Supplementary material for the paper "Population viscosity can promote the evolution of altruistic sterile helpers and eusociality": evaluation of the invasion conditions

Equilirbrium probabilities of identity by descent

Vector gathering all the probabilities of identity coefficients needed for the diploid genetic system:

$$\mathbf{Solve}\Big[\Big\{Q_{J,ff} == \frac{1}{4}\left(\frac{1}{n}\frac{(1+F)}{2}+\left(1-\frac{1}{n}\right)(1-m)^2 Q_{J,ff}\right)+\frac{1}{2}(1-m)^2 Q_{J,fm}+\frac{1}{4}\left(\frac{1}{n}\frac{(1+F)}{2}+\left(1-\frac{1}{n}\right)(1-m)^2 Q_{J,mm}\right),$$
$$Q_{J,fm} == \frac{1}{4}\left(\frac{1}{n}\frac{(1+F)}{2}+\left(1-\frac{1}{n}\right)(1-m)^2 Q_{J,ff}\right)+\frac{1}{2}(1-m)^2 Q_{J,fm}+\frac{1}{4}\left(\frac{1}{n}\frac{(1+F)}{2}+\left(1-\frac{1}{n}\right)(1-m)^2 Q_{J,mm}\right),\ Q_{J,mm} ==$$
$$\frac{1}{4}\left(\frac{1}{n}\frac{(1+F)}{2}+\left(1-\frac{1}{n}\right)(1-m)^2 Q_{J,ff}\right)+\frac{1}{2}(1-m)^2 Q_{J,fm}+\frac{1}{4}\left(\frac{1}{n}\frac{(1+F)}{2}+\left(1-\frac{1}{n}\right)(1-m)^2 Q_{J,mm}\right),$$
$$F == (1-m)^2 Q_{J,fm}\Big\},\ \{Q_{J,ff}, Q_{J,fm}, Q_{J,mm}, F\}\Big];$$

$$\mathbf{vecQDiplo = Flatten}\Big[\Big\{\Big\{Q_f \text{->} \frac{(1+F)}{2},\ Q_m \to \frac{(1+F)}{2},\ Q_{ff} \text{->} (1-m)^2 Q_{J,ff},\ Q_{fm} \text{->} (1-m)^2 Q_{J,fm},$$
$$Q_{mm} \text{->} (1-m)^2 Q_{J,mm},\ Q_{R,ff} \text{->} \frac{1}{n}\frac{(1+F)}{2}+\left(\frac{n-1}{n}\right)(1-m)^2 Q_{J,ff}\Big\}\ /.\ \%,\ \%\Big\}\Big]\ //\ \mathbf{FullSimplify}$$

$$\Big\{Q_f \to \frac{-1+(-2+m)\,m\,(-1+2n)}{-1+(-2+m)\,m\,(-1+4n)},\ Q_m \to \frac{-1+(-2+m)\,m\,(-1+2n)}{-1+(-2+m)\,m\,(-1+4n)},\ Q_{ff} \to -\frac{(-1+m)^2}{-1+(-2+m)\,m\,(-1+4n)},$$
$$Q_{fm} \to -\frac{(-1+m)^2}{-1+(-2+m)\,m\,(-1+4n)},\ Q_{mm} \to -\frac{(-1+m)^2}{-1+(-2+m)\,m\,(-1+4n)},\ Q_{R,ff} \to \frac{-1+(-2+m)\,m}{-1+(-2+m)\,m\,(-1+4n)},$$
$$Q_{J,ff} \to -\frac{1}{-1+(-2+m)\,m\,(-1+4n)},\ Q_{J,mm} \to -\frac{1}{-1+(-2+m)\,m\,(-1+4n)},$$
$$Q_{J,fm} \to -\frac{1}{-1+(-2+m)\,m\,(-1+4n)},\ F \to -\frac{(-1+m)^2}{-1+(-2+m)\,m\,(-1+4n)}\Big\}$$

