Supporting Information

Emission data: See supporting information in Fuglestvedt et al., 2008 (ref. 4); www.pnas.org/cgi/content/full/0702958104/DC1

The analytical climate model

This section describes a simple two-box analytical climate model similar to the model described by Schneider and Thompson (1981), which is used to calculate the long-term changes in global mean surface temperature due to emissions from the transport sector. By including a deep ocean, the analytical solution will include a long-term response of the climate system that is not represented in one-box models as used by Shine et al. (2005) for calculating global temperature change potentials (GTP). Using the approach described below, the derived time constants for the responses are consistent with the chosen values for the equilibrium climate sensitivity.

The global mean surface temperature response to radiative forcing perturbations following pulse emissions from the transport sector is derived for this two-box model. Equation R12 below gives the general expression for an exponentially decaying RF pulse (all components, except CO_2), while equation R19 gives the response to a CO_2 perturbation.

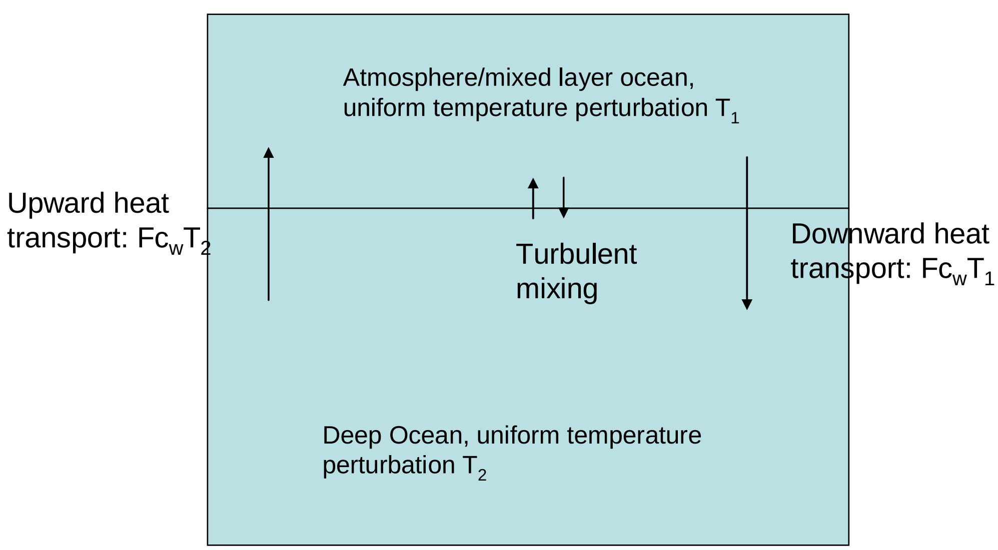

Figure 1: Schematic illustration of the two-box climate model

Energy balance equations for atmosphere/mixed layer ocean (subscript 1):

$$C_1 \frac{\partial T_1}{\partial t} = Q - \lambda^{-1} T_1 - F \cdot c_w \cdot (T_1 - T_2) - K_z \cdot \rho \cdot c_w \frac{(T_1 - T_2)}{\Delta z} \qquad \text{R1}$$

And for the deep ocean

$$C_2 \frac{\partial T_2}{\partial t} = F \cdot c_w \cdot (T_1 - T_2) + K_z \cdot \rho \cdot c_w \frac{(T_1 - T_2)}{\Delta z} \qquad \text{R2}$$

T_1 and T_2 denotes the temperature perturbations caused by a radiative forcing Q(t). The climate sensitivity of the coupled system is denoted by λ (K/Wm^{-2}). As the simpler atmosphere/mixed layer ocean the temperature (in the case of a constant Q) approach the equilibrium temperature change $T_E = Q\lambda$ as system approach equilibrium (T_1-T_2 approach zero).

With the following parameters

F: Advective mass flux of water from boundary layer to the deep ocean. It is assumed that F is constant in time, i.e. no feedback on the ocean circulation.
K_z : Diffusion coefficient for turbulent mixing of heat between the mixed layer of the ocean and the deep ocean.
Δz: Mixing depth for the turbulent mixing of heat
C_1: Heat capacity of mixed layer of the ocean
C_2: Heat capacity of the deep ocean
c_w: Specific heat of liquid water.
The choice of numerical values for these parameters is discussed at the end of this section.

