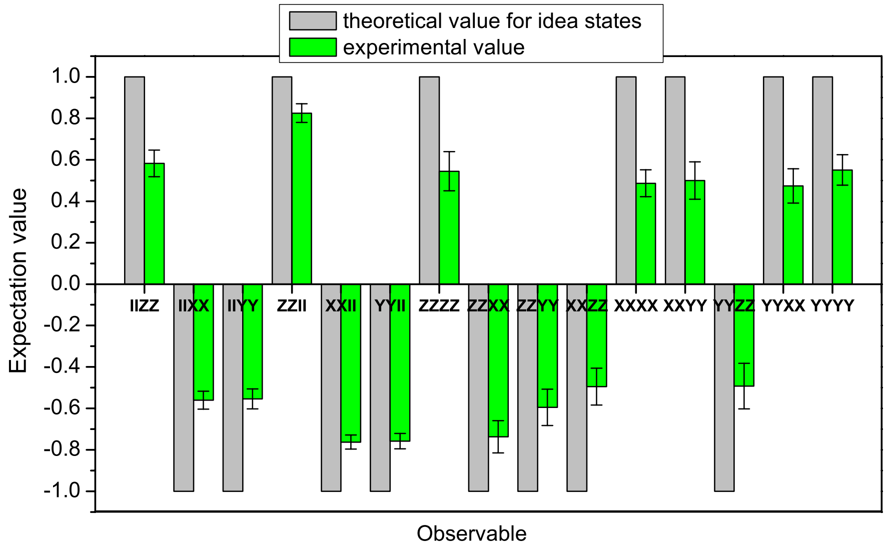

Fig. S1. Experimental result of the four-photon QEEC code $|V\rangle_I$. Expectation values of the 15 observables (*IIZZ*, *IIXX*,. . ., *YYYY*) are listed. Each of them is derived from a complete set of 16 fivefold coincidence events. The error bars denote one-standard deviation, deduced from propagated Poissonian counting statistics of the raw detection events.

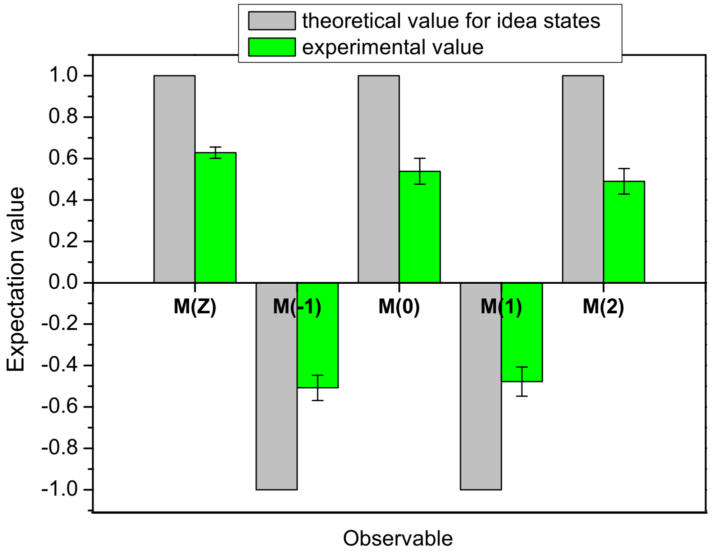

Fig. S2. Experimental result of the four-photon QEEC code $|+\rangle_l$. Here, $M(Z) = (|HHHH\rangle\langle HHHH| + |VVVV\rangle\langle VVVV|)$, $M(-1) = ((X - Y)/\sqrt{2})^{\otimes 4}$, $M(0) = X_2X_3X_4X_5$, $M(1) = ((X + Y)/\sqrt{2})^{\otimes 4}$, $M(-2) = Y_2Y_3Y_4Y_5$.

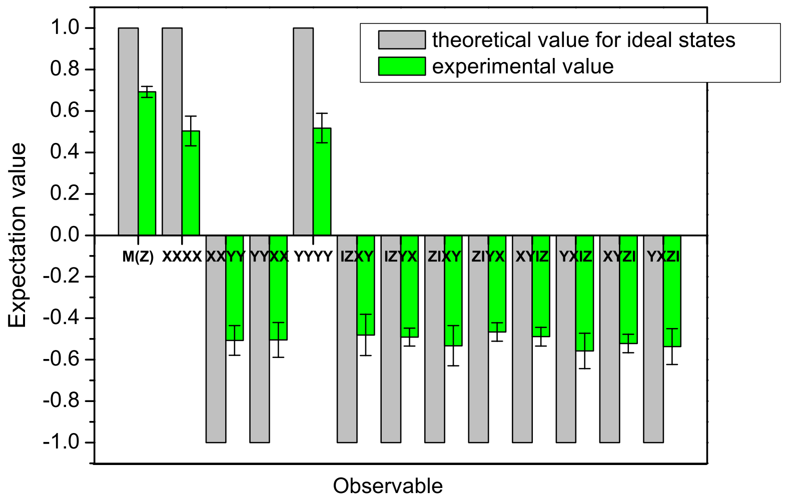

Fig. S3. Experimental result of the four-photon QEEC code $|R\rangle_I$. Here, $M(Z) = (|HHHH\rangle\langle HHHH| + |VVVV\rangle\langle VVVV| + |HHVV\rangle\langle HHVV| + |VVHH\rangle\langle VVHH|)$.