$$\left\{Q_f \to \frac{-1+(-2+m)\,m\,(-1+2n)}{-1+(-2+m)\,m\,(-1+4n)},\ Q_m \to \frac{-1+(-2+m)\,m\,(-1+2n)}{-1+(-2+m)\,m\,(-1+4n)},\ Q_{ff} \to -\frac{(-1+m)^2}{-1+(-2+m)\,m\,(-1+4n)},\right.$$
$$Q_{fm} \to -\frac{(-1+m)^2}{-1+(-2+m)\,m\,(-1+4n)},\ Q_{mm} \to -\frac{(-1+m)^2}{-1+(-2+m)\,m\,(-1+4n)},\ Q_{R,ff} \to \frac{-1+(-2+m)\,m}{-1+(-2+m)\,m\,(-1+4n)},$$
$$Q_{J,ff} \to -\frac{1}{-1+(-2+m)\,m\,(-1+4n)},\ Q_{J,mm} \to -\frac{1}{-1+(-2+m)\,m\,(-1+4n)},$$
$$\left.Q_{J,fm} \to -\frac{1}{-1+(-2+m)\,m\,(-1+4n)},\ F \to -\frac{(-1+m)^2}{-1+(-2+m)\,m\,(-1+4n)}\right\}$$

$$\left\{Q_f \to \frac{-1+(-2+m)\,m\,(-1+2n)}{-1+(-2+m)\,m\,(-1+4n)},\ Q_m \to \frac{-1+(-2+m)\,m\,(-1+2n)}{-1+(-2+m)\,m\,(-1+4n)},\ Q_{ff} \to -\frac{(-1+m)^2}{-1+(-2+m)\,m\,(-1+4n)},\right.$$
$$Q_{fm} \to -\frac{(-1+m)^2}{-1+(-2+m)\,m\,(-1+4n)},\ Q_{mm} \to -\frac{(-1+m)^2}{-1+(-2+m)\,m\,(-1+4n)},\ Q_{R,ff} \to \frac{-1+(-2+m)\,m}{-1+(-2+m)\,m\,(-1+4n)},$$
$$Q_{J,ff} \to -\frac{1}{-1+(-2+m)\,m\,(-1+4n)},\ Q_{J,mm} \to -\frac{1}{-1+(-2+m)\,m\,(-1+4n)},$$
$$\left.Q_{J,fm} \to -\frac{1}{-1+(-2+m)\,m\,(-1+4n)},\ F \to -\frac{(-1+m)^2}{-1+(-2+m)\,m\,(-1+4n)}\right\}$$

Vector gathering all the probabilities of identity coefficients needed for the haplo-diploid genetic system:

$$\text{Solve}\Big[$$
$$\left\{Q_{J,ff} == \frac{1}{4}\left(\frac{1}{n}\,\frac{(1+F)}{2} + \left(1-\frac{1}{n}\right)(1-m)^2\,Q_{J,ff}\right) + \frac{1}{2}(1-m)^2\,Q_{J,fm} + \frac{1}{4}\left(\frac{1}{n} + \left(1-\frac{1}{n}\right)(1-m)^2\,Q_{J,mm}\right),\right.$$
$$Q_{J,fm} == \frac{1}{2}\left(\frac{1}{n}\,\frac{(1+F)}{2} + \left(1-\frac{1}{n}\right)(1-m)^2\,Q_{J,ff}\right) + \frac{1}{2}(1-m)^2\,Q_{J,fm},$$
$$Q_{J,mm} == \frac{1}{n}\,\frac{(1+F)}{2} + \left(1-\frac{1}{n}\right)(1-m)^2\,Q_{J,ff},$$
$$\left.F == (1-m)^2\,Q_{J,fm}\right\},\ \{Q_{J,ff},\ Q_{J,fm},\ Q_{J,mm},\ F\}\Big];$$

```
vecQHaploDiplo =
 Flatten[{{Qf -> (1 + F)/2, Qm → 1, Qff -> (1 - m)^2 QJ,ff, Qfm -> (1 - m)^2 QJ,fm, Qmm -> (1 - m)^2 QJ,mm,
     QR,ff -> 1/n (1 + F)/2 + ((n - 1)/n) (1 - m)^2 QJ,ff} /. %, %}] // FullSimplify
```

$$\left\{Q_f \to \frac{1}{2}\left(1 + \frac{(-1+m)^2}{1 + (-2+m)\, m\, ((-2+m)\, m\, (-1+n) - 3\, n)}\right),\ Q_m \to 1,\right.$$