To simplify the notation R1 is rewritten as

$$\frac{\partial T_1}{\partial t} = \frac{Q(t)}{C_1} - \tau^{-1} T_1 - \alpha_1 (T_1 - T_2) \qquad \text{R3}$$

And R2 as

$$\frac{\partial T_2}{\partial t} = \alpha_2 (T_1 - T_2) \qquad \text{R4}$$

Where:
$\alpha_1 = \frac{c_w}{C_1}\left(F + \frac{K_z \cdot \rho}{\Delta z} \right)$ and $\alpha_2 = \frac{c_w}{C_2}\left(F + \frac{K_z \cdot \rho}{\Delta z} \right)$

And $\tau = C_1 \cdot \lambda$

Differentiating R3 wrt. time t yields

$$\frac{\partial^2 T_1}{\partial t^2} = -\left(\frac{1}{\tau} + \alpha_1\right)\frac{\partial T_1}{\partial t} + \alpha_1 \frac{\partial T_2}{\partial t} + \frac{1}{C_1}\frac{dQ}{dt} \qquad \text{R5}$$

Using R2 to substitute in R5 gives

$$\frac{\partial^2 T_1}{\partial t^2} = -\left(\frac{1}{\tau} + \alpha_1\right)\frac{\partial T_1}{\partial t} + \alpha_1 \alpha_2 (T_1 - T_2) + \frac{1}{C_1}\frac{dQ}{dt} \qquad \text{R6}$$

We then use equation R3 to solve for T_2

$$T_2 = \frac{1}{\alpha_1}\left(\frac{\partial T_1}{\partial t} - \frac{Q(t)}{C_1} + T_1(\tau^{-1} + \alpha_1)\right) \qquad \text{R7}$$

And substitute for T_2 in R6

$$\frac{\partial^2 T_1}{\partial t^2} + \left(\frac{1}{\tau} + \alpha_1 + \alpha_2\right)\frac{\partial T_1}{\partial t} + \frac{\alpha_2}{C_1} T_1 = \frac{\alpha_2}{\tau} Q(t) + \frac{1}{C_1}\frac{dQ}{dt} \qquad \text{R8}$$

$$\Rightarrow \frac{\partial^2 T_1}{\partial t^2} + A\frac{\partial T_1}{\partial t} + BT_1 = \frac{1}{C_1}\left(\alpha_2 Q(t) + \frac{dQ}{dt}\right) \qquad \text{R9}$$

Where

$A = \frac{1}{\tau} + \alpha_1 + \alpha_2$ and $B = \frac{\alpha_2}{\tau}$

The system is now transformed from a coupled system of first order differential equation (DE) to one second order ordinary DE (R9). This can now be solved using standard methods (e.g. Sydsæter, 1990).

The corresponding homogeneous DE

$$\frac{\partial^2 T_1}{\partial t^2} + A\frac{\partial T_1}{\partial t} + BT_1 = 0$$

Has a general solution of the form (if $A^2/4-B>0$, which it is in this case)

$$T_1(t) = k_1 e^{r_1 t} + k_2 e^{r_2 t}$$

The parameters r_1 and r_2 that define the time constants for the response of the climate model are independent of the radiative forcing and given by

$$r_1 = -\frac{1}{2}A + \sqrt{\frac{1}{4}A^2 - B}$$
$$\tau_{c1} = -1/r_1$$

and

$$r_2 = -\frac{1}{2}A - \sqrt{\frac{1}{4}A^2 - B}$$
$$\tau_{c2} = -1/r_2$$

And the parameters k_1 and k_2 are determined by the initial conditions (see below).

