Appendix 1. Derivation of the formula (5)

We have

$$
\begin{aligned}
&\mathrm{Cov}\left(\frac{1}{\nu}U, \frac{1}{\nu}V\right) = \\
&\mathbb{E}\Big(\frac{1}{\nu}\sum_{s=1}^{\nu} X_s \frac{1}{\nu}\sum_{k=1}^{\nu} Y_k\Big) - \mathbb{E}\frac{1}{\nu}\sum_{s=1}^{\nu} X_s \mathbb{E}\frac{1}{\nu}\sum_{k=1}^{\nu} Y_k \\
&= \sum_{n=1}^{\infty}\frac{1}{n^2}\sum_{s=1}^{n}\sum_{k=1}^{n}\mathbb{E}\big(X_sY_k\big)\mathbb{P}\{\nu=n\} - \\
&\sum_{n=1}^{\infty}\frac{1}{n}\sum_{s=1}^{n}\mathbb{E}X_s\mathbb{P}\{\nu=n\} \qquad (1) \\
&\sum_{n=1}^{\infty}\frac{1}{n}\sum_{k=1}^{n}\mathbb{E}Y_k\mathbb{P}\{\nu=n\},
\end{aligned}
$$

where $\mathbb{P}$ and $\mathbb{E}$ are the symbols of probability and expectation, respectively. Consider the first term

$$
\begin{aligned}
&\sum_{n=1}^{\infty}\frac{1}{n^2}\sum_{s=1}^{n}\sum_{k=1}^{n}\mathbb{E}\big(X_sY_k\big)\mathbb{P}\{\nu=n\} = \\
&\sum_{n=1}^{\infty}\frac{1}{n^2}\sum_{s=1}^{n}\sum_{k=1}^{n}\mathbb{E}X_s\mathbb{E}Y_k\mathbb{P}\{\nu=n\} \\
&+\sum_{n=1}^{\infty}\frac{1}{n^2}\sum_{s=1}^{n}\mathrm{Cov}(X_s, Y_s)\mathbb{P}\{\nu=n\} = \\
&\mathbb{E}X\mathbb{E}Y + \mathrm{Cov}(X,Y)\sum_{n=1}^{\infty}\frac{1}{n}\mathbb{P}\{\nu=n\}.
\end{aligned}
$$

For the last to terms in (1), we have

$$
\begin{aligned}
&\sum_{n=1}^{\infty}\frac{1}{n}\sum_{s=1}^{n}\mathbb{E}X_s\mathbb{P}\{\nu=n\} = \mathbb{E}X, \\
&\sum_{n=1}^{\infty}\frac{1}{n}\sum_{k=1}^{n}\mathbb{E}Y_k\mathbb{P}\{\nu=n\} = \mathbb{E}Y.
\end{aligned}
$$

It follows from formula (7) that

$$\mathrm{Cov}\left(\frac{1}{\nu}U, \frac{1}{\nu}V\right)$$

$$= C \cdot \mathrm{Cov}(X, Y),$$

where

$$C = \sum_{n=1}^{\infty} \frac{1}{n} \mathbb{P}\{\nu = n\} = \mathbb{E}\left(\frac{1}{\nu}\right)$$

is an unknown constant. Considering the variances of U/ν and V/ν in a similar way, we arrive at the formulae (5)