$$Q_{ff} \to -\frac{(-1+m)^2\, (-2 + (-2+m)\, m)}{2\, (1 + (-2+m)\, m\, ((-2+m)\, m\, (-1+n) - 3\, n))},\ Q_{fm} \to \frac{(-1+m)^2}{1 + (-2+m)\, m\, ((-2+m)\, m\, (-1+n) - 3\, n)},$$

$$Q_{mm} \to -\frac{(-1+m)^2\, (-1 + (-2+m)\, m)}{1 + (-2+m)\, m\, ((-2+m)\, m\, (-1+n) - 3\, n)},\ Q_{R,ff} \to \frac{1 - (-2+m)\, m}{1 + (-2+m)\, m\, ((-2+m)\, m\, (-1+n) - 3\, n)},$$

$$Q_{J,mm} \to \frac{1 - (-2+m)\, m}{1 + (-2+m)\, m\, ((-2+m)\, m\, (-1+n) - 3\, n)},\ Q_{J,ff} \to \frac{2 - (-2+m)\, m}{2\, (1 + (-2+m)\, m\, ((-2+m)\, m\, (-1+n) - 3\, n))},$$

$$\left.Q_{J,fm} \to \frac{1}{1 + (-2+m)\, m\, ((-2+m)\, m\, (-1+n) - 3\, n)},\ F \to \frac{(-1+m)^2}{1 + (-2+m)\, m\, ((-2+m)\, m\, (-1+n) - 3\, n)}\right\}$$

Check of equation (4) of the main text (maternal control of the trait)

Inclusive fitness effect (S) obtained by summing up the components listed in Table 2 of the main text:

```
S := {-C, C (1 - m)^2, B (1 - (1 - m)^2)}.{νf (tff Qf + tfm Qfm) + νm (tmf Qf + tmm Qfm),
   νf (tff QR,ff + tfm Qfm) + νm (tmf QR,ff + tmm Qfm), νf (tff Qff + tfm Qfm) + νm (tmf Qff + tmm Qfm)}
```

Threshold cost to benefit ratio for the diploid genetic system:

```
S /. {νf -> 1/2, νm -> 1/2, tff -> 1/2, tfm -> 1/2, tmf -> 1/2, tmm -> 1/2} /. vecQDiplo;
```

```
C /. Extract[Solve[% == 0 /. B → 1, C], {1}];
```

```
Simplify[% == (1 - m)^2/n]
```

```
True
```

Threshold cost to benefit ratio for the haplo-diploid genetic system:

```
S /. {νf -> 2/3, νm -> 1/3, tff -> 1/2, tfm -> 1/2, tmf → 1, tmm → 0} /. vecQHaploDiplo;
```

```
C /. Extract[Solve[% == 0 /. B → 1, C], {1}];
```

```
Simplify[% == (1 - m)^2/n]
```

```
True
```

Check of equation (5) of the main text (offspring control of the trait)

Inclusive fitness effect (S) obtained by summing up the components listed in Table 3 of the main text:

```
S := {-C, C (1 - m)^2, B (1 - (1 - m)^2)}.
  {νf (tff Qf + tfm Qf) + νm (tmf Qm + tmm Qm), νf (tff QJ,ff + tfm QJ,ff) + νm (tmf QJ,mm + tmm QJ,mm),
   νf (tff (x Qff + (1 - x) Qfm) + tfm (x Qfm + (1 - x) Qmm)) +
    νm (tmf (x Qff + (1 - x) Qfm) + tmm (x Qfm + (1 - x) Qmm))}
```

Threshold cost to benefit ratio for the diploid genetic system:

```
S /. {νf -> 1/2, νm -> 1/2, tff -> 1/2, tfm -> 1/2, tmf -> 1/2, tmm -> 1/2} /. vecQDiplo;
```

```
C /. Extract[Solve[% == 0 /. B → 1, C], {1}];
```

```
Simplify[% == (1 - m)^2/(2 n)]
```

```
True
```

Threshold cost to benefit ratio for the haplo-diploid genetic system:

```
S /. {νf -> 2/3, νm -> 1/3, tff -> 1/2, tfm -> 1/2, tmf → 1, tmm → 0} /. vecQHaploDiplo;
```

```
C /. Extract[Solve[% == 0 /. B → 1, C], {1}];
```

```
Simplify[% == (1 - m)^2/(2 n)]
```

```
True
```