Response for a pulse with exponentially decaying RF over time

We now derive T(t) with an exponentially decaying forcing $Q(t)=Q_0e^{-t/\tau}$ following a pulse emission of gas i, by inserting Q as above in R9.

$$\frac{\partial^2 T_1}{\partial t^2} + A\frac{\partial T_1}{\partial t} + BT_1 = Q_0 e^{-t/\tau_i}\left(\frac{\alpha_2}{C_1} - \frac{1}{C_1 \cdot \tau_i}\right) \quad \text{R10}$$

We are seeking a specific solution satisfying the non homogeneous DE in R10, u*(t) of the form

$$u^*(t) = ke^{-t/\tau_i}$$

The general solution is then

$$T_1(t) = k_1 e^{r_1 t} + k_2 e^{r_2 t} + u^*(t) \quad \text{R11}$$

Inserting u*(t) in R10 to determine the coefficient k gives

$$\frac{1}{\tau_i^2} k e^{-t/\tau_i} - \frac{A}{\tau_i} k e^{-t/\tau_i} + B k e^{-t/\tau_i} = Q_0 e^{-t/\tau_i} \left(\frac{\alpha_2}{C_1} - \frac{1}{C_1 \cdot \tau_i} \right)$$

$$\Rightarrow k \left(\frac{1}{\tau_i^2} - \frac{A}{\tau_i} + B \right) = Q_0 \left(\frac{\alpha_2}{C_1} - \frac{1}{C_1 \cdot \tau_i} \right)$$

$$\Rightarrow k = \frac{Q_0 \left(\frac{\alpha_2}{C_1} - \frac{1}{C_1 \cdot \tau_i} \right)}{\frac{1}{\tau_i^2} - \frac{A}{\tau_i} + B} = Q_0 \cdot k^{'}$$

The initial conditions are:

$$T_1(t=0) = 0$$

$$T_2(t=0) = 0$$

$$\frac{\partial T_1}{\partial t}(t=0) = \frac{Q_0}{C_1}$$

The coefficients k_1 and k_2 are derived from the initial conditions

$$\frac{\partial T_1}{\partial t}(t=0) = \frac{Q_0}{C_1}$$

$$\Rightarrow r_1 k_1 + r_2 k_2 - \frac{k}{\tau_i} = \frac{Q_0}{C_1}$$

And

$$T_1(t=0) = 0$$

$$\Rightarrow k_1 + k_2 + k^{'} Q_0 = 0$$

$$\Rightarrow k_1 = -k_2 - k^{'} Q_0$$

$$\Rightarrow k_2 = Q_0 \left(\frac{\frac{k^{'}}{\tau_i} + C_1^{-1} + r_1 k^{'}}{r_2 - r_1} \right) = Q_0 \cdot k_2^{'}$$

$$\Rightarrow k_1 = Q_0 \left(-k^{'} - \frac{\frac{k^{'}}{\tau_i} + C_1^{-1} + r_1 k^{'}}{r_2 - r_1} \right) = Q_0 \cdot k_1^{'}$$

$$T_1(t) = Q_0(k_1' e^{-t/\tau_{c1}} + k_2' e^{-t/\tau_{c2}} + k' e^{-t/\tau_i}) \qquad \text{R12}$$

The characteristic time constants for the climate response (τ_{c1} and τ_{c2}) are independent of the nature of the RF.

Response for CO_2 emission pulse

In the case of CO_2 the concentration response to a pulse emission is more complicated due to multiple processes with different timescales governing the removal of CO_2. Here we follow the method used by IPCC (2001) and represent this with the impulse response function R(t), derived from the Bern carbon cycle model (Joos et al., 1996) and used in IPCC (2001):

$$R(t) = a_0 + \sum_{i=1}^{4} a_i \exp\left(-\frac{t}{\tau_i}\right) \qquad \text{R13}$$

Thus the radiative forcing of the CO_2 pulse is

$$Q(t) = Q_0 \cdot R(t) \qquad \text{R14}$$

The DE for $T_i(t)$ (equation R9) is a linear DE so that if $T_{1,i}(t)$ is a solution for the RF $Q_i(t)$, and $T_{1,j}(t)$ is a solution for the RF $Q_j(t)$, then $T_1(t)= T_{1,i}(t) + T_{1,j}(t)$ is a solution to R9 with a RF of $Q(t) = Q_i(t) + Q_j(t)$. To find the temperature response for CO_2 we only need to find specific solutions corresponding to each of the a_i factors in R13, and then the response for CO_2 is the sum of the 5 specific solutions.

The first specific solution (for the constant a_0 term) is has a different form than for exponential decaying RF (R12). The response to a constant radiative forcing is derived below.

If the radiative forcing Q(t) is constant Q_o the general solution is

$$T_1(t) = k_1 e^{r_1 t} + k_2 e^{r_2 t} + \lambda Q_0 \qquad \text{R15}$$

The coefficients k_1 and k_2 are derived from the initial conditions

$$\frac{\partial T_1}{\partial t}(t=0) = \frac{Q_0}{c_1}$$
$$\Rightarrow r_1 k_1 + r_2 k_2 = \frac{Q_0}{c_1}$$
$$\Rightarrow k_2 = \lambda Q_0 \frac{\tau_1^{-1} + r_1}{r_2 - r_1} = \lambda Q_0 k_2'$$

And

$$T_1(t=0) = 0 \Rightarrow k_1 + k_2 + \lambda Q = 0$$
$$\Rightarrow k_1 = -k_2 - \lambda Q = \lambda Q(-k_2^{'} - 1) = \lambda Q k_1^{'}$$

R15 can then be written as

$$T_1(t) = \lambda Q_0 (1 + k_1^{'} e^{-t/\tau_{c1}} + k_2^{'} e^{-t/\tau_{c2}}) \qquad \text{R16}$$

$$T_{1,0}(t) = Q_0 (\lambda a_0 + \lambda a_0 k_{1,0}^{'} e^{-t/\tau_{c1}} + \lambda a_0 k_{2,0}^{'} e^{-t/\tau_{c2}}) \qquad \text{R17}$$

The specific solutions for the exponentially decaying terms (*a_1*-*a_4*) are given by equation R12:

$$T_{1,i}(t) = Q_0 a_i (k_{1,i}^{'} e^{-t/\tau_{c1}} + k_{2,i}^{'} e^{-t/\tau_{c2}} + k_i^{'} e^{-t/\tau_i}) \qquad \text{R18}$$

The overall temperature response for CO_2 is then

$$T_1(t) = T_{1,0}(t) + \sum_{i=1}^{4} T_{1,i}(t)$$

$$T_1(t) = Q_0 \left(\lambda a_0 + (\lambda a_0 k_{1,0}^{'} + \sum_{i=1}^{4} a_i k_{1,i}^{'}) \exp\left(-\frac{t}{\tau_{c1}}\right) + (\lambda a_0 k_{2,0}^{'} + \sum_{i=1}^{4} a_i k_{2,i}^{'}) \exp\left(-\frac{t}{\tau_{c2}}\right) + \sum_{i=1}^{4} a_i k_i^{'} \exp\left(-\frac{t}{\tau_i}\right) \right)$$
R19

Parameter values

λ: Climate sensitivity, i.e. temperature increase at after new equilibrium at 2xCO_2. A value of 0.9 K/Wm^{-2} is used.

F: Advective mass flux of water from boundary layer to the deep ocean, $F = 1.23 \cdot 10^{-4}$ kg m^{-2} s^{-1}. By conservation of mass F is equal in both directions. It is assumed that F is constant in time, i.e. no feedback on the ocean circulation.

K_z : Diffusion coefficient for turbulent mixing of heat between the mixed layer of the ocean and the deep ocean. $K_z = 4.4*10^{-5}$ m^{2}s^{-1}.
Δz: Mixing depth for the turbulent mixing of heat, Δz=1000 m.

C_w: Specific heat of liquid water: 4.2 10^3 J K^{-1} kg^{-1}

C_1: Heat capacity of mixed layer (70 m) of the ocean: 2.94 10^8 J K^{-1}m^{-2}
C_2: Heat capacity of the deep ocean (3000 m): 1.26 10^{10} J K^{-1}m^{-2}

Parameters in the CO_2 impulse response function (IPCC, 2001):
$a_0 = 0.1756$
$a_1 = 0.1375$
$a_2 = 0.1858$
$a_3 = 0.2423$
$a_4 = 0.2589$
τ_1= 421.093 yr
τ_2=70.5969 yr
τ_3= 21.4216 yr
τ_4= 3.4154 y

References

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